A wake-up call: information contagion and strategic uncertainty

Toni Ahnert† and Christoph Bertsch‡

March 2014 (First version: June 2012)

Abstract

A financial crisis in one region is a wake-up call for investors in other regions. If the correlation across regional fundamentals is potentially positive but uncertain ex-ante, investors acquire information about this correlation to determine their exposure. Financial contagion can occur in the absence of ex-post exposure, due to elevated strategic uncertainty among informed investors. This novel wake-up call theory of contagion explains how currency crises, bank runs, and debt crises spread across regions without a common investor base, ex-post correlated fundamentals or interconnectedness. Our wake-up call theory generates testable implications for laboratory experiments and new empirical predictions.

Keywords: contagion, information acquisition, wake-up call, mixture distribution.

JEL classifications: C7, D83, G01.

---

†Financial Studies Division, Bank of Canada, 234 Laurier Avenue W, Ottawa, ON K1A 0G9, Canada and Financial Markets Group, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, United Kingdom. E-mail: tahnert@bankofcanada.ca. The author conducted part of this research during his PhD at LSE and while visiting the Department of Economics at New York University and the Federal Reserve Board of Governors.

‡Research Division, Sveriges Riksbank, SE-103 37 Stockholm, Sweden. E-mail: christoph.bertsch@riksbank.se. The author conducted part of this research during a PostDoc at the Department of Economics, University College London, United Kingdom. An earlier version of this paper was part of his Ph.D. thesis at the European University Institute, Florence, Italy.
1 Introduction

The global financial crisis of 2007–09 illustrated that contagion poses an important systemic risk. Historically, there are numerous contagious crises episodes, such as the Asian financial crisis of 1997 and the more recent European sovereign debt crisis. Forbes (2012) distinguishes four different, but not mutually exclusive, channels of contagion: trade, banks and financial institutions, portfolio investors, and wake-up calls. Goldstein (1998) introduced the wake-up call hypothesis, whereby additional information about one region’s fundamental leads to the reappraisal of risk in another region. This hypothesis is supported by empirical evidence, such as Bekaert et al. (2014).

Despite the empirical evidence that a wake-up call plays an important role in transmitting crises, there has been surprisingly little theoretical work on this mechanism. Our paper attempts to close this gap by providing a theory of contagion based on the information acquisition after observing a crisis elsewhere, which is a wake-up call to investors. We define contagion as an increase in the probability of a financial crisis in one region due to a crisis in another region.

We develop a model of two regions with initial uncertainty about a potentially positive cross-regional correlation of fundamentals. In each region, investors play a standard global coordination game of regime change with incomplete information about the regional fundamental (Morris and Shin 2003). A crisis in the first region is observed by investors in the second region. This wake-up call induces investors to acquire costly information about the actual correlation. Intuitively, informed investors can tailor their strategy to the information obtained, while uninformed investors play an invariant strategy. Consequently, informed investors obtain a higher expected payoff by acting more aggressively upon information that suggests a regime change is more likely.

Our model generates two contagion results. First, in case of uncorrelated regional fundamentals, contagion can be more likely when investors are informed about the correlation than when uninformed (ex-post contagion). Second, contagion can be more likely after observing a crisis in the first region and learning about a zero correlation than after observing no crisis (ex-ante contagion). Bekaert et al. (2014) find that a wake-up call was the key driver of equity market contagion during the global financial crisis. They show that at the core of contagion was the reassessment of risks by investors, rather than financial or trade linkages. See also Didier et al. (2010). Analysing bond market contagion during the Asian crisis in 1997, Basu (2002) finds evidence for contagion based on the reassessment of risks in some South-East Asian countries. Van Rijckeghem and Weder (2003) view the Russian crisis as the outcome of a wake-up call in emerging markets. Studying the euro zone sovereign bond markets, Giordano et al. (2013) find empirical evidence for contagion based on the wake-up call of the Greek crisis.
For sufficiently low information costs, a strategic complementarity in information choices generates a unique equilibrium, in which all investors acquire information after a wake-up call.

Upon observing a crisis in the first region, learning that fundamentals are uncorrelated has two effects on the prior about the fundamental in the second region. First, the prior mean is higher. This mean effect reduces the probability of a crisis in the second region (Morris and Shin (2003)). Second, the prior variance is also higher, since observing a crisis in the first region is uninformative. Metz (2002) shows that the variance effect increases the probability of a crisis if the prior is strong. Intuitively, the variance effect increases the strategic uncertainty among informed investors, since they are more concerned about other informed investors attacking the regime. As a result, investors attack more aggressively and contagion becomes more likely. Our contribution is to analyze how the interaction of these two effects leads to contagion and to link this contagion result to the information choice of investors after a wake-up call.

The wake-up call theory of contagion applies to a range of phenomena. To illustrate the mechanism, we use a model of speculative currency crises (e.g. Morris and Shin (1998) and Corsetti et al. (2004)). The initial uncertainty about the correlation of fundamentals across regions reflects the unknown magnitude of trade or financial links and the unknown institutional similarity. Other applications include bank runs (Rochet and Vives (2004) and Goldstein and Pauzner (2005)) and sovereign debt crises (Corsetti et al. (2006)). In the first case, uninsured bank creditors observe a run elsewhere but are uncertain about interbank linkages. In the second case, sovereign debt holders observe a default elsewhere but are uncertain about the macroeconomic and financial links.

Our theoretical predictions are consistent with empirical findings. Eichengreen et al. (1996), for example, find evidence that a crisis elsewhere increases the likelihood “of a speculative attack by an economically and statistically significant amount” (p. 2). This observation is in line with the ex-ante contagion mechanism. Our results also help to rationalize several contagion phenomena, such as the unexpected spread of the Russian crisis to Brazil in 1998 (Bordo and Murshid (2000) and Forbes (2012)) and similar instances during the Asian crisis in 1997 (Radelet and Sachs (1998) and Corsetti et al. (1999)). Higher strategic uncertainty after a wake-up call is at the core of the ex-post contagion mechanism and is consistent with, for instance, the view of “an enhanced perception

\[\text{See also the earlier literature of e.g. Krugman (1979), Flood and Garber (1984), and Obstfeld (1986).}\]

Our wake-up call theory of contagion generates a novel set of empirical predictions described in section 6.1. The empirical literature on the contagion of banking and currency crises studies the channels of contagion and the characteristics that make the second region susceptible to contagion. Our theory suggests that the likelihood of contagion depends in a non-linear way on the characteristics of the first region. In particular, after controlling for the fundamentals of the second region, a crisis in the first region due to extremely low fundamentals is less likely to spread if fundamentals are uncorrelated. Conversely, a crisis in the first region due to moderately low fundamentals is more likely to spread if fundamentals are uncorrelated. Testing these predictions about the non-linear role of the first region’s fundamental promises to improve our understanding of contagion. Existing empirical evidence shows the importance of non-linearities.

Moreover, our theory of contagion has testable implications for laboratory experiments. Since the acquisition of information after a wake-up call can be observed, laboratory experiments are suitable for testing our model’s predictions regarding ex-post contagion and the information choice of investors. Building on the work of Heinemann et al. (2004, 2009), examining contagion within the global games framework in the laboratory is a promising yet little explored avenue for future research. Specifically, we derive three testable implications of our theory in section 6.2.


---


4For instance, extreme returns can have different implications for the transmission process across markets (Forbes and Rigobon (2002), Bekaert et al. (2014)). As for the propagation of financial shocks, Favero and Giavazzi (2002) contrast contagion with “flight-to-quality” episodes.
theory of contagion based on information acquisition after a wake-up call. We show that contagion can arise even if investors learn that fundamentals are unrelated across regions.\(^5\)

Finally, our paper makes a technical contribution to the global games literature. The ex-ante uncertainty about the correlation of fundamentals results in heterogeneous priors between informed and uninformed investors that follow a mixture distribution. Our first technical contribution is to analyze the information choice regarding a signal about the correlation of fundamentals in this setup.\(^6\) There is initial uncertainty about a homogeneous prior in Chen and Suen (2013) who study crisis waves based on learning about the prior (mean effect). By contrast, we analyze the information choice of investors after receiving a wake-up call, thereby allowing for heterogeneous priors. We also study the changes in strategic uncertainty among informed investors (variance effect) that underpins our theory of contagion.

Hellwig and Veldkamp (2009) show that information choices inherit the strategic complementarity or substitutability from the underlying beauty contest game. Our second technical contribution is to show that this “inheritance result” extends to a regime change game with ex-ante uncertainty about the correlation of fundamentals. By contrast, Szkup and Trevino (2012b) study a model with a common prior, continuous private information choice and a convex cost of acquiring information. However, strategic complementarity in information choices may not occur.\(^7\) In contrast to these papers, we study the acquisition of a publicly available signal about the cross-regional correlation of fundamentals and allow for heterogeneous priors. In our model, information about the correlation can either increase or decrease the precision of the prior about the fundamental in the second region, which is crucial for the ex-post contagion mechanism.

This paper is organized as follows. We describe a global coordination game of regime change with ex-ante uncertainty about the correlation of regional fundamentals in section\(^2\). Using mixture distributions, we solve the model in the case of exogenous information in section\(^3\). We establish a novel contagion mechanism in section\(^4\) from both an ex-ante and an ex-post perspective. Allowing

---

\(^5\) Contagion arises in Calvo and Mendoza (2000) since globalization shifts the incentives of investors from costly information acquisition to imitation and detrimental herding. By contrast, financial contagion arises in our paper because investors have an incentive to acquire information after a wake-up call.

\(^6\) Another global games paper that works with mixture distributions is Chen et al. (2012), who develop a theory of rumors in a political regime change. Apart from the different focus, they do not consider information acquisition. Ahnert and Kakhboi (2014) obtain strategic complementarity in information choices in a one-region global coordination game of regime change with a common prior, a discrete private information choice and heterogeneous information costs. They show that information acquisition amplifies the probability of financial crisis.
for endogenous information in section 5, we show that information choices exhibit strategic complementarity and that our contagion results occur in equilibrium. Linking our theoretical results to the empirical literature in section 6, we derive novel empirical predictions as well as testable implications for laboratory experiments. Section 7 concludes. We document an excellent case study of our wake-up call theory of contagion in Appendix A. Figures are in Appendix B while derivations and proofs are in Appendix C. Robustness checks and extensions are considered in Appendix D.

2 Model

We study a sequential game of speculative currency crises (Morris and Shin (1998)). There are two dates and two countries, both indexed by \( t \in \{1, 2\} \) because currency speculators in country \( t \) only move at date \( t \). Each country is inhabited by a unit continuum of risk-neutral speculators indexed by \( i \in [0, 1] \). A fundamental \( \theta_t \) measures the ability of country \( t \) to defend its currency.\(^{10}\)

**Actions and payoffs** At each date, currency speculators move simultaneously. Each speculator either attacks the currency \( (a_{it} = 1) \) or does not attack \( (a_{it} = 0) \). The outcome of the attack depends on both the aggregate attack size, \( A_t = \int_0^1 a_{it} \, di \), and the fundamental, \( \theta_t \). A currency crisis occurs if enough speculators attack relative to the fundamental, \( A_t > \theta_t \). In that case, the attack is successful from the perspective of speculators, since it leads to the abandonment of a currency’s peg. Following Vives (2005), an individual speculator’s benefit from participating in a successful currency attack is \( b > 0 \), while the loss of participating in an unsuccessful attack is \( l > 0 \):

\[
\begin{align*}
u(a_i = 1, A, \theta) & = b 1\{A > \theta\} - l 1\{A \leq \theta\}. 
\end{align*}
\]

The constant payoff from not attacking is normalized to zero, \( u(a_{it} = 0, A_t, \theta_t) = 0 \). This payoff structure is consistent with currency speculation by short-selling.\(^{11}\) The payoff differential from...
attacking increases in the aggregate attack size $A_t$ and decreases in the fundamental $\theta_t$. Therefore, the attack decisions of individual speculators exhibit global strategic complementarity.\footnote{Consequently, complete information about the fundamental leads to multiple equilibria that are sustained by self-fulfilling expectations for interim values of the fundamental $\theta_t \in (0, 1)$. By contrast, a unique equilibrium exists for extreme values of the fundamental in dominant strategies: speculators attack if the fundamental is low, $\theta_t \leq 0$, and do not attack if the fundamental is high, $\theta_t \geq 1$.}

**Information**  
The key feature of our model is the initial uncertainty about the correlation between fundamentals across countries $\rho \equiv \text{corr}(\theta_1, \theta_2)$. This correlation captures real, financial, or institutional links between countries.\footnote{Glick and Rose (1999) find empirical evidence for the role of geographic proximity and trade in the contagion of currency crises. In contrast, Van Rijckeghem and Weder (2001) and Van Rijckeghem and Weder (2003) find that spillovers through bank lending played a more important role for more recent episodes of currency crises. Finally, Dasgupta et al. (2011) find that institutional similarity to the “ground zero country”, from which the wave of crises emerged, is an important determinant for the direction of financial contagion.} The cross-country correlation of fundamentals is zero with probability $p \in (0, 1)$ or takes the positive value $\rho_H > 0$:\footnote{We consider the case of negative correlation of fundamentals for robustness in Appendix D.}

\[
\rho = \begin{cases} 
0 & \text{w.p. } p \\
\rho_H & \text{w.p. } 1 - p.
\end{cases}
\] (2)

Fundamentals in both countries follow a bivariate normal distribution with mean $\mu_t \equiv \mu$, precision $\alpha_t \equiv \alpha \in (0, \infty)$, and realized correlation $\rho$. As in the global games literature pioneered by Carlsson and van Damme (1993), each speculator receives a private signal, $x_{it}$, about the country’s fundamental before deciding whether to attack:

\[
x_{it} \equiv \theta_t + \epsilon_{it}
\] (3)

where the idiosyncratic noise $\epsilon_{it}$ is identically and independently normally distributed across speculators and countries with zero mean and precision $\gamma > 0$. The random variables for the cross-country correlation, the countries’ fundamentals, and the sequences of idiosyncratic noise are independent.

If a currency crisis occurs in country 1, the realization of the fundamental $\theta_1$ becomes common knowledge for speculators in country 2. This assumption is motivated by the public scrutiny of the monetary authority after a currency crisis in country 1. In contrast, the realized fundamental $\theta_1$
remains unobserved if no currency crisis occurs in country 1. To ensure that an informed currency speculator of country 2 cannot completely infer the fundamental $\theta_2$ after observing a currency crisis in country 1, we assume $\rho_H < 1$. The information structure is common knowledge.

The currency speculation game in country 2 has two stages. At stage 2, speculators decide simultaneously whether to attack the currency. This coordination stage is preceded by an information stage. We start by studying exogenous information, whereby a commonly known proportion of speculators, $n \in [0, 1]$, learns the correlation $\rho$. Next, we consider information acquisition, whereby currency speculators play an information acquisition game after observing a crisis in country 1. At stage 1, each speculator simultaneously decides whether to purchase a perfectly revealing and publicly available information about the correlation of the fundamentals at a cost $c > 0$. That is, every speculator can purchase the same signal but observes it privately. Apart from the private information about the second country’s fundamental, speculators are potentially heterogeneously informed about the cross-country correlation. The timeline in table 1 summarizes the model.

**Date 1:**
- The correlation of fundamentals $\rho$ is drawn.
- The fundamentals $\theta_t$ are drawn from a bivariate distribution with correlation $\rho$.
- Speculators receive private signals $x_{i1}$ and simultaneously decide whether to attack.
- Payoffs are realized. If a currency crisis occurs, $\theta_1$ becomes public knowledge.

**Date 2:**
- Information stage
  - Upon observing a crisis in country 1, speculators simultaneously decide whether to purchase a signal about $\rho$ at cost $c > 0$.
- Coordination stage
  - Speculators receive private signals $x_{i2}$ and simultaneously decide whether to attack.
  - Payoffs are realized.

**Table 1: Timeline**

---

15 Only the result of Proposition 3 on ex-ante contagion depends on this assumption (see section 4.2).

16 Corsetti et al. (2004) study the impact of a large, and potentially asymmetrically informed, currency speculator. By contrast, our speculators are of equal size and our theory of contagion does not require signalling or herding.

17 For instance, the purchase of an international newspaper that contains information about country 1, its institutional similarities with country 2, as well as information about other factors that influence the likelihood of a cross-country exposure. The international newspaper is costly as it takes money to buy and time to absorb. In terms of wholesale investors or currency speculators, costly information acquisition could be access to Bloomberg and Datastream terminals or the hiring of analysts to understand and interpret the publicly available information.
3 Equilibrium with exogenous information

This section analyzes the existence and uniqueness of an equilibrium in the currency attack coordination game in both countries. Upon briefly revising the standard equilibrium in country 1, we focus on the equilibrium in country 2 in case of exogenous information about the cross-country correlation of fundamentals. In particular, we study the consequences of exogenous public information about the correlation and establish the contagion effects in section 4. Thereafter, we analyze information acquisition in section 5 establishing the wake-up call theory of contagion.

Country 1

We start by briefly revising the equilibrium in country 1 that is well established in the literature (e.g., Vives (2005)). A Bayesian equilibrium in the currency attack coordination game in country 1 consists of an attack decision \( a(x_{1i}) \) for each speculator \( i \in [0, 1] \) and an aggregate attack size \( A_1 \) that satisfies the conditions for both individual optimality of attacking and aggregation:

\[
a(x_{1i}) \in \arg \max_{a_{1i} \in \{0, 1\}} E[u(a_{1i}, A_1, \theta_1)|x_{1i}] \quad \forall i
\]

\[
A_1 = \int_{-\infty}^{+\infty} a(x_{1i})\sqrt{\gamma} \phi(\sqrt{\gamma}(x_{1i} - \theta_1))dx_{1i} \equiv A(\theta_1)
\]

where \( \phi(x) \) is the probability distribution function of the standard Gaussian random variable.

As shown by Morris and Shin (2003), a unique Bayesian equilibrium exists if the private signal is sufficiently precise, \( \gamma > \gamma_0 \equiv \frac{\alpha^2}{\pi} \in (0, \infty) \). There are two equilibrium conditions. First, the proportion of attacking speculators equals the fundamental threshold, \( A_1^* = \theta_1^* \). Second, a speculator with the threshold signal \( x_{1i} = x_1^* \) is indifferent between attacking and not attacking. This yields one equation that implicitly defines the fundamental threshold \( \theta_1^* \) (see Appendix C.1):

\[
F_1(\theta_1^*) = \Phi\left(\frac{\alpha}{\sqrt{\alpha + \gamma}}(\theta_1^* - \mu) - \sqrt{\frac{\gamma}{\alpha + \gamma}}\Phi^{-1}(\theta_1^*)\right) = \frac{1}{1 + b/l}.
\]

Lemma 1 [Morris and Shin (2003)] If private information is sufficiently precise, \( \gamma > \gamma_0 \), then a unique Bayesian equilibrium exists in country 1. This equilibrium is in threshold strategies, whereby a speculator attacks if and only if \( x_{1i} < x_1^* \) and a currency attack occurs if and only if \( \theta_1 < \theta_1^* \), where threshold of the fundamental \( \theta_1^* \) is implicitly defined by equation (6).
In Appendix C.1 the threshold of the private signal $x^*_1$ is defined by equation (36).

**Corollary 1** If the prior about the fundamental is strong compared to the relative benefit from attacking, a currency crisis in country 1 only occurs for realized fundamentals $\theta_1$ below the prior $\mu$:

$$\mu > \Phi \left( \frac{\sqrt{\alpha + \gamma}}{\gamma} \Phi^{-1} \left( \frac{b/l}{b/l + 1} \right) \right) \Rightarrow \theta_1 \leq \theta^*_1 < \mu.$$  \hspace{1cm} (7)

**Country 2** Suppose there is a currency crisis in country 1, $\theta_1 < \theta^*_1$, such that speculators in country 2 observe the realized fundamental that satisfies $\theta_1 < \mu$ by Corollary 1. A known fraction of speculators learns the realized cross-country correlation at the beginning of date 2. Let $d_i \in \{I, U\}$ denote whether speculator $i$ is informed (I) or uninformed (U) about the cross-country correlation. To demonstrate our contagion results in section 4, it suffices to restrict attention to symmetric information about the cross-country correlation. Hence, currency speculators are either fully informed about the cross-country correlation, $n = 1$, or uninformed, $n = 0$.

