A Wake-up Call Theory Of Contagion

Toni Ahnert† Christoph Bertsch‡

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Abstract

Empirical evidence in several fields has identified wake-up calls as an important channel of contagion. We propose a theory of contagion based on the information choice of investors after a wake-up call. We study global coordination games of regime change with two regions where local fundamentals have an unobserved common component. A crisis in the first region is a wake-up call to investors in the second region. This wake-up call induces investors to re-assess the local fundamental and to acquire information about the common component. We show that (i) information acquisition occurs only after a wake-up call; and (ii) contagion occurs even if investors learn that the second region has no exposure to the first region. These results do not rely on common investors or balance sheet links across regions. Our theory of contagion applies to currency crises, runs on financial intermediaries, and sovereign debt crises. A policymaker with favorable news about the local fundamental can mitigate contagion by enhancing transparency. JEL D83, F3, G01.

Keywords: wake-up call, contagion, financial crises, information choice, fundamental re-assessment, global games, transparency.

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‡Financial Studies Division, Bank of Canada, 234 Laurier Avenue W, Ottawa, ON K1A 0G9, Canada. The author conducted part of this research during his Ph.D. at the London School of Economics and Political Science and while visiting New York University and the Federal Reserve Board of Governors. E-mail: tahnert@bankofcanada.ca

‡Corresponding author: Research Division, Sveriges Riksbank, SE-103 37 Stockholm, Sweden. The author conducted part of this research during a PostDoc at the University College London, United Kingdom. An earlier version of this paper was part of his Ph.D. thesis at the European University Institute, Florence, Italy. E-mail: christoph.bertsch@riksbank.se Tel.: +46 8 787 04 98. Fax: +46 8 21 05 31.
1 Introduction

Understanding the causes of financial contagion is an important question in banking and international finance. For example, Forbes (2012) distinguishes four channels of contagion: trade, banks, portfolio investors, and wake-up calls. According to the wake-up call hypothesis, a popular explanation for contagion put forward by Goldstein (1998), a financial crisis in region 1 is a wake-up call to investors in region 2 that induces them to re-assess and inquire about the fundamentals of region 2. The re-appraisal of risk can lead to a contagious spread of a crisis to region 2.


Despite the empirical evidence, there has been little theoretical work on the wake-up call hypothesis. We propose a wake-up call theory of contagion based on a re-assessment of local fundamentals and information acquisition after observing a crisis elsewhere. Building on global games (Carlsson and van Damme 1993), we develop a model with two regions that move sequentially and where regional fundamentals share an initially unobserved macro shock. Contagion is defined as a higher probability of a crisis in region 2 after a crisis in region 1 than after no crisis in region 1. We show that contagion occurs even if investors learn that the macro shock realization is zero and region 2 has no exposure to the crisis in region 1. Since common investors and balance sheet links across regions are absent, our theory isolates the wake-up call component in the transmission of financial crises.
We consider a standard global coordination game of regime change with incomplete information about the fundamental (Morris and Shin 2003) in each region. A financial crisis occurs in a given region if sufficiently many investors act against the regime (attack a currency peg, withdraw funds from a bank, or refuse to roll over debt). In contrast to the standard game, our model is sequential. Investors in region 1 decide whether to act, which determines the outcome in region 1. Afterwards, investors in region 2 observe the public signal of whether a regime change occurred in region 1 and update their beliefs about the macro shock. Subsequently, investors decide whether to learn the macro shock at a cost.

We show that investors in region 2 have a higher incentive to acquire information after a wake-up call if the macro shock is sufficiently negatively skewed (Lemma 4). For an intermediate range of information costs, investors thus acquire information about the macro shock if and only if a crisis occurred in region 1. This differential information choice arises because investors face an elevated risk of a strongly negative macro shock after a wake-up call, while the negative macro shock is less likely after no crisis. After a crisis in region 1, investors in region 2 have a higher incentive to acquire information in order to understand whether regional fundamentals are linked. That is, an investor’s benefit of tailoring its attack rule to the realized macro shock is higher after a crisis in region 1 than after no crisis.

We also show that contagion can occur even if all investors learn that the macro shock is zero (Proposition 4). That is, the probability of a crisis in region 2 after a crisis in region 1 and learning that region 2 has no exposure to region 1 is higher than the probability of a crisis in region 2 after no crisis in region 1. This contagion result arises from the endogenous information choice of investors and Bayesian updating about the macro shock. In the absence of a crisis in region 1, investors in region 2 choose not to acquire information and form a more optimistic belief about the macro shock. As a result, the probability of a crisis in region 2 is lower after no crisis in region 1.

Empirical evidence supports our model’s empirical prediction of an endogenously higher incentive to acquire information about a common macro shock after observing a wake-up call. A recent empirical literature suggests that investor attention and information demand, proxied by internet search intensity, play an important role (Da et al. 2011). Specifically, Vlastakis and Markellos (2012) study the U.S.
stock market and find that the demand for information at the market level is positively associated with measures of volatility, after controlling for the market return and information supply. This elevated demand for information at the market level closely mirrors the information acquisition about the macro shock in our model.

A key assumption for the result on the differential information choice of investors is the negatively skewed distribution of the macro shock. There is an extensive empirical literature on the negative skewness of important macroeconomic variables, including GDP growth, individual stock returns, and the aggregate stock market. See Campbell and Hentschel (1992) and Bae et al. (2007) for the sources of the negative skewness of stock returns. An extensive empirical and theoretical literature on asymmetric business cycles studies the occurrence of sharp recessions and slow booms (for example, Neftci 1984). Theoretical explanations for the negative skewness of output and total factor productivity include Acemoglu and Scott (1997), Veldkamp (2005), and Jovanovic (2006). For developing countries, Ranciere et al. (2003) find that financially liberalized countries exhibit a negatively skewed growth of both GDP and credit. Ranciere et al. (2008) argue that a negative skewness captures systemic risk and is therefore a systemic component.

Building on a standard global coordination game of regime change, the wake-up call theory of contagion has several applications. For currency crises, speculators observe a currency attack and are uncertain about the magnitude of trade or financial links or institutional similarity. For rollover risk and bank runs, wholesale investors observe a run elsewhere and are uncertain about interbank exposures. For sovereign debt crises, bond holders observe a sovereign default elsewhere and are uncertain about the macroeconomic links, the commitment of the international lender of last resort, or the resources of multilateral bail-out funds. For political regime change, activists observe a revolution, for example during the Arab spring, and are uncertain about the impact on their government’s ability to stay in power.

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1 See also Morris and Shin (1998) and Corsetti et al. (2004) for a one-regional global game that builds on the earlier works of Krugman (1979), Flood and Garber (1984), and Obstfeld (1986).  
2 See also Rochet and Vives (2004) and Goldstein and Pauzner (2005) for a one-regional global game that builds on the earlier work of Diamond and Dybvig (1983).  
3 See also Corsetti et al. (2006). See Drazen (1999) for membership contagion.  
4 For a one-regional global game of political regime change with endogenous information manipulation or dissemination, see Edmond (2013) and Shadmehr and Bernhardt (2015), respectively.

We also describe additional implications of our model and relate these to the literatures on global coordination games and information choice. To evaluate the incentives of investors to acquire information, one has to consider the optimal attack behavior for any given proportion of informed investors. Since some investors may be informed while others are uninformed, the prior beliefs about the macro shock may be heterogeneous across investors. Specifically, the prior of uninformed investors follows a mixture distribution. While uninformed investors play an invariant strategy, informed investors can tailor their strategy to the observed macro shock. We show that a unique optimal attacking rules exist if private information about the fundamental is sufficiently precise (