To distinguish between informed and uninformed speculators in country 2, let $a_{iI} \equiv a_{i2}(d_i = I)$ and $a_{iU} \equiv a_{i2}(d_i = U)$ denote the individual attacking decision, respectively. Likewise, $A_{2I}$ and $A_{2U}$ are the proportions of informed and uninformed speculators who attack. Furthermore, let the equilibrium thresholds be denoted as $\theta^*_2 = \theta^*_2(\rho)$ if informed and $\theta^*_2$ if uninformed, where we highlight that the cross-country correlation $\rho$ is known to informed speculators.

**Bayesian updating of prior about the fundamental** Upon observing the currency crisis in country 1, both informed and uninformed speculators update their prior about the fundamental in country 2. First, the update of an informed speculator is affected by both the observed fundamental $\theta_1$ and the known exposure to the crisis country $\rho$. The conditional mean is $\mu_2(\rho, \theta_1) \equiv \mu_2(\rho, \theta_1) = \rho \theta_1 + (1 - \rho) \mu$, while the conditional variance is $\alpha_2(\rho) \equiv \alpha_2(\rho) = \frac{\alpha}{1 - \rho^2}$, and normality is preserved. An informed speculator forms the following updated prior about the fundamental in country 2:

$$\theta_2 | \rho = 0 \sim N \left( \mu, \frac{1}{\alpha} \right)$$  \hspace{1cm} (8)

$$\theta_2 | \rho = \rho_H, \theta_1 \sim N \left( \rho_H \theta_1 + (1 - \rho_H) \mu, \frac{1 - \rho_H^2}{\alpha} \right).$$  \hspace{1cm} (9)

\footnote{We also show the comparative statics of the fundamental threshold $\theta^*_1$ and the dependence of the ranking of equilibrium thresholds on the strength of the prior about the fundamental in Appendix C.1.}
Relative to the prior, a positive cross-country correlation lowers both the mean and the variance of the updated prior. These reductions are the more pronounced, the larger the correlation, thereby pushing the updated prior towards $\theta_1$ and away from $\mu$, as shown in Figure 1 in Appendix B.1.

Second, an uninformed speculator forms an updated prior about $\theta_2$ by using the ex-ante distribution of the cross-country correlation only. As a result, the updated prior is a mixture distribution: an uninformed speculator believes that $\theta_2$ is drawn from the distribution described in (8) with probability $p$ and from the distribution described in (9) with probability $1 - p$.

### 3.1 Informed speculators

Observing a crisis in country 1 is no news if speculators learn that there is zero cross-country correlation. The analysis of country 1 applies directly with $\theta_{2I}(\rho = 0, \theta_1) = \theta_1^*$. In contrast, if all speculators learn that there is positive correlation, $\rho = \rho_H$, an adaption is required to obtain a corollary of Lemma 1. The modified threshold for the private signal precision is $\gamma_1 \equiv \frac{\alpha_2^2}{2\pi(1-\rho_H^2)} \in (\gamma_0, \infty)$. Furthermore, the unique threshold fundamental $\theta_{2I}^* = \theta_{2I}^*(\rho, \theta_1)$ is implicitly defined by:

$$F_2(\theta_{2I}^*, \rho) \equiv \Phi\left(\frac{\alpha_2(\rho) [\theta_{2I}^* - \mu_2(\rho, \theta_1)]}{\alpha_2(\rho) + \gamma} - \sqrt{\frac{\gamma}{\alpha_2(\rho) + \gamma}} \Phi^{-1}(\theta_{2I}^*)\right) = \frac{1}{1 + b/l}$$

for any realization of the correlation $\rho \in \{0, \rho_H\}$ and any observed fundamental $\theta_1 < \theta_1^*$.\(^{19}\)

**Corollary 2** Suppose there is a crisis in country 1, $\theta_1 < \theta_1^*$, and speculators are informed about the cross-country correlation, $n = 1$. If private information is sufficiently precise, $\gamma > \gamma_4$, then there exists a unique Bayesian equilibrium in country 2. This equilibrium is in threshold strategies. A crisis occurs if the realized fundamental is below the threshold $\theta_{2I}^*(\rho, \theta_1)$ defined by equation (10).

We characterize the strength of the prior about the fundamental in Definition 1. A strong prior about the fundamentals in country 2 (Definition 1) implies a strong prior in country 1 (Corollary 1). As shown in Appendix C.2, a weak prior makes a currency crisis more likely,\(^{20}\)

\(^{19}\)Figure 1 depicts the updated prior distributions about the fundamental in country 2. The updated prior distribution of informed speculators, who learn about a zero correlation, has the highest mean and variance. In contrast, learning about positive correlation leads to an updated prior distribution with the lowest mean and variance. Whereas the mixture distribution can be a unimodal distribution similar to a normal distribution with fat tails (illustrated in the first panel), it may also be bimodal for a sufficiently small $\theta_1$ (illustrated in the second panel).

\(^{20}\)We provide a detailed discussion of comparative statics in Appendix C.2.
\[ \mu_2(\rho, \theta_1) < \theta^*_2I(\rho, \theta_1) < 1, \]  
while a strong prior makes a crisis less likely, \[ 0 < \theta^*_2I(\rho, \theta_1) < \mu_2(\rho, \theta_1). \] 

**Definition 1** The prior about the fundamental is strong if \( \mu \in S_1 \) and it is weak if \( \mu \in S_2 \). This holds independently of the realization of \( \rho \in \{0, \rho_H\} \). In contrast, whether the prior is strong or weak depends on the realization of \( \rho \) if \( \mu \notin \{S_1, S_2\} \), where:

\[
S_1 = \left\{ \mu, \theta_1, \alpha, \gamma, \rho_H, b, l : \mu \in S_1 \right\} \\
S_2 = \left\{ \mu, \theta_1, \alpha, \gamma, \rho_H, b, l : \mu \in S_2 \right\}
\]

\[
X(\rho) = \Phi\left( -\frac{\sqrt{\alpha_2(\rho) + \gamma}}{\sqrt{\gamma}} \Phi^{-1}\left( \frac{1}{1 + b/l} \right) \right), \quad Y(\rho) = \frac{1}{2} - \frac{\sqrt{\alpha_2(\rho) + \gamma}}{\alpha_2(\rho)} \Phi^{-1}\left( \frac{1}{1 + b/l} \right).
\]

If speculators learn that the correlation is positive, \( \rho = \rho_H \), then both the mean and the variance of the updated prior about \( \theta_2 \) are lower after a currency crisis in country 1. Therefore, the relative size of these mean and variance effects determines whether the equilibrium threshold increases or decreases relative to the case of zero cross-country correlation, \( \theta^*_2I(\rho_H, \theta_1) \leq \theta^*_2I(0, \theta_1) \).

Since this ranking of thresholds is crucial for the subsequent analysis, we establish conditions sufficient for \( \theta^*_2I(0, \theta_1) > \theta^*_2I(\rho_H, \theta_1) \). Lemma 2 states that an ordering of thresholds requires two conditions: (i) a strong prior about the fundamental in country 2; and (ii) an intermediate level of the fundamental in country 1. The existence of a unique equilibrium threshold is again ensured by sufficiently precise private information, \( \gamma > \gamma_1 \).

**Lemma 2 Fundamental threshold ranking.** Suppose private information is sufficiently precise, \( \gamma > \gamma_1 \), and investors are informed, \( n = 1 \). Then, the threshold ranking \( \theta^*_2I(0, \theta_1) > \theta^*_2I(\rho_H, \theta_1) \) is ensured by a strong prior about the fundamental in country 2 and an intermediate level of the fundamental in country 1. If the relative benefit from attacking is large, i.e. \( b > l \), then bounds involving \( X(\rho) \) are sufficient. However, a low relative benefit from attacking, i.e. \( b \leq l \), requires the strengthening of these bounds involving \( Y(\rho) \). Mirroring the definition of the prior, these results hold independently of the realized cross-country correlation. Furthermore, a weak prior is associated with an incidence of attacks above 50% and \( \theta^*_2I(\rho_H, \theta_1) > \frac{1}{2} \), while a strong prior is associated with an incidence of attacks below 50% and \( \theta^*_2I(\rho, \theta_1) > \frac{1}{2} \).

However, no direct conclusion for the overall probability of a currency crisis in country 2 conditional on \( \theta_1 \) can be drawn since the conditional distribution of \( \theta_2 \) varies across these cases. In particular, the distribution of \( \theta_2|\rho = \rho_H, \theta_1 \) places greater weight on lower realizations than the distribution of \( \theta_2|\rho = 0, \theta_1 \).

---

\[21\] If the relative benefit from attacking is large, i.e. \( b > l \), then bounds involving \( X(\rho) \) are sufficient. However, a low relative benefit from attacking, i.e. \( b \leq l \), requires the strengthening of these bounds involving \( Y(\rho) \). Mirroring the definition of the prior, these results hold independently of the realized cross-country correlation. Furthermore, a weak prior is associated with an incidence of attacks above 50% and \( \theta^*_2I(\rho_H, \theta_1) > \frac{1}{2} \), while a strong prior is associated with an incidence of attacks below 50% and \( \theta^*_2I(\rho, \theta_1) > \frac{1}{2} \).

\[22\] However, no direct conclusion for the overall probability of a currency crisis in country 2 conditional on \( \theta_1 \) can be drawn since the conditional distribution of \( \theta_2 \) varies across these cases. In particular, the distribution of \( \theta_2|\rho = \rho_H, \theta_1 \) places greater weight on lower realizations than the distribution of \( \theta_2|\rho = 0, \theta_1 \).
fundamental in country 1, $\theta_1 \in [\theta_1, \theta_1^*]$, where the lower bound is defined as follows:

$$
\hat{\theta}_1 \equiv \mu + \left( \frac{(\theta_1^* - \mu)[1 - \frac{\alpha}{\rho H} \sqrt{\frac{\sigma_2(\rho H)^{1+\gamma}}{\alpha + \gamma}}]}{\rho H} \right) + \sqrt{\gamma} \Phi^{-1}(\theta_1^*) \left( \frac{\alpha_2(\rho H)^{1+\gamma}}{\alpha_2(\rho H)^{1+\gamma} - 1} \right) < \mu,
$$

(12)

with $\theta_1^*$ as defined in equation (6). A necessary condition for $\hat{\theta}_1 < \theta_1^*$ is:

$$
\mu < \hat{\theta}_1 - \frac{\sqrt{\gamma}}{\alpha} \Phi^{-1}(\hat{\theta}_1) - \sqrt{\frac{\alpha + \gamma}{\gamma}} \Phi^{-1}\left( \frac{1}{1 + b/l} \right).
$$

(13)

If $b \geq 1$, then condition (13) is also sufficient. However, if $b < 1$, then condition (13) and:

$$
\Phi\left( - \frac{\alpha + \gamma}{\gamma} \Phi^{-1}\left( \frac{1}{1 + b/l} \right) \right) < \hat{\theta}_1 - \frac{\sqrt{\gamma}}{\alpha} \Phi^{-1}(\hat{\theta}_1) - \sqrt{\frac{\alpha + \gamma}{\gamma}} \Phi^{-1}\left( \frac{1}{1 + b/l} \right)
$$

(14)

are necessary and sufficient for $\hat{\theta}_1 < \theta_1^*$.

**Proof** See Appendix C.3.

Under the sufficient conditions of Lemma 2 there is a positive mass of fundamnetals, $\theta_1 \in [\theta_1, \theta_1^*]$, that is conducive to both a currency crisis in country 1 and the threshold ranking $\theta_1^*(0, \theta_1) > \theta_1^*(\rho H, \theta_1)$ in country 2. To avoid that the equilibrium threshold $\theta_1^*$ is too small, i.e. $\theta_1^* > \hat{\theta}_1$, the prior about country 1’s fundamental must not be too strong, as ensured by equation (13), and the relative benefit from attacking $b/l$ must not be too small, as ensured by equation (14).

At the core of Lemma 2 is the change in the variance of the updated prior about the fundamental in country 2 as the realized cross-country correlation changes. We discuss this variance effect in detail in Appendix C.2.2. Under the circumstances described in Lemma 2 a decrease in the relative precision of public signals due to a lower realization of $\rho$ increases strategic uncertainty and therefore induces speculators to attack more aggressively, $\theta_1^*(0, \theta_1) > \theta_1^*(\rho H, \theta_1)$. This ranking of equilibrium thresholds is crucial for the contagion effects established in section 4, where we also provide a detailed intuition for the implications of changes in strategic uncertainty.

---

23The threshold ranking reverses for low $\theta_1$, $\theta_1^*(0, \theta_1) < \theta_1^*(\rho H, \theta_1)$ for all $\theta_1 < \hat{\theta}_1$. See also the proof of Lemma 4.
3.2 Uninformed speculators

Consider now the symmetric case of uninformed speculators, \( n = 0 \). We start by analysing the updating of an uninformed speculator in country 2 after observing a crisis in country 1 and state conditions for the existence of a unique monotone equilibrium in Proposition [1].

**Bayesian updating** Speculators in country 2 have observed a currency crisis in country 1, \( \theta_1 < \theta_1^* \), but are uninformed about the realized cross-country correlation of fundamentals \( \rho \). Using Bayes’ rule, uninformed speculators use their private signal \( x_{i2} \) to form a posterior about the distribution of the cross-country correlation. Let \( \hat{p} \equiv \Pr\{ \rho = 0 | \theta_1, x_{i2} \} \) denote the probability of zero correlation of fundamentals. Its posterior distribution and comparative statics are derived in Appendix C.4.1.

**Equilibrium analysis** There are again two equilibrium conditions. First, the critical mass condition states that the proportion of uninformed speculators who are attacking equals the fundamental threshold, \( A_{2U} = \theta_{2U}^* \), where the equilibrium threshold in case of uninformed speculators only depends on the observed fundamental in country 1 after a crisis, \( \theta_{2U}^* = \theta_{2U}^*(\theta_1) \):

\[
x_{2U}^* = \theta_{2U}^* + \sqrt{\frac{T}{\gamma}} \Phi^{-1}(\theta_{2U}^*).
\] (15)

Second, an uninformed speculator with the threshold signal \( x_{2i} = x_{2U}^* \) is indifferent between attacking and not attacking the currency. As shown in Appendix C.4.2, this yields one equation that implicitly defines the fundamental threshold \( \theta_{2U}^* \), where the posterior belief about the no-correlation probability is evaluated at the threshold signal, such that \( \hat{p}(\theta_{2U}^*) \equiv \Pr\{ \rho = 0 | \theta_1, x_{2i} = x_{2U}^*(\theta_{2U}^*) \} \):

\[
G(\theta_{2U}^*, \theta_1) = \hat{p}(\theta_{2U}^*) F_2(\theta_{2U}^*(\theta_1), 0) + (1 - \hat{p}(\theta_{2U}^*)) F_2(\theta_{2U}^*(\theta_1), \rho_H) = \frac{1}{1 + b/l}.
\] (16)

Therefore, \( G(\theta_{2U}^*, \theta_1) \) is a mixture of \( F_2(\theta_{2U}^*(0, \theta_1), 0) \) and \( F_2(\theta_{2U}^*(\rho_H, \theta_1), \rho_H) \) with \( \hat{p}(\theta_{2U}^*) \) as weight. Since the speculators are uninformed about the cross-country correlation, the expression is evaluated at the same fundamental threshold \( \theta_{2U}^* \) throughout.

The case of uninformed speculators is more complicated due to the mixture distribution. Using the results of [Milgrom (1981)] and [Vives (2005)], we can show that the best-response function...
of an individual speculator is strictly increasing in $\theta_{2U}^*$ (see Appendix C.4.3). Therefore, there exists a unique equilibrium in threshold strategies if private information is sufficiently precise. The main insight of the uniqueness proof is that $G(\theta_{2U}^*, \theta_1)$ monotonically decreases in $\theta_{2U}^*$ if $\gamma$ exceeds a finite threshold $\gamma_2 \in (0, \infty)$, despite its dependence on $\hat{p}(\theta_{2U}^*)$.

**Proposition 1 Unique monotone Bayesian equilibrium.** Suppose there is a crisis in country 1, $\theta_1 < \theta_1^*$, and speculators are uninformed about the cross-country correlation, $n = 0$. If private information is sufficiently precise, $\gamma > \gamma_2$, then there exists a unique monotone equilibrium in country 2. Each uninformed speculator attacks if and only if the private signal is smaller than the threshold $x_{2U}^*$. A currency crisis occurs if and only if the realized fundamental in country 2 is smaller than the threshold $\theta_{2U}^*(\theta_1)$ defined by equation (16), which is a weighted average of the thresholds that prevail if speculators were informed:

$$\min\{\theta_{2I}^*(0, \theta_1), \theta_{2I}^*(\rho_H, \theta_1)\} < \theta_{2U}^*(\theta_1) < \max\{\theta_{2I}^*(0, \theta_1), \theta_{2I}^*(\rho_H, \theta_1)\}.$$  

(17)

**Proof** See Appendix C.5.

This concludes the equilibrium analysis when speculators are endowed with symmetric information about the cross-country correlation. The equilibrium behavior is summarized in Corollary 2 (if informed) and Proposition 1 (if uninformed). These results, together with the threshold ranking in Lemma 2, allow us to establish two contagion results in case of exogenous information below.

### 4 Contagion

Contagion is defined as an increase in the probability of a crisis in country 2 after a crisis in country 1. We show that contagion occurs even if speculators learn, prior to their currency attack decision, that the fundamentals of both countries are unrelated. Therefore, a crisis can spread contagiously across countries even if speculators in country 2 are completely insulated from the crisis in country 1. In this section we establish two results on contagion.

The first result is that contagion can occur ex-post, that is for a realized cross-country correlation of zero, $\rho = 0$. In section 4.1 we show that contagion is more likely when speculators are
informed about the correlation than when they are uninformed. Despite learning ‘good news’ about country 2’s mean of the fundamental, namely no exposure to the crisis country, speculators attack the currency in country 2 more aggressively than without news about the cross-country correlation. Since this effect holds for a given realization \( \rho = 0 \), we call it \textit{ex-post contagion} (Proposition \textsuperscript{2}).

The second result is that contagion can also occur ex-ante. Specifically, we show in section \textsuperscript{4.2} that the probability of a crisis in country 2 can be higher after observing a crisis in country 1 than after observing no crisis. This result holds although speculators learn that there is zero exposure to the crisis country. In the no-crisis case, we allow for both correlated and uncorrelated fundamentals by integrating over these realizations, so we call it \textit{ex-ante contagion} (Proposition \textsuperscript{3}).

Our novel contagion results contrast sharply with the previously established information contagion channel.\textsuperscript{24} We analyze the implications of ex-ante uncertainty about the correlation of fundamentals. In the context of our model, this information contagion channel arises after a crisis in country 1 if fundamentals are correlated ex-post. In contrast, our contagion results arise even if speculators learn that there is zero correlation ex-post, so exposure to the crisis country is absent.

\subsection*{4.1 Ex-post contagion}

If there is a crisis in country 1, the ability of its monetary authority to defend the currency must be low, \( \theta_1 < \theta_1^* < \mu \), where the second inequality follows from Corollary \textsuperscript{1}. Since the cross-country correlation of fundamentals is potentially positive, country 2’s fundamental may also be low. Hence, learning that the fundamentals are uncorrelated may induce speculators to attack the currency of country 2 less aggressively. We show that this conjecture is incorrect: contagion can occur even if fundamentals are uncorrelated ex-post. If fundamentals are uncorrelated, \( \rho = 0 \), the likelihood of a currency crisis in case of informed speculators \((n = 1)\) can be higher than in case of uninformed speculators \((n = 0)\). Proposition \textsuperscript{2} states conditions for the presence of ex-post contagion.

\textbf{Proposition 2 Ex-post contagion.} Suppose there is a crisis in country 1, \( \theta_1 < \theta_1^* \), and fundamentals are uncorrelated, \( \rho = 0 \). If the fundamental in country 2 is strong, the realized fundamental

\textsuperscript{24}Acharya and Yorulmazer (2008) show that the funding cost of one bank increases after bad news about another bank if the banks’ loan portfolio returns have a common factor. To avoid information contagion ex-post, banks herd their investment ex-ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures. Adverse news about aggregate solvency of the banking system leads to runs on multiple banks.
in country 1 satisfies $\theta_1 > \bar{\theta}_1$, and $\gamma > \max\{\gamma_1, \gamma_2\}$, then a crisis in country 2 is more likely when speculators are informed about the cross-country correlation than when they are uninformed:

$$\Pr (\theta_2 < \theta^*_2(\rho, \theta_1) | \rho = 0, \theta_1) > \Pr (\theta_2 < \theta^*_{2U}(\theta_1) | \rho = 0, \theta_1), \forall \theta_1 \in (\bar{\theta}_1, \theta^*_1). \quad (18)$$

**Proof** See Appendix C.6.

The key to understanding Proposition 2 is Figure 1 in Appendix B.1. If the correlation is potentially positive, then a crisis in country 1 reduces both the mean and the variance of the updated prior about the fundamental in country 2. The overall effect depends on the relative size of the mean and variance effects, since these move in opposite directions. Thus, a strong variance effect, as described in Lemma 2, is at the heart of the ordering of equilibrium thresholds. It leads to a large degree of strategic uncertainty among speculators, giving rise to the ex-post contagion result. Below, we provide additional details of, and intuition for, the mean and variance effects.

**Mean effect** It is well known that adverse public information, a lower prior $\mu_2(\rho, \theta_1)$, raises the equilibrium threshold (Morris and Shin (2003), Vives (2005), and Manz (2010)). As such, a higher correlation causes a decrease in the updated prior of the fundamental in country 2, conditional on observing $\theta_1 < \mu$ as insured by the strong prior (see Corollary 1 and Definition 1). Consequently, observing zero correlation increases the mean of the updated prior and lowers $\theta^*_2I(0)$ relative to $\theta^*_2I(\rho_H)$, so the mean effect always works against our ex-post contagion result.

**Variance effect** The equilibrium thresholds also depend on the precisions of the private and public information (Metz (2002)). The variance effect arises because the correlation $\rho$ affects the precision of the updated prior $\alpha_2(\rho)$. If the prior about the fundamental is strong, then the equilibrium threshold is below the mean of the updated prior about the fundamentals in country 2, $\theta^*_2I(\rho, \theta_1) < \mu_2(\rho, \theta_1)$. Given the strong prior about fundamentals (Definition 1), speculators do not attack the currency of country 2 unless their private signal is very low. Then, a relative decrease in the precision of public information (because speculators learn $\rho = 0$) increases strategic uncertainty, as reflected by a more dispersed belief about other speculators’ posteriors. Thus, speculators are

---

25See Appendix C.2 for comparative statics results and their dependence on the mean and variance effects.
more concerned about other speculators receiving adverse private signals and attacking the currency. This shift in beliefs about other speculators’ posteriors induces more aggressive speculative attacks and raises $\theta^*_U(0, \theta_1)$ relative to $\theta^*_U(\rho_H, \theta_1)$. If the conditions in Lemma 2 hold, then the variance effect outweighs the mean effect and the ordering of equilibrium fundamental thresholds is $\theta^*_U(0, \theta_1) > \theta^*_U(\rho_H, \theta_1)$. Given that the true state of the world is $\rho = 0$, there is a one-to-one mapping between the ordering of equilibrium thresholds and the likelihood of a crisis.

A crisis can spread contagiously from one country to another despite the public information about zero correlation ex-post (Proposition 3). Contagion is less likely if speculators do not learn about the (zero) cross-country correlation and believe that a positive correlation, and thus exposure to the crisis country, is possible. Therefore, information can lead to contagion, which is an uninformed-is-bliss feature. Ex-post contagion also arises with information acquisition about the cross-country correlation analyzed in section 4.

4.2 Ex-ante contagion

Next we show that contagion can also occur ex-ante. A crisis in country 1 spreads contagiously despite speculators learning ‘good news’ about country 2’s fundamental mean (no exposure to the crisis country). More precisely, learning $\rho = 0$ after observing a crisis in country 1 results in a higher likelihood of a crisis in country 2 than if no crisis had been observed. Not observing a crisis in country 1 implies that country 1’s fundamentals must have been rather good. Thus, country 2’s fundamental is more likely to be good because of a potentially positive cross-country correlation.

Proposition 3 summarizes the result. The left-hand side of the inequality in equation (20) is unchanged relative to Proposition 2 while the right-hand side now allows for either realization of the correlation $\rho \in \{0, \rho_H\}$ and is only conditional on not observing a crisis in country 1, $\theta_1 \geq \theta^*_1$.

---

26 Despite the risk-neutrality of all speculators, the variance effect matters because of strategic uncertainty. Higher-order moments help to predict the equilibrium behavior of other speculators and are therefore payoff-relevant information in the incomplete-information coordination game between speculators.

27 More information can lead to adverse outcomes in Hirshleifer (1971). Information acquisition can be privately optimal but has a negative public value, since it makes co-insurance for risk-averse agents infeasible. Instead, Morris and Shin (2007) analyze optimal communication and provide a rationale for coarse information, for instance in credit ratings. Dang et al. (2012) provide an “ignorance-is-bliss” argument, whereby information insensitivity is key to security design in the money market. More transparency can also be harmful in an expert model with career concerns (Prat (2005)).

28 Using the events $E_1 = \theta_2 < \theta^*_U$ and $E_2 = \theta_1 \geq \theta^*_1$, the ex-ante probability of a crisis in country 2 after not observing a crisis in country 1 is decomposed by the law of total probability: $\Pr\{E_1|E_2\} = p \Pr\{E_1|\rho = 0, E_2\} + (1 -$
Here we use the assumption that $\theta_1$ is publicly observed only in the case of a crisis.

Proposition 3  **Ex-ante contagion**  Suppose the fundamental in country 2 is strong, private information sufficiently precise, and $\theta^*_1 \geq \theta_1$, as ensured by conditions (13) and (14) of Lemma 2. If the following sufficient condition holds:

$$\frac{1 - \rho_H^2}{\alpha} \leq [\theta^*_{2f}(0) + \rho_H \theta_1 - (1 + \rho_H)\mu]^2,$$

with $\theta^*_{2f}(0)$ defined by equation (14), then a crisis in country 2 is more likely after observing a crisis in country 1 than after observing no crisis, even if all speculators in country 2 learn $\rho = 0$:

$$\text{Pr}\{\theta_2 < \theta^*_{2f} | \rho = 0, \theta_1 < \theta^*_1\} > \text{Pr}\{\theta_2 < \theta^*_{2f} | \theta_1 \geq \theta^*_1\}. \tag{20}$$

*Proof*  See Appendix C.7.

We can again use the mean and variance effects to gather intuition for the ex-ante contagion result. First, if there is zero realized cross-country correlation, then the conditioning on a crisis does not matter and the probabilities are the same. Second, if the fundamentals are correlated, then the conditional probability on the right-hand side differs from the left-hand side for two reasons. On the one hand, the public information about the fundamental is more precise, such that strategic uncertainty is reduced under the sufficient conditions of Lemma 2. Therefore, the variance effect always works in favor of the stated inequality. On the other hand, learning that no crisis occurred in country 1 shifts the mean of the updated prior. However, the mean effect is ambiguous in general since realizations $\theta_1 \in [\theta^*_1, \mu]$ are consistent with both no crisis in country 1 and a reduced fundamental mean in country 2. The sufficient condition ensures that this part of the mean effect for $\theta_1 \in [\theta^*_1, \mu]$ is always dominated by the variance effect and the other part of the mean effect for $\theta_1 \in [\mu, \infty)$, which in turn ensures ex-ante contagion. Inequality (14) is more likely to hold, the stronger the prior belief about the fundamentals in country 2 and the less precise public information, since both are associated with a low $\theta^*_{2f}(0)$.
5 Information acquisition

We study information acquisition in this section and present the wake-up call theory of contagion. Observing a crisis in country 1 is a wake-up call for speculators in country 2. This induces speculators to acquire information about the cross-country correlation in order to determine the exposure to the crisis country. Contagion can occur even if speculators observe zero cross-country correlation. Therefore, our ex-post and ex-ante contagion results stated in Propositions 2 and 3 occur in equilibrium when information is endogenous.

Our solution concept is perfect Bayesian equilibrium (PBE) defined below. Date 2 is divided into an information stage (stage 1) and a coordination stage (stage 2). To analyze the private incentives to acquire information, we extend our previous analysis of the coordination stage to the case of asymmetrically informed speculators. We solve for the optimal attacking rule \( a_2^* \) in the coordination stage in section 5.1. We prove the existence and uniqueness of such a rule for a given proportion of informed speculators in Lemma 3. Next, we analyze the dependence of the optimal attacking rule on the proportion of informed speculators in Lemma 4. In section 5.2, we solve for the optimal information acquisition rule \( d_i^* \) and establish the main results in Proposition 4.

**Definition 2** A pure-strategy perfect Bayesian equilibrium in threshold strategies consists of an information acquisition rule \( d_i^* \in \{I, U\} \) for each speculator \( i \), an aggregate proportion of informed speculators \( n^* \in [0, 1] \), an attacking decision rule \( a_{2I}^*(n^*; \theta_1, x_i) \in \{0, 1\} \) for each speculator \( i \), and an aggregate proportion of attacking speculators \( A_2^* \in [0, 1] \) such that:

1. All speculators optimally choose \( d_i \) in the information stage.
2. The proportion \( n^* \) is consistent with the individually optimal information choices \( \{d_i^*\}_{i \in [0,1]} \).
3. For any given realization of \( \rho \in \{0, \rho_H\} \), informed speculators make optimal attack decisions \( a_{2I}^*(n^*; \theta_1, \rho, x_i) \). Uninformed speculators make optimal attacking decisions \( a_{2U}^*(n^*; \theta_1, x_i) \).
4. The proportion \( A_2^* \) is consistent with the individually optimal attacking decisions:

\[
A_2^* \equiv A(n^*; \theta_2, \rho) = n^* \int_{-\infty}^{+\infty} a_{2I}^*(n^*; \theta_1, \rho, x_i) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_i - \theta_2)) dx_i + (1 - n^*) \int_{-\infty}^{+\infty} a_{2U}^*(n^*; \theta_1, x_i) \sqrt{\gamma} \phi(\sqrt{\gamma}(x_i - \theta_2)) dx_i, \quad \forall \rho \in \{0, \rho_H\}.
\] (21)
5.1 Stage 2: optimal attacking rules

We generalize our analysis of section 3.2 by allowing for asymmetrically informed speculators. That is, the proportion $n^*$ acquires information about the realized cross-country correlation $\rho$ (informed speculators), while the remainder is uninformed. In equilibrium, the proportion $n^*$ is known at the coordination stage and unaffected by the individual information choice.\[29\]

Maintaining our focus on monotone equilibria, each speculator attacks if the private signal is below a threshold specific to the information choice. When $n \in [0, 1]$ is the proportion of informed speculators to be determined in stage 1, there are now three thresholds in stage 2: one for uninformed speculators $x^{*U}_{2U}(n, \theta_1)$ and two for informed speculators, $x^{*I}_{2I}(n, 0, \theta_1)$ and $x^{*I}_{2I}(n, \rho_H, \theta_1)$, that is one for each realization of the correlation. Next, the fundamental threshold also depends on the proportion of informed speculators $n$ and on the realized correlation $\rho$, yielding two fundamental thresholds $\theta^*_2(n, \rho, \theta_1)$. The equilibrium in the coordination stage can be described by two equations in two unknowns $\theta^*_2(n, 0, \theta_1)$ and $\theta^*_2(n, \rho_H, \theta_1)$, which we describe in detail in Appendix C.8.\[30\]

We now analyze the existence of a unique optimal attacking rule. The presence of asymmetrically informed speculators complicates the analysis, as we must keep track of the interaction of uninformed speculators, characterized by a mixture distribution, with informed speculators. However, we are able to prove existence and uniqueness for sufficiently precise private information:

**Lemma 3 Optimal attacking rule in stage 2.** Suppose there is a crisis in country 1, $\theta_1 < \theta^*_1$, and the proportion $n$ is informed about the correlation $\rho$. If private information is sufficiently precise, then there exists a unique attacking rule. Each uninformed speculator attacks if and only if $x_{i2} < x^{*U}_{2U}(n, \theta_1)$. Similarly, each informed speculator attacks if and only if $x_{i2} < x^{*I}_{2I}(n, \rho, \theta_1)$ after learning the realization of $\rho$. For each $\rho \in \{0, \rho_H\}$, a crisis occurs if and only if fundamentals are sufficiently low, $\theta_2 \leq \theta^*_2(n, \rho, \theta_1)$.

**Proof** See Appendix C.9

How do the fundamental thresholds depend on the proportion of informed speculators? This

\[29\] In contrast to section 4, common knowledge about $n$ is not required. Given the conditions of Proposition 4, information acquisition is a dominant action in stage 1.

\[30\] Intuitively, we recover our previous results for the limiting cases of uninformed (informed) speculators as the proportion of informed speculators converges to zero (one).
question is crucial for understanding the private incentives to acquire information at stage 1. We have already analyzed the polar cases of completely uninformed and informed speculators, \( n \in \{0, 1\} \). Proposition\[\textbf{1}\] shows that the fundamental threshold is independent of the realized cross-country correlation if all speculators are uninformed \( \theta_{21}^*(\theta_1) \). The fundamental threshold varies with the realized correlation if all speculators are informed (Corollary\[\textbf{2}\]), where Lemma\[\textbf{2}\] establishes sufficient conditions for \( \theta_{21}^*(0, \theta_1) > \theta_{21}^*(\rho_H, \theta_1) \), including \( \theta_1 > \theta_1 \). By contrast, the threshold ranking is reversed, \( \theta_{21}^*(0, \theta_1) < \theta_{21}^*(\rho_H, \theta_1) \), if \( \theta_1 < \theta_1 \), while \( \theta_{21}^*(0, \theta_1) = \theta_{21}^*(\rho_H, \theta_1) \) if \( \theta_1 = \theta_1 \).

We now analyze the general case of changes in the proportion of informed speculators. In Lemma\[\textbf{4}\] we establish conditions sufficient for the fundamental thresholds to evolve continuously and monotonically as \( n \) changes\[\textbf{31}\]. As a consequence, the distance between both fundamental thresholds as well as between both attacking thresholds evolve monotonically as well. More specifically, the distance between the fundamental thresholds, \( |\theta_{21}^*(n, 0, \theta_1) - \theta_{21}^*(n, \rho_H, \theta_1)| \), continuously and monotonically increases in \( n \). This property is crucial for establishing the strategic complementarity in information choices in section\[\textbf{5.2}\].

**Lemma 4** Proportion of informed speculators and equilibrium thresholds. Suppose there is a crisis in country 1, \( \theta_1 < \theta_1 \), and strong fundamentals in country 2. If private information is sufficiently precise, \( \gamma < \gamma < \infty \), and public information is sufficiently imprecise, \( 0 < \alpha < \alpha \), then:

(A) **Boundedness.** The fundamental thresholds in the polar case of informed speculators bound the fundamental thresholds in the general case of asymmetrically informed speculators:

if \( \theta_1 \geq \theta_1 \):

\[\theta_{21}^*(\rho_H, \theta_1) \leq \theta_{21}^*(n, \rho, \theta_1) \leq \theta_{21}^*(0, \theta_1) \forall \rho \in \{0, \rho_H\} \forall n \in [0, 1]\]

if \( \theta_1 < \theta_1 \):

\[\theta_{21}^*(0, \theta_1) \leq \theta_{21}^*(n, \rho, \theta_1) \leq \theta_{21}^*(\rho_H, \theta_1) \forall \rho \in \{0, \rho_H\} \forall n \in [0, 1].\]

(B) **Monotonicity.** The fundamental threshold in the case of zero (positive) cross-country correlation increases (decreases) in the proportion of informed speculators. Strict monotonicity is attained if and only if the fundamental thresholds are strictly bounded:

\[\text{ex-post contagion occurs as described in Proposition}\[\textbf{2}\] \theta_{21}^*(0, \theta_1) > \theta_{21}^*(\rho_H, \theta_1) > \theta_{21}^*(\rho_H, \theta_1). \text{ As depicted in Figure 2, this threshold ranking holds for any strictly positive proportion of informed speculators.} \]
\[
\frac{d\theta_2^*(n, 0, \theta_1)}{dn} \begin{cases} 
> 0 & \text{if } \theta_2^*(\rho_H, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(0, \theta_1) \\
< 0 & \text{if } \theta_2^*(0, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(\rho_H, \theta_1) \\
= 0 & \text{if } \theta_2^*(\rho, \theta_1) = \theta_2^*(n, \rho, \theta_1)
\end{cases} \quad \forall \rho, n \in [0, 1). \quad (22)
\]

\[
\frac{d\theta_2^*(n, \rho_H, \theta_1)}{dn} \begin{cases} 
< 0 & \text{if } \theta_2^*(\rho_H, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(0, \theta_1) \\
> 0 & \text{if } \theta_2^*(0, \theta_1) < \theta_2^*(n, \rho, \theta_1) < \theta_2^*(\rho_H, \theta_1) \\
= 0 & \text{if } \theta_2^*(\rho, \theta_1) = \theta_2^*(n, \rho, \theta_1)
\end{cases} \quad \forall \rho, n \in [0, 1). \quad (23)
\]

(C) Monotonicity in attacking thresholds. As a consequence of the monotonicity in fundamentals thresholds:

\[
\frac{d|x_2^*(n, 0, \theta_1) - x_2^*(n, \rho_H, \theta_1)|}{dn} \geq 0 \quad \forall n \in [0, 1). \quad (24)
\]

Proof See Appendix C.10

Figure B.2 in Appendix B visualizes the results of Lemma 4. The fundamental thresholds for \(\rho = 0\) and \(\rho = \rho_H\) diverge as the proportion of informed speculators increases. This is a consequence of informed speculators capitalizing on their information advantage. While uninformed speculators must use the same attack threshold irrespective of the realized correlation, informed speculators adjust their attack thresholds accordingly. Intuitively, a larger proportion of informed speculators raises the fundamental threshold \(\theta_2^*(n, 0, \theta_1)\) because informed speculators attack more aggressively after learning \(\rho = 0\), compared with uninformed speculators (Part (a); see thick dotted line in Figure 2). The opposite is true in the case of positive correlation, \(\rho = \rho_H\), because informed speculators attack less aggressively compared to uninformed speculators. Consequently, the \(\theta_2^*(n, \rho_H, \theta_1)\) decreases in the proportion of informed speculators (see thick dashed line in Figure 2), whereas the difference between the thresholds increases in \(n\) (Part (b)).
5.2 Stage 1: optimal information acquisition

We now turn to the information stage to solve for the pure-strategy PBE. First, we show that the information choices of speculators exhibit strategic complementarity. Second, we demonstrate that there exists a unique equilibrium in which all speculators acquire information if the information cost is sufficiently small. This equilibrium features both contagion effects.

We continue to concentrate on the case of a crisis in country 1, \( \theta_1 < \theta_1^* \). Thus, speculators in country 2 observe the fundamental \( \theta_1 \) before deciding whether to acquire costly and publicly available information about the cross-country correlation \( \rho \). Such information helps a speculator assess the exposure to the crisis country. We also maintain the sufficient conditions of Lemma 2, so \( \underline{\theta}_1 < \theta_1 < \theta_1^* \) and \( \theta_{2I}(0) > \theta_{2I}(\rho_H) \).

Next, we show that the monotonicity in attack thresholds established in Lemma 4 (C) leads to strategic complementarity in information choices. A speculator’s incentive to acquire information increases in the proportion of informed speculators. Consider the information choice of an individual speculator \( i \), who compares the expected payoffs from becoming informed and from remaining uninformed (both defined in Appendix C.11). The expected utility of an informed speculator, \( \text{E}[u(d_i = I, n)] \equiv EU_I - c \), comprises the benefit of attacking if a currency crisis occurs, the cost of attacking if no crisis occurs, and the information cost. The possible realizations of the cross-country correlation are also taken into account, as it affects the attacking threshold of an informed speculator, \( x_{2I}(n, 0, \theta_1) \) and \( x_{2I}(n, \rho_H, \theta_1) \). By contrast, the expected utility of an uninformed speculator, \( \text{E}[u(d_i = U, n)] \equiv EU_U \), does not comprise the information cost. Furthermore, an uninformed speculator cannot tailor her attacking strategy and must use the same attacking threshold \( x_{2U}(n, \theta_1) \) irrespective of the realized cross-country correlation.

Optimality at the information stage requires that a given speculator acquires information if the expected utility differential \( EU_I - EU_U \) is no smaller than the information cost:

\[
d^*_i = I \iff EU_I - EU_U \geq c. \tag{25}
\]

In other words, it pays to acquire information about the cross-country correlation if the benefit from using tailored attacking thresholds covers at least the information cost. Let the highest information
cost that makes a speculator indifferent between the information choices be \( \tilde{c}(n, \theta_1) \equiv EU_I - EU_U: \)

\[
\tilde{c}(n, \theta_1) = p \left( \int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} g(x_{2I}(\theta_2)) dx_{2I} \right) - \int_{\theta_2^*(n, 0, \theta_1)}^{\infty} g(x_{2I}(\theta_2)) dx_{2I} - \int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} g(x_{2I}(\theta_2)) dx_{2I} \right) - \left(1 - p \right) \left( \int_{-\infty}^{\theta_2^*(n, \rho_H, \theta_1)} g(x_{2I}(\theta_2)) dx_{2I} \right) - \int_{\theta_2^*(n, \rho_H, \theta_1)}^{\infty} g(x_{2I}(\theta_2)) dx_{2I} \right),
\]

(26)

where the fundamental of country 2 conditional on the realized correlation, \( f(\theta_2|\rho, \theta_1) \), is normally distributed with mean \( \mu_2(\rho, \theta_1) \) and precision \( \alpha_2(\rho) \), while the private signal conditional on the fundamental, \( g(x|\theta_2) \), is normally distributed with mean \( \theta_2 \) and precision \( \gamma \).

**Intuition** Consider the benefit from using a tailored attacking threshold. An informed speculator’s marginal benefit of increasing the attack threshold \( x_{2I}(n, \rho, \theta_1) \) is given by:

\[
b \int_{-\infty}^{\theta_2^*(n, \rho, \theta_1)} g(x_{2I}(n, \rho, \theta_1)|\theta_2) f(\theta_2|\rho, \theta_1) d\theta_2 - f \int_{\theta_2^*(n, \rho, \theta_1)}^{\infty} g(x_{2I}(n, \rho, \theta_1)|\theta_2) f(\theta_2|\rho, \theta_1) d\theta_2,
\]

(27)

which is zero when evaluated at \( x_{2I}(n, \rho, \theta_1) = x_{2I}^*(n, \rho, \theta_1) \forall \rho \in \{0, \rho_H\} \) by optimality. Furthermore, equation (27) decreases monotonically in \( x_{2I}(n, \rho, \theta_1) \):

\[
\frac{dg(x_{2I}(n, \rho, \theta_1)|\theta_2)}{dx_{2I}(n, \rho, \theta_1)} = \begin{cases} 
> 0 & \text{if } x_{2I}(n, \rho, \theta_1) < \theta_2 \\
\leq 0 & \text{if } x_{2I}(n, \rho, \theta_1) \geq \theta_2.
\end{cases}
\]

(28)

The attacking threshold converges to the fundamental threshold as private noise vanishes:

\[
\lim_{\gamma \to \infty} x_{2I}^*(n, \rho, \theta_1) = \theta_2^*(n, \rho, \theta_1) \forall \rho \in \{0, \rho_H\},
\]

(29)

Recall that \( x_{2I}^*(n, 0, \theta_1) > x_{2U}^*(n, \theta_1) > x_{2I}^*(n, \rho_H, \theta_1) \) when \( \theta_2^*(n) > \theta_2^*(\rho_H) \). Hence, the marginal benefit from increasing \( x_{2I}^*(n, 0, \theta_1) \) above \( x_{2U}^*(n, \theta_1) \) is:

\[
p \left( b \int_{-\infty}^{\theta_2^*(n, 0, \theta_1)} g(x_{2I}^*) dx_{2I} \right) f(\theta_2) d\theta_2 > 0,
\]

(30)
while the marginal benefit from increasing $x_{2I}^*(n, \rho_H, \theta_1)$ above $x_{2U}^*(n, \theta_1)$ is given by:

\[
(1 - p) \left( b \int_{-\infty}^{\theta_2^*(n, \rho_H, \theta_1)} g(x_{2U}^*(\theta_2)) f(\theta_2 | \rho_H, \theta_1) d\theta_2 - l \int_{\theta_2^*(n, \rho_H, \theta_1)}^{+\infty} g(x_{2U}^*(\theta_2)) f(\theta_2 | \rho_H, \theta_1) d\theta_2 \right) < 0. \tag{31}
\]

These expressions are best understood in terms of type-I and type-II errors. Let the null hypothesis be that there is a crisis in country 2, such that $\theta_2 < \theta_2^*$. Each of the expressions in equations (30) and (31) have two components. The first component in each equation represents the marginal benefit from attacking when a crisis occurs. (Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error)). The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 4 together with Proposition 1 imply the following. After a crisis in country 1, we have for strong fundamentals in country 2, a sufficiently precise private information, and a sufficiently imprecise public information that $\theta_2^*(n, \rho_H, \theta_1) < \theta_2^*(n, 0, \theta_1) \forall n \in [0, 1]$ if $\theta_1 \in (\theta_1, \theta_1^*)$. Hence, the marginal benefit from increasing $x_{2I}^*(n, \rho_H, \theta_1)$ above $x_{2U}^*(n, \theta_1)$ is positive because the type-I error is more costly than the type-II error. By contrast, the marginal benefit from decreasing $x_{2I}^*(n, \rho_H, \theta_1)$ below $x_{2U}^*(n, \theta_1)$ is positive because the type-II error is relatively more costly. In sum, informed agents attack more aggressively upon learning that fundamentals are uncorrelated.

Lemma 5 states how the threshold information cost, $\bar{c}(n, \theta_1)$, changes with the proportion of informed speculators. Hellwig and Veldkamp (2009) show that information choices inherit the strategic complementarity or substitutability from the underlying beauty contest game. We show that this “inheritance” result extends to a global coordination game of regime change with ex-ante uncertainty about the correlation of fundamentals, in which speculators can acquire a publicly available signal. Specifically, Lemma 5 describes a strategic complementarity in information choices.

**Lemma 5** Strategic complementarity in acquiring information. Suppose there is a crisis in country 1, $\theta_1 < \theta_1^*$, and strong fundamentals in country 2. If private information is sufficiently precise, $\gamma < \gamma < \infty$, and public information is sufficiently imprecise, $0 < \alpha < \alpha$, then the incentives to acquire information increase in the proportion of informed speculators $n$:

\[
\frac{dc(n, \theta_1)}{dn} \geq 0 \forall \theta_1 < \theta_1^*. \tag{32}
\]
Furthermore, for all \( n \in [0, 1] \):

\[
\begin{align*}
\bar{c}(n, \theta_1) = \begin{cases} 
> 0 & \forall \theta_1 \neq \bar{\theta}_1, \\
0 & \text{if } \theta_1 = \bar{\theta}_1.
\end{cases}
\end{align*}
\]  

(33)

**Proof** See Appendix C.11.

Strategic complementarity in information choices implies conditions sufficient for the existence of a unique monotone pure-strategy PBE. For any \( \theta_1 \in (\bar{\theta}_1, \theta_1^*) \), there exists a range of positive information costs, \( 0 < c < \bar{c}(0, \theta_1) \), such that acquiring information about the cross-country correlation is a dominant strategy for all speculators, \( n^* = 1 \). Furthermore, both the ex-ante contagion and ex-post contagion results, described previously for exogenous information, occur in equilibrium with endogenous information acquisition. These results are summarized in Proposition 4.

**Proposition 4** *Existence of a unique monotone PBE with ex-ante and ex-post contagion.* Suppose there is a crisis in country 1, \( \theta_1 < \theta_1^* \), strong fundamentals in country 2, sufficiently precise private information, \( \bar{\gamma} < \gamma < \infty \), and sufficiently imprecise public information, \( 0 < \alpha < \beta \).

If the information cost is sufficiently small, \( c < \bar{c}(0, \theta_1) \), then there exists a unique monotone pure-strategy PBE. All speculators acquire information about the cross-country correlation, \( n^* = 1 \), and use the attacking threshold \( x^*_{2I}(1, \rho, \theta_1) \) for \( \rho \in \{0, \rho_H\} \). By contrast, if the information cost is sufficiently large, \( \bar{c}(1, \theta_1) < c \), then there exists a unique monotone pure-strategy PBE, in which no information is acquired, \( n^* = 0 \), and speculators use the attacking threshold \( x^*_{2U}(0, \theta_1) \).

Contagion occurs even if each speculator learns that countries are uncorrelated, \( \rho = 0 \):

1. **Ex-ante contagion** arises if \( c \leq \int_{-\infty}^{\theta_1^*} \bar{c}(0, \theta_1)f(\theta_1)d\theta_1 \), without further restrictions on the realized fundamental in country 1, \( \theta_1 \).

2. **Ex-post contagion** arises, given conditions (13) and (14) of Lemma 2, if \( \theta_1 \in (\bar{\theta}_1, \theta_1^*) \): the probability of a crisis is higher if \( c < \bar{c}(0, \theta_1) \) (informed) than if \( \bar{c}(1, \theta_1) < c \) (uninformed).

**Proof** See Appendix C.12.
6 Testable implications

We relate our results to the empirical literature on contagion and interdependency in section 6.1 where we develop novel empirical predictions. Furthermore, we describe testable predictions for laboratory experiments in section 6.2.

6.1 Empirical literature and novel predictions

There is a large literature on interdependence and contagion in financial economics and international finance (see [Forbes (2012)] for a recent survey). The approaches differ and range from probability models (e.g. [Eichengreen et al. (1996)]) to the correlation analysis (e.g. [Forbes and Rigobon (2002)]) as well as the closely related VAR models (e.g. [Favero and Giavazzi (2002)]), latent factor/GARCH models (e.g. [Bekaert et al. (2014)]), and extreme value analysis (e.g. [Bae et al. (2003)]). Important to our paper is the empirical literature that investigates the channels of contagion and how these depend on the fundamental characteristics of the second region (see [Glick and Rose (1999), Van Rijckeghem and Weder (2001, 2003), and Dasgupta et al. (2011)]). This literature suggests that the likelihood of contagion is higher with stronger trade and financial links, as well as with a higher institutional similarity. The cross-regional correlation in our model captures such factors.

We contribute to this literature by accounting for the non-linear effects of the fundamental in the initially affected region. The empirical literature studies the channels by which contagion spreads and the characteristics of the second region that makes it susceptible to contagion. Our theory complements this literature by studying how the likelihood of contagion depends on the realized fundamental in the first region, as directly implied by Proposition 2 and Lemma 2.

Let \( P \geq 0 \) measure the intensity of trade links, financial links, and institutional similarities with the crisis region. Consistent with our model, \( \theta_1 \) is the fundamental in the first region that is affected by a crisis. Let \( 1(\theta_2, P) \) be an indicator function that measures whether another region with the characteristics \( \theta_2 \) (fundamentals of the other region, such as key macro variables) and \( P \) is affected by the crisis in the first region. Our model implies the following two empirical predictions:

**Empirical prediction 1.**

\[
\frac{d^21(\theta_2, P)}{dP d\theta_1} < 0.
\] (34)
**Empirical prediction 2.** Under the conditions of Proposition 2 there exists some $\theta_1$ such that:

\[
\frac{d1(\theta_2, P)}{dP} < 0 \quad \text{if} \quad \theta_1 > \theta_1,
\]

\[
\frac{d1(\theta_2, P)}{dP} > 0 \quad \text{if} \quad \theta_1 < \theta_1.
\]

(35)

Prediction 1 states that a crisis in the first region due to a worse realization of the fundamental $\theta_1$ is more likely to spread contagiously, the stronger the exposure observed by the empiricist (i.e., a higher $P$). This prediction hinges on updating and the mean effect. It is also implied by other models that build on the mean effect, such as an extension of Acharya and Yorulmazer (2008).

Based on the variance effect, our ex-post contagion mechanism stated in Proposition 2 gives rise to Prediction 2. In particular, after controlling for the contemporaneous fundamentals of the second region (the realization of $\theta_2$), there is a non-linear effect of the realized fundamental in the first region, $\theta_1$. A crisis in the first region due to *moderately low* fundamentals is more likely to spread if the empiricist observes no linkages ($P = 0$). This is implied by $\theta_2^*(0) > \theta_2^*(\rho_H) \forall \theta_1 \in (\theta_1, \theta_1^*)$. By contrast, a crisis due to *extremely low* fundamentals is less likely to spread if the empiricist observes no linkages. This is implied by $\theta_2^*(0) < \theta_2^*(\rho_H) \forall \theta_1 < \theta_1$.

When studying the channels of contagion, analyzing the characteristics of the second region is insufficient. Specifically, our theory suggests a non-linearity in $\theta_1$, so an empiricist should discriminate between moderately low and extremely low realized fundamentals in the first region. These empirical implications of our theory might improve the understanding of financial contagion.

### 6.2 Predictions for laboratory experiments

Our theory of contagion generates three implications that can be tested in laboratory experiments. Such experiments have the advantage of observing the information choice of participants after a wake-up call. The literature on laboratory experiments features several experimental studies on financial contagion as well as on bank runs. While there has been a substantial interest in studying global games models in the laboratory following Heinemann et al. (2004, 2009), contagion within global games models in the laboratory following Heinemann et al. (2004, 2009), contagion within

\[32\]See e.g. Cipriani and Guarino (2008) and Cipriani et al. (2013) for an experiment on financial contagion and Schotter and Yorulmazer (2009) and Garratt and Keister (2009) for an experiment on bank runs.
the global games framework has only recently begun to attract attention.\footnote{To our knowledge, \cite{Trevino2013} is the only experimental study on contagion within the global games framework. The author investigates a fundamental and a social learning channel of contagion.}

The first implication is the ex-post contagion effect (Proposition 2). When the actual correlation is zero, this experiment contrasts the cases in which all participants are informed or uninformed about this correlation. The ex-post contagion mechanism predicts that the variance effect (elevated strategic uncertainty) can overturn the mean effect. Thus, contagion can occur even if all investors observe that there is no exposure or interconnectedness ex-post, relative to uninformed investors.

A second set of theoretical predictions relate to the acquisition of publicly available information about the correlation.\footnote{While there is some experimental work on the acquisition of private signals, for instance by \cite{SzkupTrevino2012a}, the acquisition of publicly available signals is still unexplored.} We suggest to investigate three questions. How do the incentives to acquire information change with the known number of informed participants (Lemma 5)? This question can be analyzed for a given $\theta_1 < \theta_1^*$ by exogenously varying $n$ and eliciting the willingness to pay for information. Furthermore, which thresholds of the information cost $\tau$ induce all (no) participant(s) to acquire information (Proposition 4)? This question can be analyzed by setting $n = 0$ ($n = 1$) in the previous exercise. Similarly, the information choices of participants are recorded for various information costs. Finally, under what conditions do the contagion results arise in laboratory experiments with endogenous information? This question requires to record, for various information costs, the information choices of participants after a wake-up call.

A third implication concerns the fundamental of the first region, $\theta_1$, in the previous two experiments. Proposition 2 and Lemma 2 imply an important role for the threshold $\theta_1$, highlighting the non-linear effect of fundamentals in the first region (see also section 6.1.)

7 Conclusion

We have proposed a wake-up call theory of financial contagion. When the correlation across regional fundamentals is potentially positive but uncertain ex-ante, a crisis in one region is a wake-up call for investors in another region. To determine their exposure, investors acquire information. Due to a strategic complementarity in information choices, all investors acquire information about the cross-regional correlation if the information cost is sufficiently low.
We show that contagion can be more likely when speculators are informed about the zero cross-regional correlation than when they are uninformed (ex-post contagion). The key driver of this result is elevated strategic uncertainty among informed investors. We also show that contagion can be more likely after observing a crisis in the first region and learning about a zero correlation than after observing no crisis (ex-ante contagion).

We provide novel empirical predictions on contagion as well as testable implications for laboratory experiments. A previously unexplained banking crisis is a compelling case study of our wake-up call theory of contagion. Our theory helps to improve our understanding of “unexpected contagion” across regions with seemingly unrelated fundamentals and no interconnectedness.

Our contagion result is robust to variations of the model. Appendix D discusses the robustness of our results and contains interesting extensions, including heterogeneous regions, imperfect learning about the correlation, a different timing assumption, and potentially negative correlation.
References


A The Latvian crisis of 2011


The ex-ante contagion mechanism (Proposition 3) suggests that a bank run against the Latvian subsidiary of Swedbank is more likely after observing the failure of Krajbanka, even if all the depositors of Swedbank learn that Swedbank is not exposed to Krajbanka. According to our theory, this result holds for strong fundamentals of the potential victim (Definition 1), a characteristic that was arguably appropriate for Swedbank.

Furthermore, Krajbanka had no significant direct exposure to Swedbank. Instead, the cause for the failure of Krajbanka is related to its owners, but not to Swedbank. Consequently, the depositors of Swedbank were unlikely to be concerned about the possibility of direct exposure or interconnectedness. Instead, our theory suggests that they experienced a wake-up call that led to contagion based on elevated strategic uncertainty.

---

35 We would like to thank David Farelius and Juks Reimo for sharing their insights.
36 Source: BBC news on Dec. 12, 2011, "Panic fuels Latvian run on bank". The withdrawal size was verified ex-post and is consistent with information form official sources. This corresponds to 4% of Latvian households deposits in Swedbank that have been withdrawn during a weekend were limited ATM withdrawals are the only option to receive cash. (Source: Swedbank’s Annual Report 2011).
37 First statements from Latvian and Swedish regulators were given to Latvian media on Sunday evening. See also press release from the Latvian regulator Financial and Capital Market Commission on Dec. 12, 2011.
38 Swedbank had an A+ rating by Standard&Poors. It was considered systemically important for Sweden and was the largest Swedish bank by number of customers. Moreover, a strong commitment by Swedbank to support its Latvian subsidiary was likely, not only because of the interest to maintain its leading market position in Latvia, but also because of the international commitment to assist Latvia. The international support for Latvia was also reflected in the USD 2.5 billion in December 2008, which was part of a larger package for Latvia supported by the EU and others, totalling USD 10.5 billion. In addition, the international support is also documented in the Vienna Initiative from 2009 who states on their website the key objective to prevent “a large-scale and uncoordinated withdrawal of cross-border bank groups from the region” (see http://vienna-initiative.com/vienna-initiative-part-1/).
39 Krajbanka was a local lender that is one ninth of the size of the Latvian subsidiary of Swedbank when measured by total assets. (Source: Financial reports by Swedbank and KPMG Baltics who managed the liquidation of Krajbanka.) The Latvian assets of Swedbank represented only 2.5% of the total assets held by its Swedish parent bank. Moreover, the funding to Krajbanka did not come from Swedbank. Until November 2011, the Russian Vladimir Antonov was a majority shareholder of the leading Lithuanian bank Bankas Snoras and of Krajbanka. He was chairman of Bankas Snoras and in the Supervisory Board of Krajbanka. Bankas Snoras was nationalized in November 2011 and held big stakes in Krajbanka.
B Figures

B.1 Updated Prior distributions

Figure 1: The updated prior distributions about the fundamental in country 2: the dashed brown and dotted blue lines represent the updated prior distribution for an informed speculator who observes no correlation and positive correlation, respectively. The solid red line represents an uninformed speculator’s updated prior distribution. Parameters: \( \mu = 0.6, \alpha = 1, p = 0.7, \rho_H = 0.8 \) and \( \theta_1 = 0.07 (\theta_1 = -1) \) for the first (second) panel.

B.2 The incentives to acquire information

Figure 2: The fundamental thresholds as a function of the proportion of informed speculators \( n \). Parameters are \( \mu = 0.6, \alpha = 1, \gamma = 1.5, b/l = 1/3, p = 0.7, \rho_H = 0.8 \) and \( \theta_1 = 0.07 \).
C Derivations and proofs

C.1 Bayesian equilibrium in country 1

The first equilibrium condition states that the equilibrium proportion of attacking speculators equals the critical fundamental threshold below which a currency crisis occurs:

\[ A(\theta^*_1) = \Pr\{x_{i1} < x^*_1|\theta^*_1\} = \Phi(\sqrt{\gamma}(x^*_1 - \theta^*_1)) = \theta^*_1 \]

\[ \Rightarrow x^*_1 = \theta^*_1 + \frac{1}{\sqrt{\gamma}} \Phi^{-1}(\theta^*_1). \] (36)

The second equilibrium condition is an indifference condition for currency speculators. Upon receiving the threshold private signal \( x_{i1} = x^*_1 \), a speculator is indifferent between attacking and not attacking the currency:

\[ b \Pr\{\theta_1 < \theta^*_1|x_{i1} = x^*_1\} - l \Pr\{\theta_1 > \theta^*_1|x_{i1} = x^*_1\} = 0 \] (37)

where:

\[ \Pr\{\theta_1 < \theta^*_1|x_{i1}\} = \Phi\left(\frac{\theta^*_1 - \mathbb{E}[\theta_1|x_{i1}]}{\sqrt{\text{Var}[\theta_1|x_{i1}]}}\right) = \Phi\left(\frac{\sqrt{\alpha + \gamma}\left[\theta^*_1 - \frac{\alpha \mu + \gamma x_{i1}}{\alpha + \gamma}\right]}{\alpha + \gamma}\right), \]

which is decreasing in \( x_{i1} \). Therefore, it is indeed optimal for a speculator to attack if and only if \( x_{i1} < x^*_1 \). Combining the two equilibrium conditions leads to equation (36), in which the right-hand side is constant and the left-hand side changes according to:

\[ \frac{dF_1(\theta_1)}{d\theta_1} = \frac{\varphi(\cdot)}{\sqrt{\alpha + \gamma}} \left[ \alpha - \frac{\sqrt{\gamma}}{\varphi(\Phi^{-1}(\theta_1))} \right]. \] (38)

In the limit, as \( \theta_1 \to 0 \), then \( F(\theta_1) \to 1 \). Likewise, as \( \theta_1 \to 1 \), then \( F_1(\theta_1) \to 0 \). Given that \( 0 < \frac{1}{1+b/l} < 1 \), a sufficiently precise private signal, \( \gamma > \gamma_0 \equiv \frac{\alpha^2}{2\pi} \), ensures \( \frac{dF_1(\theta_1)}{d\theta_1} \) < 0 and thus the existence and uniqueness of \( \theta^*_1 \).

**Equilibrium properties** If the private signal is sufficiently precise, i.e. \( \gamma > \gamma_0 \), then the equilibrium threshold \( \theta^*_1 \) decreases in the mean of the fundamental \( \mu \) but increases in the relative gain from attacking \( \frac{b}{l} \). Thus, the probability of a currency crisis increases if either the prior about the
fundamental is lower or the relative benefit from attacking is higher:

\[ \frac{d\theta_1^*}{d\mu} = \frac{\alpha}{\alpha - \frac{\sqrt{\gamma}}{\Phi(\Phi^{-1}(\theta_1^*))}} < 0 \]  
\[ \frac{d\theta_1^*}{d(\frac{b}{l})} = -\frac{\sqrt{\alpha + \gamma}}{\phi(\cdot)(1 + \frac{b}{l})^2} \left[ \alpha - \frac{\sqrt{\gamma}}{\phi(\Phi^{-1}(\theta_1^*))} \right] > 0. \]

There are two possible rankings of the equilibrium thresholds depending on the prior about the fundamental. The prior is defined as strong if it is high compared to the relative gain from attacking \( \frac{b}{l} \), which corresponds to the condition in Corollary 1. A strong prior prevents speculative currency attacks for realized fundamentals close to but below the prior: \( 0 < \theta_1^* < \mu \). In contrast, the prior about the fundamental \( \mu \) is weak if the above inequality is reversed. A weak prior leads to speculative currency attacks for realized fundamentals close to but above the prior: \( \mu < \theta_1^* < 1 \).

In the simplifying case of \( b = l \), a strong prior means \( \frac{1}{2} < \mu \), while a weak prior means \( \mu < \frac{1}{2} \). This implies \( 0 < x_1^* < \theta_1^* < \frac{1}{2} < \mu \) for a strong prior and \( \mu < \frac{1}{2} < \theta_1^* < x_1^* < 1 \) for a weak prior.

### C.2 Informed speculators

This section establishes the comparative static results when all speculators are informed about the realization of the cross-country correlation after observing a crisis in country 1, i.e. \( \theta_1 < \theta_1^* \leq \mu \).

We discuss in turn the role of the public and private signal precision in section C.2.1 and the implications for the ordering of equilibrium thresholds for either realization of the cross-country correlation in section C.2.2.

Definition 1 formalizes the distinction between a weak and a strong prior belief about the fundamental. The sets \( S_1 \) and \( S_2 \) can be derived by reformulating equation (10) to:

\[ \Phi^{-1}(\theta_2^*) - \frac{\alpha_2(\rho)}{\sqrt{\gamma}}(\theta_2^* - \mu_2(\rho, \theta_1)) = -\frac{\sqrt{\alpha_2(\rho) + \gamma}}{\sqrt{\gamma}} \Phi^{-1}\left( \frac{1}{1 + b/l} \right). \]

Then, \( X(\rho) \) can be derived by setting \( \theta_2^* = \mu_2(\rho, \theta_1) \) and isolating \( \mu_2(\rho, \theta_1) \). A sufficient condition assuring that strong (weak) prior beliefs are associated with a low (high) incidence of attacks below (above) 50% are derived from equation (11) by setting \( \theta_2^* = \frac{b}{l} = \frac{1}{2} \), which leads to \( Y(\rho) \).
C.2.1 Comparative statics 1: precision of public and private signals

The subsequent discussion draws in parts from Bannier and Heinemann (2005). We have:

\[
\frac{d\theta^*_2}{d\alpha} \begin{cases}
< 0 & \text{if } \theta^*_2 < \mu_2(\rho, \theta_1) + \frac{1}{2\sqrt{\alpha_2(\rho)+\gamma}} \Phi^{-1}\left(\frac{1}{1+b/l}\right) \\
\geq 0 & \text{otherwise}
\end{cases}
\]

and:

\[
\frac{d\theta^*_2}{d\gamma} \begin{cases}
> 0 & \text{if } \theta^*_2 < \mu_2(\rho, \theta_1) + \frac{1}{\sqrt{\alpha_2(\rho)+\gamma}} \Phi^{-1}\left(\frac{1}{1+b/l}\right) \\
\leq 0 & \text{otherwise}
\end{cases}
\]

If \( b \leq l \), then a prior belief that fundamentals are relatively strong, i.e. \( \theta^*_2(\rho, \theta_1) < \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\} \), implies that \( \frac{d\theta^*_2}{d\alpha} < 0 \) and \( \frac{d\theta^*_2}{d\gamma} > 0 \). If \( b > l \), then a prior belief that fundamentals are relatively weak, i.e. \( \theta^*_2(\rho, \theta_1) > \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\} \), implies that \( \frac{d\theta^*_2}{d\alpha} > 0 \) and \( \frac{d\theta^*_2}{d\gamma} < 0 \).

Instead, if \( b > l \), then \( \theta^*_2(\rho, \theta_1) < \mu_2(\rho, \theta_1) \forall \rho \in \{0, \rho_H\} \) does not necessarily imply that \( \frac{d\theta^*_2}{d\alpha} < 0 \) and \( \frac{d\theta^*_2}{d\gamma} > 0 \). In other words, the inequalities involving \( X(\rho) \) in Definition \( \text{I} \) are no longer sufficient if \( b > l \). However, Definition \( \text{II} \) provides a more restrictive definition of a strong (weak) prior about fundamentals by imposing additional conditions involving \( Y(\rho) \), which assure that a strong (weak) prior belief is associated with a low (high) incidence of attacks below (above) 50%.

In this way, Definition \( \text{I} \) also ensures that a strong prior belief implies that \( \frac{d\theta^*_2}{d\alpha} < 0 \) and \( \frac{d\theta^*_2}{d\gamma} > 0 \) even if \( b > l \). Similarly, Definition \( \text{II} \) ensures that a weak prior belief implies that \( \frac{d\theta^*_2}{d\alpha} > 0 \) and \( \frac{d\theta^*_2}{d\gamma} < 0 \) even if \( b \leq l \).

C.2.2 Comparative statics 2: Ordering of the equilibrium thresholds

The aim of this section is to shed light on the interplay between the mean effect and the variance effect, which crucially influences the ordering of equilibrium thresholds \( \theta^*_2(0, \theta_1) \) and \( \theta^*_2(\rho_H, \theta_1) \).

Here, our focus is exclusively on the ordering of equilibrium thresholds and not on the ordering of likelihoods of successful currency attacks.

\footnote{As mentioned in the main text, there is no one-to-one mapping between the ordering of equilibrium thresholds and the ordering of likelihoods of a crisis since the realization of the cross-country correlation affects the conditional}
Prior belief | Effect of an increase in $\rho$ on $\theta^*_{2f}(\rho, \theta_1)$ | Ordering of thresholds
--- | --- | ---
Mean effect | $\frac{d\theta^*_{2f}(\rho, \theta_1)}{d\mu_2}$ | $\rho_H > 0$
Variance effect | $\frac{d\theta^*_{2f}(\rho, \theta_1)}{d\alpha_2}$ | $\rho_H < 0$

| strong | $> 0$ | $\theta^*_{2f}(\rho_H, \theta_1) < \theta^*_{2f}(0, \theta_1)$ if VE > ME |
| weak | $\forall \rho \in (-1, 1)$ | $\theta^*_{2f}(\rho_H, \theta_1) > \theta^*_{2f}(0, \theta_1)$ if VE > ME |

Table 2: Effect of an increase in $\rho$ on $\theta^*_{2f}(\rho, \theta_1)$ and equilibrium threshold ordering given $\theta_1 < \theta^*_1 \leq \mu$.

One of the first papers that examine the dependence of equilibrium thresholds on the precision of the private signal $\gamma$ and the public signal $\alpha$ was Metz (2002). An inspection of equation (10) for the special case $b = l = \frac{1}{2}$ reveals that the equilibrium threshold $\theta^*_{2f}(0, \theta_1)$ increases (decreases) in the precision of the private signal $\gamma$ when the prior belief is that fundamentals are strong (weak). This result is consistent with the findings of Rochet and Vives (2004). A related result is that the above relationship is opposite when considering a change in the precision of the public signal $\alpha$.

Table 2 summarizes the effects of an increase in the cross-country correlation $\rho$ if $\theta_1 < \theta^*_1$. Both the mean $\mu_2(\rho, \theta_1)$ and the precision $\alpha_2(\rho)$ of the updated prior about the fundamental in country 2 are affected by a change in $\rho$. The effect of an increase in $\rho$ on $\theta^*_{2f}(\rho, \theta_1)$ and its impact on the ordering of equilibrium thresholds depends on the prior about the fundamentals. Therefore, the cases where the mean effect (ME) and the variance effect (VE) go in opposite directions are emphasized in bold. If the cross-country correlation is potentially positive, i.e. $\rho_H > 0$, this requires a strong prior about the fundamental.

To understand the mechanics behind the results presented in table 2 recall that $\frac{d\alpha_2(\rho)}{d\rho} > 0$. As a result, the precision of the public signal is lowest in the state where there is no correlation ($\alpha < \alpha_2(\rho_H)$). Consequently, the variance effect tends to decrease (increase) $\theta^*_{2f}(\rho, \theta_1)$ if the prior belief is that fundamentals are strong (weak). For a prior belief that fundamentals are strong, there is a clear tension between the mean and the variance effect if $\rho_H > 0$, which plays a crucial role in Lemma 2. Furthermore, this tension vanishes if $\theta_1 \geq \theta^*_1$ since the mean and variance effects go in the same direction. This last result is used in the proof of Proposition 3.
C.3 Proof of Lemma 2

If private information is sufficiently precise, then $F_2(\theta^*_2, \rho)$ decreases in $\theta^*_2$ for a given $\rho$. Hence, equation (10) implies that $\theta^*_2(0, \theta_1) > \theta^*_2(\rho_H, \theta_1)$ if $F_2(\theta^*_2(0, \theta_1), 0) > (\theta^*_2(0, \theta_1), \rho_H)$, where $\alpha_2(0) = \alpha$ and $\mu_2(0, \theta_1) = \mu$:

$$\frac{\alpha}{\sqrt{\alpha + \gamma}} [\theta^*_2(0, \theta_1) - \mu] - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta^*_2(0, \theta_1)) > 0$$

Solving this inequality for $\theta_1$, which is implicit in $\mu_2(\rho_H, \theta_1)$, results in condition (12).

Next, $\theta_1 < \mu$ arises because $\theta^*_2 < \mu$, $[1 - \frac{\alpha}{\alpha_2(\rho_H)} \sqrt{\frac{\alpha_2(\rho_H) + \gamma}{\alpha_2 + \gamma}}] > 0$, and $[\sqrt{\frac{\alpha_2(\rho_H) + \gamma}{\alpha_2 + \gamma}} - 1] > 0$. Finally, $\Phi^{-1}(\theta^*_2(0, \theta_1)) < 0$ if $\mu_2(\rho, \theta_1) < Y(\rho) \forall \rho \in \{0, \rho_H\}$. Therefore, $\theta_1 \in [\theta_1, \mu]$ is non-empty and the inequality in equation (12) follows.

We now present a condition to ensure $\theta_1 \in [\theta^*_1, \mu]$ is non-empty. This requires intermediate values of $\theta^*_1$. From equation (10), $\theta_1 < \theta^*_1$ if:

$$\frac{\alpha}{\sqrt{\alpha + \gamma}} (\theta_1 - \mu) - \sqrt{\frac{\gamma}{\alpha + \gamma}} \Phi^{-1}(\theta_1) > \Phi^{-1}\left(\frac{1}{1 + b/l}\right).$$

which can be reformulated to equation (13).

Finally, we must verify that the constraint in equation (13) is consistent with Definition 1, which requires:

$$\mu > \Phi\left(-\sqrt{\frac{\alpha + \gamma}{\gamma}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right)\right)$$

$$\mu > \frac{1}{2} - \sqrt{\frac{\alpha + \gamma}{\alpha}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right).$$

First, inequality (45) is more restrictive than inequality (44) if $-\sqrt{\frac{\alpha + \gamma}{\gamma}} \Phi^{-1}\left(\frac{1}{1 + b/l}\right) > 0$ or $b > l$. A combination of inequality (45) and inequality (13) leads to:

\[41\text{If we were to use just } X, \text{ and not also } Y, \text{ in the definition of the bounds that characterize a weak or a strong prior, then } [\theta_1, \mu] \text{ may be empty under some parameter values.}\]
\[ \Phi(\frac{\gamma - \alpha \Phi^{-1}(\theta_1) - \sqrt{\alpha + \gamma \Phi^{-1}}}{\alpha - \sqrt{\alpha + \gamma \Phi^{-1}}}, \frac{1}{1 + b/l}) > \frac{1}{2} - \frac{\sqrt{\alpha + \gamma \Phi^{-1}}}{\alpha - \sqrt{\alpha + \gamma \Phi^{-1}}}, \] (46)

The above inequality is always satisfied since the left-hand side is decreasing in \( \theta_1 \) given our assumption on \( \gamma \) for uniqueness. Furthermore, we have already established that \( \theta_1 < 0.5 \) because of \( \mu > Y(0) \) in Definition 1.

Second, inequality (44) is more restrictive than inequality (45) if \( b < l \). A combination of inequality (44) and inequality (13) leads to the condition in equation (14), which holds if \( b/l \) is sufficiently high. (q.e.d.)

C.4 Uninformed speculators

C.4.1 Bayesian updating

The relationship between the posterior probability of zero cross-country correlation \( \hat{\rho} \) and the private signal \( x_{i2} \) is non-monotone. Here we show that \( \frac{d\hat{\rho}}{dx_{i2}} > 0 \) if the private signal is relatively high. Intuitively, a speculator places more weight on the probability of zero cross-country correlation after receiving a relatively good private signal about the fundamental in country 2. Instead, after a low private signal, \( \frac{d\hat{\rho}}{dx_{i2}} > 0 \) is not guaranteed. For extremely low signals, an even worse signal makes an uninformed speculator infer that \( \rho = 0 \) is more likely.

Uninformed speculators use Bayes’ rule to form a posterior belief about the probability of uncorrelated fundamentals:

\[ \hat{\rho} \equiv \Pr\{\rho = 0|\theta_1, x_{i2}\} = \frac{p \Pr\{x_{i2}|\theta_1, \rho = 0\}}{p \Pr\{x_{i2}|\theta_1, \rho = 0\} + (1 - p) \Pr\{x_{i2}|\theta_1, \rho = \rho_H\}, \] (47)

We now compute \( \Pr\{x_{i2}|\theta_1, \rho\} \) for both realizations of \( \rho \). Since the variance terms are unconditional
on $\theta_1$, we find the sum of $\text{Var}[\epsilon_2]$ and $\text{Var}[\theta_2]$, which is $\frac{1}{\alpha}$ and $\frac{1-\rho^2_H}{\alpha}$. Therefore:

$$\Pr\{x_{i2}|\theta_1, \rho = 0\} = \frac{1}{\sqrt{\text{Var}[x_{i2}|\rho = 0]}} \phi \left( \frac{x_{i2} - \text{E}[x_{i2}|\theta_1, \rho = 0]}{\sqrt{\text{Var}[x_{i2}|\rho = 0]}} \right) = \left( \frac{1}{\alpha} + \frac{1}{\gamma} \right)^{-\frac{1}{2}} \phi \left( \frac{x_{i2} - \mu}{\sqrt{\frac{1}{\alpha} + \frac{1}{\gamma}}} \right)$$

(48)

$$\Pr\{x_{i2}|\theta_1, \rho = \rho_H\} = \frac{1}{\sqrt{\text{Var}[x_{i2}|\rho = \rho_H]}} \phi \left( \frac{x_{i2} - \text{E}[x_{i2}|\theta_1, \rho = \rho_H]}{\sqrt{\text{Var}[x_{i2}|\rho = \rho_H]}} \right) = \left( \frac{1-\rho_H^2}{\alpha} + \frac{1}{\gamma} \right)^{-\frac{1}{2}} \phi \left( \frac{x_{i2} - \text{E}[x_{i2}|\theta_1, \rho = \rho_H]}{\sqrt{\frac{1-\rho_H^2}{\alpha} + \frac{1}{\gamma}}} \right).$$

(49)

Since we maintain $\rho_H > 0$ throughout this paper (with the exception of Appendix section D), we find the following derivatives for the posterior belief about the no-correlation probability $\hat{p}$.

$$\frac{d\hat{p}}{d\theta_1} \begin{cases} 
\geq 0 & \text{if } x_{i2} \leq \rho_H \theta_1 + (1-\rho_H)\mu \\
< 0 & \text{otherwise.}
\end{cases}$$

(50)

If the private signal about the fundamental in country 2, $x_{i2}$, is sufficiently low, an increase in the fundamental of country 1, $\theta_1$, leads uninformed speculators to put a larger probability on uncorrelated fundamentals across countries.

How does $\hat{p}$ vary with the private signal $x_{i2}$? We find that:

$$\frac{d\hat{p}}{dx_{i2}} \begin{cases} 
> 0 & \text{if } \rho_H > 0 \text{ and } x_{i2} \geq \rho_H \theta_1 + (1-\rho_H)\mu \\
< 0 & \text{if } \rho_H < 0 \text{ and } x_{i2} \leq \rho_H \theta_1 + (1-\rho_H)\mu \\
\leq 0 & \text{otherwise.}
\end{cases}$$

(51)

Speculators in country 2 observed a currency crisis in country 1, that is, $\theta_1 \leq \theta_1^* < \mu$, but are uninformed about the cross-country correlation between fundamentals which potentially is positive. Therefore, after receiving a relatively good private signal, that is, $x_{i2} \geq \rho_H \theta_1 + (1-\rho_H)\mu$, a speculative places more weight on the probability of zero cross-country correlation.

However, the relationship between $\hat{p}$ and $x_{i2}$ is non-monotone after an uninformed speculator

---

42The signs of the derivative are reversed for $\rho_H < 0$. 

44
observes a relatively bad private signal, that is, 

\[ x_i^2 < \rho H \theta_1 + (1 - \rho_H) \mu. \]

If the private signal takes an intermediate value, \( \frac{d\hat{p}}{dx_i^2} > 0 \) still holds. Instead, if the private signal is very low, then \( \frac{d\hat{p}}{dx_i^2} \leq 0 \) due to the more dispersed prior distribution if \( \rho = 0 \). For the same reason, an extremely high or low private signal induces uninformed speculators to believe that fundamentals are uncorrelated across regions, i.e. \( \lim_{x_i^2 \to +\infty} \hat{p} = 1 = \lim_{x_i^2 \to -\infty} \hat{p}. \)

C.4.2 Equilibrium analysis

Two equilibrium conditions have to be satisfied. First, \( A_{2U}^* = \theta_{2U}^* \), which leads to the first equilibrium condition given by equation (15). Second, a speculator with the threshold signal \( x_{2U}^* \) is indifferent between attacking the currency or not, given \( \theta_{2U}^* \):

\[
\begin{align*}
    b \Pr\{\theta_2 \leq \theta_{2U}^* | \theta_1, x_{2U}^*\} - t \Pr\{\theta_2 > \theta_{2U}^* | \theta_1, x_{2U}^*\} &= 0 \\
    \end{align*}
\]

(52)

where:

\[
\Pr\{\theta_2 \leq \theta_{2U}^* | \theta_1, x_{2U}^*\} = \hat{p}(\theta_{2U}^*)\Phi(\rho = 0, \theta_{2U}^*) + (1 - \hat{p}(\theta_{2U}^*))\Phi(\rho = \rho_H, \theta_{2U}^*). \\
\]

(53)

The indifference condition is a mixture between the indifference conditions for the informed speculators that observe no correlation, \( n = 1 \) and \( \rho = 0 \), and the informed speculators that observe positive correlation, \( n = 1 \) and \( \rho = \rho_H \). Combining equations (15) and (52) leads to the equilibrium condition stated in equation (16).

C.4.3 Monotonicity

In contrast to the previous standard analysis, \( G(\theta_{2U}^*, \theta_1) \) is harder to characterize since the weights of the mixture and the posterior beliefs about the probability of cross-country correlation now depend on the threshold signal \( x_{2U}^* \). Therefore, the question arises as to whether or not our focus on monotone equilibria is justified despite the global non-monotonicity of \( \hat{p}(x_{2U}^*(\theta_{2U}^*)) \) in \( x_{2U}^* \) and, hence, \( \theta_{2U}^* \), as established above. Fortunately, the best-response function of an individual speculator
i proves to be strictly increasing in the fundamental thresholds $\theta_{2U}^*$ used by other speculators:

$$ r' = -\frac{d \Pr(\theta_2 < \hat{\theta}_{2U}(\hat{x}_2) | \theta_1, x_{i2})}{d \hat{x}_2} \frac{d \Pr(\theta_2 < \hat{\theta}_{2U}(\hat{x}_2) | \theta_1, \hat{x}_2)}{d \hat{x}_2} > 0, $$

(54)

where $\hat{x}_{i2}$ is the critical threshold of the private signal used by player i, $\hat{x}_2$ is the threshold used by all other speculators, and $\hat{\theta}_{2U}(\hat{x}_2)$ is the critical threshold of the fundamental in country 2. This is because $\Pr(\theta_2 < \theta_{2U}^* | \theta_1, x_{i2})$ is monotonically decreasing in $x_{i2}$ using a result of Milgrom (1981) (see below). Furthermore, given all other speculators use a threshold strategy, $\Pr(\theta_2 < \hat{\theta}_{2U}(\hat{x}_2) | \theta_1, x_{i2})$ increases in $\hat{x}_2$. Following Vives (2005), the best response of player i is to use a threshold strategy with attack threshold $\hat{x}_{i2}$, where $\Pr(\theta_2 < \hat{\theta}_{2U}(\hat{x}_2) | \theta_1, \hat{x}_{i2}) = \frac{1}{1 + \frac{\phi(\sqrt{\gamma}(x_1 - \theta))}{\phi(\sqrt{\gamma}(x_j - \theta))}}$, implying $r' > 0$. Therefore, our focus on monotone equilibria is valid and we now determine conditions sufficient for a unique monotone Bayesian equilibrium.

The conditional density function $f(x | \theta)$ is normal with mean $\theta$ and satisfies the monotone likelihood ratio property (MLRP). That is, for all $x_i > x_j$ and $\theta' > \theta$, we have:

$$ \frac{f(x_i | \theta')}{f(x_j | \theta)} \geq \frac{f(x_j | \theta')}{f(x_j | \theta)} \Leftrightarrow \frac{\phi(\sqrt{\gamma}(x_i - \theta'))}{\phi(\sqrt{\gamma}(x_j - \theta'))} \geq \frac{\phi(\sqrt{\gamma}(x_i - \theta))}{\phi(\sqrt{\gamma}(x_j - \theta))}. $$

(55)

Using Proposition 1 of Milgrom (1981), we conclude that $\Pr(\theta_2 < \theta_{2U}^* | \theta_1, x_{i2})$ monotonically decreases in $x_{i2}$. Furthermore, notice that $\frac{d \Pr(\theta_2 < \theta_{2U}^* | \theta_1, \hat{x}_2)}{d \hat{x}_2} > 0$. From equation (15) we can derive:

$$ 0 \leq \frac{d \hat{\theta}_2(\hat{x}_2)}{d \hat{x}_2} \leq \frac{1}{1 + \sqrt{\frac{\gamma}{\theta}}} $$

(56)

C.5 Proof of Proposition 1

The result in Proposition 1 is proven in three steps. First, we show that $G(\theta_{2U}, \theta_1) \rightarrow 1 > \frac{1}{1 + b/l}$ as $\theta_{2U} \rightarrow 0$, as well as $G(\theta_{2U}, \theta_1) \rightarrow 0 < \frac{1}{1 + b/l}$ as $\theta_{2U} \rightarrow 1$. Second, we show that $\frac{d G(\theta_{2U}, \theta_1)}{d \theta_{2U}} < 0$ for some sufficiently high but finite values of $\gamma$, such that $G$ strictly decreases in $\theta_{2U}$. Therefore, if $\theta_{2U}^*$ exists, it is unique. Third, by continuity, there exists a $\theta_{2U}^*$ that solves $G(\theta_{2U}, \theta_1) = \frac{1}{1 + b/l}$.

**Step 1: limiting behavior** Observe that $G(\theta_{2U}, \theta_1)$ is a weighted average of $F_2(\theta_{2U}^*(\theta_1), 0)$ and $F_2(\theta_{2U}^*(\theta_1), \rho_H)$. As $\theta_{2U} \rightarrow 0$, then $F_2(\theta_{2U}^*(\theta_1), \rho) \rightarrow 1$ for any $\rho \in \{0, \rho_H\}$, so $G(\theta_{2U}, \theta_1) \rightarrow 1 >
Likewise, as \( \theta_{2U} \to 1 \), then \( F_2(\theta_{2U}^*(\theta_1), \rho) \to 0 \) for any \( \rho \in \{0, \rho_H\} \), so \( G(\theta_{2U}, \theta_1) \to 0 < \frac{1}{1+b/l} \).

**Step 2: strictly negative slope** Using the indifference condition of uninformed speculators \( x_{2U}^\ast \) to substitute \( x_{2U}^\ast \), we arrive at equation \((15)\). Thus, the total derivative of \( G \) is:

\[
\frac{dG(\theta_{2U}, \theta_1)}{d\theta_{2U}} = \hat{p}(\theta_{2U}) \frac{dF_2(\theta_{2U}^*(\theta_1), 0)}{d\theta_{2U}} + (1 - \hat{p}(\theta_{2U})) \frac{dF_2(\theta_{2U}^*(\theta_1), \rho_H)}{d\theta_{2U}} + \frac{d\hat{p}(\theta_1, x_{2U}(\theta_{2U}))}{dx_{2U}} \frac{dx_{2U}(\theta_{2U})}{d\theta_{2U}} \left[ F_2(\theta_{2U}^*(\theta_1), 0) - F_2(\theta_{2U}^*(\theta_1), \rho_H) \right].
\]

(57)

The proof proceeds by inspecting the individual terms of equation \((57)\). We know from our analysis of the case of informed speculators that \( \frac{dF_2(\theta_{2U}^*(\theta_1), 0)}{d\theta_{2U}} < 0 \) if \( \gamma > \gamma_0 \) and that \( \frac{dF_2(\theta_{2U}^*(\theta_1), \rho_H)}{d\theta_{2U}} < 0 \) if \( \gamma > \gamma_1 \). Moreover, these derivatives are also strictly negative in the limit when \( \gamma \to \infty \). Thus, the first two components of the sum are negative and finite in the limit of vanishing private noise. By continuity, these terms are also negative for a sufficiently high but finite private noise by continuity.

The sign of the third summand in equation \((57)\) is ambiguous: \( F_2(\theta_{2U}^*(\theta_1), 0) \leq F_2(\theta_{2U}^*(\theta_1), \rho_H) \) whenever \( \theta_{2U}^*(0) \leq \theta_{2U}(\rho_H) \) and \( F_2(\theta_{2U}^*(\theta_1), 0) > F_2(\theta_{2U}^*(\theta_1), \rho_H) \) otherwise. However, the difference vanishes in the limit when \( \gamma \to \infty \).

The last term to consider is \( \frac{d\hat{p}(\theta_1, x_{2U}(\theta_{2U}))}{dx_{2U}(\theta_{2U})} \frac{dx_{2U}(\theta_{2U})}{d\theta_{2U}} \). Given the previous sufficient conditions on the relative precision of the private signal:

\[
0 < \frac{dx_{2U}}{d\theta_{2U}} = 1 + \frac{1}{\sqrt{\gamma \hat{p}(\Phi^{-1}(\theta_{2U}))}} < 1 + \frac{\sqrt{2\pi}}{\alpha}.
\]

Finally, from section \((C.4.1)\) we know that the sign of \( \frac{d\hat{p}}{dx_{2U}} \) is ambiguous. However, the derivative is finite for \( \gamma \to \infty \). Taken together with the zero limit of the first factor of the third term, this term vanishes in the limit.

As a result, by continuity, there must exist a finite level of precision \( \gamma > \gamma_2 \in (0, \infty) \) such that \( \frac{dG(\theta_{2U}, \theta_1)}{d\theta_{2U}} < 0 \) for all \( \gamma > \gamma_2 \). This concludes the second step of the proof and therefore the overall proof of Proposition \((q.e.d.)\)
C.6 Proof of Proposition 2

The proof has five steps. First, $\gamma > \max\{\gamma_1, \gamma_2\} < \infty$ meets the sufficient conditions of Corollary 2 and Proposition 1, so $\theta_*^{2I}(\rho, \theta_1)$ and $\theta_*^{2U}(\theta_1)$ are unique. Second, we have the threshold ranking $\theta_*^{2I}(0, \theta_1) > \theta_*^{2I}(\rho_H, \theta_1)$ under the sufficient conditions of Lemma 2, i.e., an intermediate realized fundamental in country 1, $\theta_1 \in (\bar{\theta}_1, \theta_*^1)$, and a strong prior about the fundamental in country 2 according to Definition 1. Third, using the result of the weighted average of Proposition 1 and noting that the weight satisfies $\hat{p} \in (0, 1)$, it follows directly that $\min\{\theta_*^{2I}(0, \theta_1), \theta_*^{2I}(\rho_H, \theta_1)\} < \theta_*^{2U}(\theta_1) < \max\{\theta_*^{2I}(0, \theta_1), \theta_*^{2I}(\rho_H, \theta_1)\}$. Combined with the second point, we have the following ranking of thresholds: $\theta_*^{2I}(0, \theta_1) > \theta_*^{2U}(\theta_1) \forall \theta_1 \in (\bar{\theta}_1, \theta_*^1)$. Finally, given that the realized cross-country correlation of fundamentals is $\rho = 0$, the ordering of thresholds implies that the probability of a crisis in country 2 is higher when speculators are informed than when they are uninformed. (q.e.d.)

C.7 Proof of Proposition 3

Recall that speculators observe $\theta_1$ only if there was a crisis in country 1, i.e. if $\theta_1 < \theta_*^1$. Recall our notation, where $\theta_*^{2U} \equiv \theta_*^{2U}|_{\theta_1 \geq \theta_*^1}$ denotes the equilibrium threshold of country 2 after no crisis in country 1 is observed and $\theta_*^{2U}(\theta_1) \equiv \theta_*^{2U}|_{\theta_1 < \theta_*^1}$ if a crisis in 1 is observed.

The proof is constructed in five steps. First, it is most intuitive to decompose the right-hand side of equation (20) for $E_3 \equiv \theta_2 < \theta_*^{2U}$ by the law of total probability:

$$
\Pr\{E_3|\theta_1 \geq \theta_*^1\} = p \Pr\{E_3|\rho = 0, \theta_1 \geq \theta_*^1\} + (1 - p) \Pr\{E_3|\rho = \rho_H, \theta_1 \geq \theta_*^1\}. \quad (58)
$$

Since $p \in (0, 1)$, it then suffices to show the following two conditions: (i) $\Pr\{\theta_2 < \theta_*^{2I}(0)|\rho = 0, \theta_1 < \theta_*^1\} > \Pr\{\theta_2 < \theta_*^{2U}|\rho = 0, \theta_1 \geq \theta_*^1\}$; and (ii) $\Pr\{\theta_2 < \theta_*^{2I}(0)|\rho = 0, \theta_1 < \theta_*^1\} > \Pr\{\theta_2 < \theta_*^{2U}|\rho = \rho_H, \theta_1 \geq \theta_*^1\}$, which we do below. In other words, we construct sufficient conditions without resorting to the ex-ante probability of positive cross-country correlation.

Inequality (i) Second, since the sufficient conditions of Corollary 2 and Proposition 1 are met (sufficiently precise private information), $\theta_*^{2U}(\theta_1)$ and $\theta_*^{2I}(0, \theta_1)$ are unique. Furthermore, since
the sufficient conditions of Lemma 2 are met (intermediate realized fundamental in country 1 and strong prior about the fundamental in country 2), $\theta_{2U}^*(\theta_1) < \theta_{2I}^*(0, \theta_1)$ for all $\theta_1 \in (\underline{\theta}_1, \overline{\theta}_1)$, where the interval is non-empty given the sufficient conditions in Lemma 2. As a result, the true distribution is the same, while the thresholds are ranked as stated. Therefore, the stated inequality (i) follows directly.

**Inequality (ii)** Third, following an argument similar to the proof of Proposition 1, it can be shown that $\theta_{2U}^*$ is unique, which is sketched here. Absent a crisis in country 1, the realization of $\theta_1$ is unobserved, so the indifference condition of speculators integrates over all possible $\theta_1 \geq \theta_1^*$ weighted by an updated distribution of $\theta_1$ that uses the private signal, $f(\theta_1|x_2^*)$:

\[
\begin{align*}
\Pr\{\theta_1|\rho = 0, x_2^*(\theta_{2U}^*)\} &= \sqrt{\alpha} \phi\left(\sqrt{\alpha}(\theta_1 - \mu)\right) \\
\Pr\{\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)\} &= \frac{1}{\sqrt{\text{Var}[\theta_1|\rho_H, x_2^*(\theta_{2U}^*)]}} \phi\left(\frac{\theta_1 - E[\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)]}{\sqrt{\text{Var}[\theta_1|\rho_H, x_2^*(\theta_{2U}^*)]}}\right) \\
&= \left(1 + \frac{1 - \rho_H^2}{\gamma}\right)^{-\frac{1}{2}} \phi\left(\frac{\theta_1 - [\rho_H x_2^*_{2U} + (1 - \rho_H)\mu]}{\sqrt{\frac{1}{\alpha} + \frac{1 - \rho_H^2}{\gamma}}}\right).
\end{align*}
\]

Thus $\theta_{2U}^*$ solves:

\[
\begin{align*}
&\int_{\theta_1^*}^{\infty} \left(\hat{p}(\theta_{2U}^*) \Pr\{\theta_1|\rho = 0, x_2^*(\theta_{2U}^*)\} F_2(\theta_{2U}^*, 0) + (1 - \hat{p}(\theta_{2U}^*)) \Pr\{\theta_1|\rho = \rho_H, x_2^*(\theta_{2U}^*)\} F_2(\theta_{2U}^*, \rho_H)\right) f(\theta_1|x_2^*(\theta_{2U}^*)) d\theta_1 \\
&= \frac{1}{1 + b/l},
\end{align*}
\]

Following the steps in the proof of Proposition 1 one can show that there exists a unique $\theta_{2U}^*$ that solves the above system if $\gamma$ is sufficiently high. The integration over $\theta_1$, which effectively constructs a weighted average, does not alter the analysis qualitatively.

Fourth, how do the thresholds compare? We want to show that $\theta_{2U}^* < \theta_{2I}^*(0)$ if $\theta_1 \geq \theta_1^*$, which is sketched here. For each realization of $\theta_1 > \theta_1^*$, the $\theta_{2U}^*$ that solves

\[
\hat{p}(\theta_2) F_2(\theta_{2U}^*, 0) + (1 - \hat{p}(\theta_2)) F_2(\theta_{2U}^*, \rho_H) = \frac{1}{1 + b/l},
\]
is smaller than \( \theta^*_2(0) \) since \( \theta^*_{2U}(\rho_H, \theta_1) < \theta^*_{2I}(0) \ \forall \ \theta_1 > \theta^*_1 \) by a generalization of Lemma 2 that allows for \( \theta_1 > \theta^*_1 \) (see also comparative statics in Appendix C.2.2). As a result, the integration over all different possible realizations of \( \theta_1 \), which is \( \theta_1 \geq \theta^*_1 \), must yield \( \theta^*_{2U} < \theta^*_{2I}(0) \). Thus, we have established the same ranking of equilibrium thresholds as for inequality (i). However, the true distribution differs between the left-hand and right-hand sides of inequality (ii), which we discuss below.

Fifth, observe that \( \theta_2 \) is drawn from a less favorable distribution whenever \( \rho = \rho_H \) and \( \theta_1 \in [\theta^*_1, \mu] \). Likewise, the distribution is more favorable if \( \rho = \rho_H \) and \( \theta_1 > \mu \) holds. The argument below relies on cancelling out the part \( \theta_1 \in [\theta^*_1, \mu] \), which works against inequality (ii), with \( \theta_1 \in [\mu, \mu + (\mu - \theta^*_1)] \), which works for it. This will allow us to provide a condition sufficient for inequality (ii), where \( \theta^*_{2U} \) is replaced by \( \theta^*_{2I}(0) \).

Because of the threshold ranking \( \theta^*_{2U} < \theta^*_{2I}(0) \) (step 4), the following condition is sufficient for inequality (ii):

\[
\Pr\{\theta_2 < \theta^*_{2I}(0) | \rho = 0, \theta_1 < \theta^*_1 \} \geq \Pr\{\theta_2 < \theta^*_{2I}(0) | \rho = \rho_H, \theta_1 \geq \theta^*_1 \}. \tag{63}
\]

Since the crisis probability in country 2 is low for a high fundamental in country 1, \( \theta_1 \geq 2\mu - \theta^*_1 \), because of positive cross-country correlation, \( \rho = \rho_H \), we have yet another sufficient condition:

\[
\Pr\{\theta_2 < \theta^*_{2I}(0) | \rho = 0, \theta_1 < \theta^*_1 \} \geq \Pr\{\theta_2 < \theta^*_{2I}(0) | \rho = \rho_H, \theta^*_1 \leq \theta_1 \leq 2\mu - \theta^*_1 \}. \tag{64}
\]

The idea of this reformulation is to generate a condition, \( \theta^*_1 \leq \theta_1 \leq 2\mu - \theta^*_1 \), that is symmetric around \( \mu \), and focusing on \( \theta^*_1 \) (which is fine, given that \( \theta^*_1 \geq \theta^*_1 \)). This allows us to disregard the ex-ante distribution of \( \theta_1 \) in the subsequent proof. More precisely, for any \( \theta^*_1 \in [\theta^*_1, \mu] \), there exists a \( \theta^*_1 \equiv 2\mu - \theta^*_1 \in [\mu, 2\mu - \theta^*_1] \) such that both of these values are equally likely, \( \phi(\sqrt{\alpha}[\theta^*_1 - \mu]) = \phi(\sqrt{\alpha}[\theta^*_1 - \mu]) \). Also, the left-hand side is \( \Pr\{\theta_2 < \theta^*_{2I}(0) | \rho = 0, \theta_1 < \theta^*_1 \} = \phi(\sqrt{\alpha}[\theta_2 - \mu]) \).

Then, inequality (64) must hold if for any pair \( (\theta^*_1, \theta^*_1) \):

\[
\phi\left(\sqrt{\alpha^2(\rho_H)}[\theta_2 - \mu_2(\rho_H, \theta^*_1)]\right) + \phi\left(\sqrt{\alpha^2(\rho_H)}[\theta_2 - \mu_2(\rho_H, \theta^*_1)]\right) \leq \phi\left(\sqrt{\alpha}[\theta_2 - \mu]\right) \forall (\theta^*_1, \theta^*_1). \tag{65}
\]
By construction, $\theta_i^+$ and $\theta_i^-$ are equidistant to $\mu$, so the above inequality holds if $m(\theta_1)$ strictly decreases and is weakly convex, where:

$$m(\theta_1) \equiv \sqrt{\frac{\alpha^2(\rho_H)}{2\pi}} \exp\{-\left(\frac{\alpha^2(\rho_H)}{2} (\theta_2^+(0) - [\rho_H\theta_1 + (1 - \rho_H)\mu])^2\right)\}. \quad (66)$$

By the Lemma 2, $m'(\theta_1) < 0$ for all $\theta_1 \in (\theta_*^+; 2\mu - \theta_*^-)$. Furthermore, $m''(\theta_1) \geq 0$ for all $\theta_1^- \in [\theta_*^+, \mu]$ if the inequality in equation (19) of Proposition 3 holds. This is because $\theta_*^+ \geq \underline{\theta}_1$, which completes the proof. (q.e.d.)

C.8 Derivations for the coordination stage in country 2

Recall that $\theta_1$ is observed by speculators in country 2 if and only if there is a currency crisis in country 1, $\theta_1 < \theta_*^+$. Furthermore, a fraction $n$ is informed about the cross-country correlation $\rho$, where $n$ is chosen optimally at the information stage. As before, the equilibrium conditions comprise an indifference condition and a critical mass condition. First, the equilibrium proportion of attacking speculators $A_2^*$ equals the fundamental threshold $\theta_*^2(\rho)$, where we use the short-hand $\theta_*^2(\rho) \equiv \theta_*^2(n, \rho, \theta_1)$. Another short-hand notation is $x_*^{2I}(\rho) \equiv x_*^{2I}(n, \rho, \theta_1)$ and $x_*^{2U} \equiv x_*^{2U}(n, \theta_1)$ for the attacking thresholds of informed and uninformed speculators, respectively.

First, for all $\rho \in \{0, \rho_H\}$ the critical mass condition is:

$$\theta_*^2(\rho) = n\Phi\left(\sqrt{\gamma}[x_*^{2I}(\rho) - \theta_*^2(\rho)]\right) + (1 - n)\Phi\left(\sqrt{\gamma}[x_*^{2U} - \theta_*^2(\rho)]\right). \quad (67)$$

Second, in contrast to the cases of $n \in \{0, 1\}$, the critical mass condition can no longer be used to express the attacking threshold as a function of the fundamental threshold. In these polar cases, we inserted this relationship in the indifference condition, which then simplified to the function $F_1$, $F_2$, and $G$ that fully characterized the equilibrium threshold $\theta^*$. By contrast, the interaction between informed and uninformed speculators implies that the attacking threshold of uninformed investors, $x_*^{2U}$, cannot be separated. Therefore, we define the following useful short-hand $\Psi$ to
characterize the indifference conditions:

\[ \Psi(\theta^*_2, x^*_{2d}, \rho) \equiv \Phi \left( \theta^*_2 \sqrt{\frac{\alpha^2(\rho) + \gamma}{\alpha^2(\rho) + \gamma}} \right) \] (68)

for \( d \in \{I, U\} \) and \( \rho \in \{0, \rho_H\} \). Therefore, the indifference condition for an uninformed speculator is:

\[ J(n, \theta^*_2(0), \theta^*_2(\rho_H), x^*_{2U}) \equiv \hat{p}\Psi(\theta^*_2(0), x^*_{2U}, 0) + (1 - \hat{p})\Psi(\theta^*_2(\rho_H), x^*_{2U}, \rho_H) = \frac{1}{1 + b/l} \] (69)

where \( \hat{p} = \hat{p}(\theta_1, x^*_{2U}) \) and an indifference condition for an informed speculator for each realization of the cross-country correlation:

\[ \Psi(\theta^*_2(\rho), x^*_{2I}(\rho), \rho) = \frac{1}{1 + b/l} \quad \forall \rho \in \{0, \rho_H\}. \] (70)

These are related to the previous functions. \( \Psi \) is the generalization of \( F_2 \), while \( J \) generalizes \( G \).

We have five equations in five unknowns. In the simplest case, in country 1, we had two thresholds \( x^*_1 \) and \( \theta^*_1 \). There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states \( x^*_1 \) in terms of \( \theta^*_1 \), into indifference condition. This yields one equation implicit in \( \theta^*_1 \). We pursue a modified strategy here, solving this system of equations in order to express the equilibrium in terms of \( \theta^*_2(0) \) and \( \theta^*_2(\rho_H) \) only.

We also use the following insight. Since uninformed speculators do not observe the realized cross-country correlation, the attacking threshold must be identical across these realizations, \( x^*_{2U}(\rho = 0) = x^*_{2U}(\rho = \rho_H) \). In the following steps, we derive this threshold for either realization of \( \rho \) by using the fundamental threshold \( \theta^*_2(\rho) \) and equalize both expressions.

First, we use the critical mass condition in equation (67) for \( \theta^*_2(0) \) to express \( x^*_{2U} \) as a function of \( \theta^*_2(0) \) and \( x^*_{2I}(0) \). Second, we use the indifference condition of informed speculators in case of \( \rho = 0 \), equation (70), to obtain \( x^*_{2I}(0) \) as a function of \( \theta^*_2(0) \). Third, we use the critical mass condition in equation (67) for \( \theta^*_2(\rho_H) \) to express \( x^*_{2U} \) as a function of \( \theta^*_2(\rho_H) \) and \( x^*_{2I}(\rho_H) \). Then, we use the indifference condition of informed speculators in case of \( \rho = \rho_H \), equation (70), to obtain
$x^*_2(\rho_H)$ as a function of $\theta^*_2(\rho_H)$. Thus, we arrive at:

$$x^*_2U(\rho) = \theta^*_2(\rho) + \frac{1}{\sqrt{\gamma}} \Phi^{-1}\left(\frac{\theta^*_2(\rho) - n \Phi{\left(\frac{\alpha_2(\rho)(\theta^*_2(\rho) - \mu_2(\rho,\theta_1)) - \sqrt{\alpha_2(\rho) + \gamma \Phi^{-1}\left(\frac{1}{1+n}\right)}}{1-n}\right)}}{1-n}\right) \forall \rho.$$  

(71)

For further reference, for all $\rho \in \{0, \rho_H\}$, a sufficient condition for the partial derivatives with respect to the equilibrium thresholds to be strictly positive is $\gamma > \gamma_1$:

$$\frac{dx^*_2U(\rho)}{d\theta^*_2(\rho)} > 0.$$  

(72)

Since the attacking threshold is the same for an uninformed speculator, subtracting equation (71) evaluated at $\rho = 0$ from the same equation evaluated at $\rho = \rho_H$ must yield zero. This yields the first of two implicit relationships between $\theta^*_2(0)$ and $\theta^*_2(\rho_H)$:

$$K(n, \theta^*_2(0), \theta^*_2(\rho_H)) \equiv x^*_2U(0) - x^*_2U(\rho_H) = 0.$$  

(73)

Now, we construct the second implicit relationship between the two aggregate thresholds $\theta^*_2(0)$ and $\theta^*_2(\rho_H)$ in two steps. First, insert equation (71) evaluated at $\rho = 0$ in $\Psi(\theta^*_2(0), x^*_2U(0), 0)$ and in $\hat{p}$ as used in $J(n, \theta^*_2(0), \theta^*_2(\rho_H), x^*_2U)$. Second, insert equation (71) evaluated at $\rho = \rho_H$ in $\Psi(\theta^*_2(\rho_H), x^*_2U(\rho_H), \rho_H)$. Combining both expressions yields:

$$L(n, \theta^*_2(0), \theta^*_2(\rho_H)) \equiv J(n, \theta^*_2(0), \theta^*_2(\rho_H), x^*_2U(0), x^*_2U(\rho_H)) - \frac{1}{1+b/l} = 0.$$  

(74)

C.9 Proof of Lemma 3

This proof establishes under which conditions there exists a unique attacking rule by analyzing a system characterized by two equations, (73) and (74), in two unknowns, $\theta_2(0)$ and $\theta_2(\rho_H)$. The proof builds heavily on the description of the coordination stage in the case of potentially asymmetrically informed speculators described in Appendix C.8. Here we show existence and uniqueness of the pair $(\theta^*_2(0), \theta^*_2(\rho_H))$. Then, the attacking thresholds are uniquely backed out from $(\theta^*_2(0), \theta^*_2(\rho_H))$.

First, we analyze the relationship between $\theta_2(0)$ and $\theta_2(\rho_H)$ as governed by $K$. Using equa-
tions (73) and (72), \( \frac{\partial K}{\partial x_2(0)} > 0 \) and \( \frac{\partial K}{\partial x_2(\rho_H)} < 0 \). Hence, \( \frac{\partial \theta_2(0)}{\partial x_2(\rho_H)} > 0 \) by the implicit function theorem. (This is for a given \( n \) and \( \theta_1 \).)

Second, we analyze the relationship between \( \theta_2(0) \) and \( \theta_2(\rho_H) \) as governed by \( L \). It can be shown that \( \gamma > \gamma_1 \) is sufficient for \( \frac{\partial L}{\partial x_2(\rho_H)} < 0 \). Thus, one can show that \( \frac{dL}{d\theta_2(0)} < 0 \) holds for a sufficiently high but finite value of \( \gamma \). This is proven by generalizing the argument of the proof of Proposition 1, so \( \lim_{\gamma \to \infty} [\Psi(\theta_2^*(0), x_{2U}, 0) - \Psi(\theta_2^*(\rho_H), x_{2U}, \rho_H)] = 0 \). Hence, \( \frac{d\theta_2(0)}{d\theta_2(\rho_H)} < 0 \) in the limit. By continuity, there exists a finite precision of private information that guarantees the inequality as well.

Taken both of these points together, \( (\theta_2^*(0), \theta_2^*(\rho_H)) \) is unique if it exists. This arises from the established strict monotonicity and the opposite sign.

Third, we establish existence of \( (\theta_2^*(0), \theta_2^*(\rho_H)) \) by making two points: (i) for the highest permissible value of \( \theta_2(0) \), the value of \( \theta_2(\rho_H) \) prescribed by \( K \) is strictly larger than the value of \( \theta_2(\rho_H) \) prescribed by \( L \); and (ii) for the lowest permissible value of \( \theta_2(0) \), the value of \( \theta_2(\rho_H) \) prescribed by \( K \) is strictly smaller than the value of \( \theta_2(\rho_H) \) prescribed by \( L \).

To make these points, consider the following auxiliary step. For any \( \theta_2(\rho) \geq \theta_{21}^*(\rho) \), it can be shown that:

\[
\frac{\partial L}{\partial \theta_2(\rho)} - n\Phi\left(\frac{\alpha_2(\rho)(\theta_2(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho)^2 + \gamma \Phi^{-1}(1 - \frac{1}{1+b/l})}}{\alpha_2(\rho) + \gamma \Phi^{-1}(1 - \frac{1}{1+b/l})}\right) > 0
\]

because \( F_2(\theta_2(\rho), \rho) \leq \frac{1}{1+b/l} \) for any \( \rho \in \{0, \rho_H\} \). Note that both the previous expression and the partial derivative hold with strict inequality if \( \theta_2(\rho) > \theta_{21}^*(\rho) \).

Inspecting the inside of the inverse of the cdf, \( \Phi^{-1} \), we define the highest permissible value of \( \theta_2(\rho) \) that is labelled \( \overline{\theta}_2(\rho, n) \) for all \( \rho \):

\[
1 = \frac{\overline{\theta}_2(\rho, n) - n\Phi\left(\frac{\alpha_2(\rho)(\theta_2(\rho) - \mu_2(\rho, \theta_1)) - \sqrt{\alpha_2(\rho)^2 + \gamma \Phi^{-1}(1 - \frac{1}{1+b/l})}}{\alpha_2(\rho) + \gamma \Phi^{-1}(1 - \frac{1}{1+b/l})}\right)}{1 - n}.
\]

Hence, \( 1 \geq \overline{\theta}_2(\rho, 1) \geq \theta_{21}^*(\rho) \) \( \forall \rho \), where the first (second) inequality binds if and only if \( n = 0 \) (\( n = 1 \)).

Next, evaluate \( K \) at the highest permissible value, \( \theta_2(0) = \overline{\theta}_2(0, n) \), which yields \( \theta_2(\rho_H) = \).
$\theta_2(\rho_H, n)$. Likewise, evaluate $L$ at the highest permissible value, $\theta_2(0) = \bar{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) < \bar{\theta}_2(\rho_H, n)$. This proves point (i).

We now proceed with point (ii). Similar to before, we can define the lowest permissible value of $\theta_2(\rho)$, which is labelled $\theta_2(\rho, n)$ for all $\rho$. Now, $0 \leq \theta_2(\rho, 1) \leq \theta_2(\rho) \forall \rho$, where the first (second) inequality binds if and only if $n = 0$ ($n = 1$).

Next, evaluate $K$ at the lowest permissible value, $\theta_2(0) = \underline{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) = \underline{\theta}_2(\rho_H, n)$. Likewise, evaluate $L$ at the lowest permissible value, $\theta_2(0) = \underline{\theta}_2(0, n)$, which yields $\theta_2(\rho_H) > \underline{\theta}_2(\rho_H, n)$. This proves point (ii) and completes the proof. (q.e.d.)

C.10 Proof of Lemma 4

We prove the results of Lemma 4 in turn. A general observation is that the updated belief on the probability of positive cross-country correlation becomes degenerate: $\hat{p} \rightarrow p$ for $\alpha \rightarrow 0$. Results (A) and (B) are closely linked, so we start by proving them below.

Results (A) and (B). This proof has three steps.

Step 1: We show in the first step that both fundamental thresholds in the case of asymmetrically informed speculators lie either within these bounds or outside of them. As a consequence of $\hat{p} \rightarrow p$, condition $L(n, \theta_2^*(0), \theta_2^*(\rho_H)) = 0$ prescribes that, for any $n$, the thresholds $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$ are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all speculators are informed, $\theta_2^*(0)$ and $\theta_2^*(\rho_H)$. This is proven by contradiction. First, suppose that $\theta_2^*(\rho_H) < \theta_2^*(\rho_H)$ and $\theta_2^*(0) < \theta_2^*(0)$. This leads to a violation of $L(\cdot) = 0$ because $J(n, \theta_2^*(0), \theta_2^*(\rho_H)) > \frac{1}{1 + b/l} \forall n$ if $\alpha \rightarrow 0$. Second, suppose that $\theta_2^*(\rho_H) > \theta_2^*(\rho_H)$ and $\theta_2^*(0) > \theta_2^*(0)$. Again, this leads to a violation because $J(n, \theta_2^*(0), \theta_2^*(\rho_H)) < \frac{1}{1 + b/l} \forall n$ if $\alpha \rightarrow 0$.

Step 2: We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed speculators, $\frac{d\theta_2^*(\rho)}{dn}$ and $\frac{d\theta_2^*(\rho)}{dn}$. Applying the implicit function theorem for
Simultaneous equations, we obtain these derivatives:

\[
\frac{d\theta^*_2(n, 0, \theta_1)}{dn} = \frac{-\frac{\partial K}{\partial n} - \frac{\partial K}{\partial L(\theta_1)}}{\frac{\partial L}{\partial n} - \frac{\partial L}{\partial L(\theta_1)}} \equiv \frac{|M_1|}{|M|} \tag{77}
\]

where \(|M| \equiv \det(M)|. We also find that:

\[
\frac{d\theta^*_2(n, \rho_H, \theta_1)}{dn} = \frac{-\frac{\partial K}{\partial n} - \frac{\partial K}{\partial L(\theta_1)}}{\frac{\partial L}{\partial n} - \frac{\partial L}{\partial L(\theta_1)}} \equiv \frac{|M_2|}{|M|}. \tag{78}
\]

To find \(|M|\), recall from the proof of Lemma \(3\) that \(\frac{\partial K}{\partial \theta_2(0)} > 0\) and \(\frac{\partial K}{\partial \theta_2(\rho_H)} < 0\). Furthermore, \(\frac{\partial L}{\partial \theta_2(\rho_H)} < 0\) and \(\frac{\partial L}{\partial \theta_2(0)} < 0\) for a sufficiently high but finite value of \(\gamma\). As a result, \(|M| < 0\) for a sufficiently high but finite value of \(\gamma\).

The proof proceeds by analyzing \(|M_1|\) and \(|M_2|\). To do this, we first examine the derivatives \(\frac{\partial K}{\partial n}\) and \(\frac{\partial L}{\partial n}\). Thereafter, we combine the results to obtain the signs of \(|M_1|\) and \(|M_2|\). We obtain:

\[
\frac{\partial K}{\partial n} = \begin{cases} 
< 0 & \text{if } \theta^*_2(n, 0, \theta_1) < \theta^*_2(0, \theta_1) \land \theta^*_2(n, \rho_H, \theta_1) > \theta^*_2(\rho_H) \\
> 0 & \text{if } \theta^*_2(n, 0, \theta_1) > \theta^*_2(0, \theta_1) \land \theta^*_2(n, \rho_H, \theta_1) < \theta^*_2(\rho_H) \\
= 0 & \text{if } \theta^*_2(n, 0, \theta_1) = \theta^*_2(0, \theta_1) \land \theta^*_2(n, \rho_H, \theta_1) = \theta^*_2(\rho_H) 
\end{cases} \quad \forall n \in [0, 1).
\]

After having found the partial derivative for one equilibrium condition \((K)\), we turn to the other equilibrium condition \((L)\). Here, we can invoke the envelope theorem in order to obtain \(\frac{\partial L}{\partial n} = 0\). The idea is the following. Since \(L\) represents the indifference condition of an uninformed speculator, the proportion of informed speculators enters only indirectly via \(x^*_2\). Therefore, we
can write:

\[
\frac{\partial L}{\partial n} = \frac{\partial J}{\partial x^*_U} \frac{\partial x^*_U}{\partial n} + \frac{\partial J}{\partial n} \quad \text{(79)}
\]

Since \(x^*_U\) is the optimal attacking threshold of an uninformed speculator, it satisfies \(J(\cdot, x^*_U) = \frac{1}{1+b/l}\). Thus, we must have \(\frac{\partial J}{\partial x^*_U} = 0\), which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

Third, we obtain the derivatives of the fundamental thresholds for sufficiently small but positive values of \(\alpha\). We find that:

\[
\frac{d\theta^*_2(n, 0, \theta_1)}{dn} = \begin{cases} 
> 0 & \text{if } \theta^*_2(n, 0, \theta_1) \text{ and } \theta^*_2(n, \rho_H, \theta_1) > \theta^*_2(\rho_H) \\
< 0 & \text{if } \theta^*_2(n, 0, \theta_1) > \theta^*_2(0) \text{ and } \theta^*_2(n, \rho_H, \theta_1) < \theta^*_2(\rho_H) \\
= 0 & \text{if } \theta^*_2(n, 0, \theta_1) = \theta^*_2(n, \rho_H, \theta_1) = \theta^*_2(\rho_H) 
\end{cases} \quad \forall n \in [0, 1). 
\]

and:

\[
\frac{d\theta^*_2(n, \rho_H, \theta_1)}{dn} = \begin{cases} 
< 0 & \text{if } \theta^*_2(n, 0, \theta_1) < \theta^*_2(0) \text{ and } \theta^*_2(n, \rho_H, \theta_1) > \theta^*_2(\rho_H) \\
> 0 & \text{if } \theta^*_2(n, 0, \theta_1) > \theta^*_2(0) \text{ and } \theta^*_2(n, \rho_H, \theta_1) < \theta^*_2(\rho_H) \\
= 0 & \text{if } \theta^*_2(n, 0, \theta_1) = \theta^*_2(0) \text{ and } \theta^*_2(n, \rho_H, \theta_1) = \theta^*_2(\rho_H) 
\end{cases} \quad \forall n \in [0, 1). 
\]

**Step 3:** In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold w.r.t. the proportion of informed speculators is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed speculators are always bounded, which proves Result (A). The distinction between the two cases arises because:

\[
\theta^*_2(0) = \begin{cases} 
> \theta^*_2(\rho_H) & \text{if } \theta_1 > \underline{\theta}_1 \\
< \theta^*_2(\rho_H) & \text{if } \theta_1 < \underline{\theta}_1 \\
= 0 & \text{if } \theta_1 = \underline{\theta}_1 
\end{cases} \quad \text{(80)}
\]

Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed,
yielding Result (B).

Now, for the case of $\theta_1 \geq \theta_\ast_1$, we prove that $\theta^*_2(\rho_H) \leq \theta^*_2(\rho_H), \theta^*_2(0) \leq \theta^*_2(0)$ for all $n$ if $\alpha$ sufficiently small. First, $\theta^*_2(\rho_H) < \theta^*_2(0) = \theta^*_2(\rho_H) = \theta^*_2(0)$ if $n = 0$, while $\theta^*_2(0) = \theta^*_2(0)$ and $\theta^*_2(\rho_H) = \theta^*_2(\rho_H)$ if $n = 1$. Second, $\frac{d\theta^*_2(\rho_H)}{dn}|_{n=0} > 0$ and $\frac{d\theta^*_2(\rho_H)}{dn}|_{n=1} = 0$. Third, by continuity $\theta^*_2(0) < \theta^*_2(0) < \theta^*_2(0)$ and $\frac{d\theta^*_2(n,\theta_1,0)}{dn} > 0$ for small values of $n$. Fourth, if for any $\hat{n} \in (0,1]$ $\theta^*_2(0) < \theta^*_2(0) \Rightarrow \theta^*_2(0)$ when $n \to \hat{n}$, then – for sufficiently small but positive values of $\alpha$ – it has to be true that $\theta^*_2(\rho) < \theta^*_2(\rho)$ when $n \to \hat{n}$. This is because of the result in step 1. Fifth, given $\frac{d\theta^*_2(n,\theta_1,0)}{dn} < 0$ if $\theta^*_2(0) > \theta^*_2(0)$ and $\theta^*_2(\rho_H) < \theta^*_2(\rho_H)$, it follows by continuity that $\theta^*_2(0) = \theta^*_2(0)$ and $\theta^*_2(\rho_H) = \theta^*_2(\rho_H)$ for all $n \geq \hat{n}$. In conclusion, $\theta^*_2(\rho_H) \leq \theta^*_2(\rho_H), \theta^*_2(0) = \theta^*_2(0)$ for all $n \in [0,1]$ if $\alpha$ sufficiently small.

For the case $\theta_1 < \theta_1$ it can be proven that $\theta^*_2(\rho_H) \geq \theta^*_2(\rho_H), \theta^*_2(0) \geq \theta^*_2(0)$ for all $n$ if $\alpha$ is sufficiently small using a similar argument (all signs in relation to fundamental thresholds flip).

**Result (C).** From equation (70),

$$x^*_2(\rho) = \frac{\alpha_2(\rho, \theta_1) + \gamma \theta^*_2(\rho)}{\gamma} - \frac{\alpha_2(\rho, \theta_1)}{\gamma} \mu_2(\rho, \theta_1) - \frac{\sqrt{\alpha_2(\rho, \theta_1) + \gamma}}{\gamma} \Phi^{-1} \left( \frac{1}{1 + b/l} \right)$$

we see that:

$$\frac{\gamma}{\alpha_2(\rho, \theta_1) + \gamma} \frac{dx^*_2(\rho)}{dn} = \frac{d\theta^*_2(\rho)}{dn}.$$  \hfill (82)

Therefore, by continuity, there exists a sufficiently small but positive value of $\alpha$ that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. Therefore, the distance between the fundamental thresholds is monotone for any $n > 0$, which implies $\frac{d(x^*_2(0) - x^*_2(\rho_H))}{dn} > 0$. (q.e.d.)
C.11 Proof of Lemma 5

The proof has three cases and builds on equation (26). Equation (26) is constructed from $EU_I$ and $EU_U$. The expected utility of an informed speculator writes:

$$E[u(d_i = I, n)] = EU_I - c$$

$$= -c + p \left( b \int_{-\infty}^{\theta_1^*(n,0,\theta_1)} f(x|\theta_2)dx f(\theta_2|0,\theta_1)d\theta_2 \right) + (1-p) \left( b \int_{-\infty}^{\theta_1^*(n,\rho_H,\theta_1)} f(x|\theta_2)dx f(\theta_2|\rho_H,\theta_1)d\theta_2 \right)$$

By contrast, the expected utility of an uninformed speculator writes:

$$E[u(d_i = U, n)] = EU_U$$

$$= p \left( b \int_{-\infty}^{\theta_1^*(n,0,\theta_1)} f(x|\theta_2)dx f(\theta_2|0,\theta_1)d\theta_2 \right) + (1-p) \left( b \int_{-\infty}^{\theta_1^*(n,\rho_H,\theta_1)} f(x|\theta_2)dx f(\theta_2|\rho_H,\theta_1)d\theta_2 \right)$$

First, for $\theta_1 = \theta_1^*$ there are no benefits from acquiring information because $x_{2I}^*(n,\rho,\theta_1) = x_{2U}^*(n,\theta_1^*) \forall \rho$. Hence, $\bar{c}(n,\theta_1^*) = 0 \forall n \in [0,1]$ from equation (26).

Second, if $\theta_1 < \theta_1^*$ then $\theta_1^*(0) > \theta_1^*(\rho_H)$ and $x_{2I}^*(n,0,\theta_1) > x_{2I}^*(n,\theta_1^*,\theta_1) > x_{2I}^*(n,\rho_H,\theta_1)$ under the sufficient conditions of Lemma 2. We will prove that $\frac{\partial c(n,\theta_1)}{\partial n} \geq 0 \forall \theta_1 \in (\theta_1^*,\theta_1^*)$ and $\bar{c}(n,\theta_1^*) > 0 \forall \theta_1 \in (\theta_1^*,\theta_1^*)$.

An increase in the proportion of informed speculators is associated with a (weak) increase in both $\theta_1^*(0)$ and $x_2^*(n,\theta_1^*) as well as a (weak) decrease in both $\theta_1^*(\rho_H)$ and $x_2^*(\rho_H)$. Furthermore, $x_{2U}^*(n,\theta_1)$ is unaffected. An increase in $n$ leads to a relative increase of the benefit component in the first summand of equation (26) and a relative increase of the loss component in the second summand. For this reason, the left-hand side of equation (26) increases in $n$. Thus, $\frac{\partial E[u(d_i = I, n)]}{\partial n} \geq 0 \forall \theta_1 \in (\theta_1^*,\theta_1^*)$.

It remains to consider the case of $\theta_1 < \theta_1^*$. Here, we have $\theta_1^*(0) < \theta_1^*(\rho_H)$ and $\theta_1^*(0,\theta_1) \leq \theta_1^*(n,\theta_1)$.
\[ \theta_2^*(n, \rho, \theta_1) \leq \theta_2^*(\rho_H, \theta_1) \quad \forall \rho \in \{0, 1\}. \]

Hence, \( x_2^*(n, 0, \theta_1) < x_2^*(n, \rho_H, \theta_1) \). We will prove that \( \frac{\Delta(n, \theta_1)}{dn} \geq 0 \quad \forall \theta_1 < \theta_1^* \). Hence, it is optimal to purchase information if the differential expected payoff is positive. Given that \( \theta_2^*(0) < \theta_2^*(\rho_H) \), the first two summands in (26) are strictly positive and, thus, \( \mathcal{V}(n, \theta_1) > 0 \quad \forall \theta_1 < \theta_1^* \). Furthermore, an increase in \( n \) is associated with a (weak) decrease in \( \theta_2^*(0) \) and \( x_2^*(0) \), and a (weak) increase in \( \theta_2^*(\rho_H) \) and \( x_2^*(\rho_H) \). For this reason, an increase in \( n \) leads to a relative increase of the loss component in the first summand of equation (26) and a relative increase in the benefit component in the second summand. As a result, we have that the left-hand side of equation (26) increases in \( n \). Thus, \( \frac{\Delta(n, \theta_1)}{dn} \geq 0 \quad \forall \theta_1 < \theta_1^* \), which concludes the proof. (q.e.d.)

C.12 Proof of Proposition 4

The first result follows from Lemma 5 in combination with Proposition 3. From Lemma 5 there exists a strictly positive cost level such that information acquisition occurs for all \( \theta_1 \neq \theta_1^* \). Conditional on \( \rho = 0 \), the distribution of \( \theta_1 \) is described by the pdf \( f(\theta_1) \), which places a strictly positive probability mass on \( \theta_1 \neq \theta_1^* \). As a result, there exists a strictly positive cost level such that information acquisition occurs if \( c < \int_{\theta_1}^{\theta_1^*} \mathcal{V}(0, \theta_1) f(\theta_1) d\theta_1 \). Hence, there exists a unique pure-strategy PBE where the ex-ante contagion effect arises if private signal is sufficiently precise and the public signal sufficiently imprecise.

The second result follows from Lemma 5 in combination with Lemma 2. Lemma 2 gives conditions such that the interval \((\theta_1, \theta_1^*)\) is non-empty. From Lemma 5 there exists a strictly positive cost level such that information acquisition occurs for all \( c \leq c(0, \theta_1) \) and does not occur for all \( c \geq c(1, \theta_1) \), provided that \( \theta_1 \in (\theta_1, \theta_1^*) \) in both cases. Hence, there does exist a unique pure-strategy PBE with \( n^* = 1 \) in the former case and with \( n^* = 0 \) in the latter case, provided the private signal is sufficiently precise and the public signal sufficiently imprecise. More specifically, for both cases there exist unique optimal attacking rules at the coordination stage and a unique information acquisition rule at the information stage. Given Proposition 2, the ex-post contagion effect occurs because the probability of a crisis is higher if \( c < \bar{c}(0, \theta_1) \) (informed) than if \( \bar{c}(1, \theta_1) < c \) (uninformed). (q.e.d.)
D Extensions and robustness

We discuss the robustness of our results to changes in assumptions as well as some extensions.

D.1 Heterogeneity across countries: a numerical illustration

Our model features symmetric countries, $\mu_1 = \mu_2$ and $b_1/l_1 = b_2/l_2$. If fundamentals are known to be uncorrelated, $\rho = 0$, country 2 is identical to country 1. However, our setup is flexible and can allow for asymmetric countries. For example, the initially affected country may be more prone to a crisis, which arises if the relative benefit from attacking is larger in country 1, $b_1/l_1 > b_2/l_2$, or the prior about the fundamental in country 2 is relatively weaker, $\mu_1 < \mu_2$.

Perhaps surprisingly, this heterogeneity across countries implies a substantially stronger contagion effect. This stronger contagion effect arises as a crisis in country 1 already occurs for higher realizations of $\theta_1$, since the mean effect is smaller:

$$\frac{d(\theta^*_2(0, \theta_1) - \theta^*_2(\rho_H, \theta_1))}{d\theta_1} > 0.$$  

(85)

Consider a numerical example with two symmetric countries using the parameters from Figure 2 in Appendix B.2: $\alpha = 1$ (substantially positive), $\gamma = 1.5$ (quite small), $\mu_1 = \mu_2 = \mu = 0.6$, $b_1/l_1 = b_2/l_2 = b/l = 1/3$, $p = 0.7$, $\rho_H = 0.8$. For these parameter values, the fundamental threshold in country 1 is $\theta^*_1 = 0.1$. Suppose a crisis occurs in country 1 after a fundamental realization of $\theta_1 = 0.07 < \theta^*_1$. When the correlation is positive, informed speculators attack more aggressively, resulting in a larger range of fundamentals consistent with a crisis in country 2, $\theta^*_2(0) = 0.1 > \theta^*_2(\rho_H) = 0.09$. In short, the variance effect outweighs the mean effect.

In case of asymmetric benefits, $b_1/l_1 = 1/3 < b_2/l_2 = 2$, $\theta^*_1$ increases from 0.1 to 0.75. While the threshold in country 2 is unchanged if $\rho = 0$, $\theta^*_2(0) = 0.1$, it can be substantially smaller if $\rho = \rho_H$. Recall that a crisis in country 1 occurs for higher realizations of $\theta_1$, such as $\theta_1 = 0.5$. However, $\theta_1 = 0.5$ implies a lower thresholds in country 2, $\theta^*_2(\rho_H, \theta_1 = 0.5) = 0.01 < \theta^*_2(\rho_H, \theta_1 = 0.1) = 0.09$. As a result, the difference in fundamental thresholds, and hence the strength of the ex-post contagion effect, increases from $\theta^*_2(0) - \theta^*_2(\rho_H, 0.1) = 0.01$ to $\theta^*_2(0) - \theta^*_2(\rho_H, 0.5) = 0.09$. 

61
D.2 Imperfect learning about the correlation

Our key insights hold if learning about the correlation is imperfect. Suppose that the signal about \( \rho \) purchased by speculators is noisy. Noise requires us to use mixture distributions when analysing the case of informed speculators. That is, the attacking thresholds of informed speculators would become more similar to those of uninformed speculators as we move from perfect to imperfect learning about the correlation. While the quantitative difference between the attacking thresholds used by informed and uninformed shrinks, our qualitative results are unaffected.

D.3 Negative cross-country correlation of fundamentals

Suppose that fundamentals may be negatively correlated, \( \rho_H < 0 \). Hence, a low realized fundamental in country 1 shifts the conditional distribution of the fundamental in country 2 after speculators learn that \( \rho = \rho_H \), implying a higher conditional mean of the prior, \( \mu_2(\rho_H, \theta_1) > \mu \). As a result, the fundamental threshold ranking from Lemma 2 are reversed, as can be seen in Table 2 in Appendix C.2.2 Hence, the reverse of the ex-post contagion effect established in section 4.1 arises when \( \rho_H < 0 \). After a crisis in country 1 and if the correlation is non-zero, \( \rho = \rho_H < 0 \), contagion is more likely if speculators are informed about the correlation than when uninformed.

D.4 Modelling ex-ante uncertainty about the correlation

What is the role of \( p \) and \( \rho_H \)? A variation in \( p \) has a quantitative effect only. In particular, a change in \( p \) leaves \( \theta_1 \) unchanged, while an increase (decrease) in \( p \) reduces (increases) the difference between \( \theta^*_2U \) and \( \theta^*_2I(0) \). Hence, the ex-post contagion effect in Proposition 2 is weakened (strengthened) when \( p \) increases (decreases), leaving our qualitative results unaffected. The effect of changing \( \rho_H \) is harder to understand because it affects both \( \underline{\theta}_1 \) and the prior in country 2. However, we can show an ex-post stability effect. The ex-post contagion result holds with opposite inequality if \( \rho = \rho_H \):

\[
\Pr(\theta_2 < \theta^*_2U(\rho, \theta_1)|\rho = \rho_H, \theta_1) < \Pr(\theta_2 < \theta^*_2U(\theta_1)|\rho = \rho_H, \theta_1) \forall \theta_1 \in (\underline{\theta}_1, \theta^*_1) \quad (86)
\]

\[
\Pr(\theta_2 < \theta^*_2U(\rho, \theta_1)|\rho = 0, \theta_1) < \Pr(\theta_2 < \theta^*_2U(\theta_1)|\rho = 0, \theta_1) \forall \theta_1 < \underline{\theta}_1. \quad (87)
\]
D.5 Different timing

Consider a reversal of the timing, whereby both countries are swapped. That is, country 1 now moves at date 2 and vice versa. First, there is no reason for agents in country 2 to acquire information about the correlation at date 1. Second, if agents in country 1 can acquire information about the cross-country correlation after observing a crisis in country 2, the setup is identical. Third, if agents in country 1 cannot acquire information about the correlation after observing a crisis in country 2, then only an information contagion channel prevails (see also section 4). In contrast, we focus on the information choice of agents after observing a crisis (a wake-up call).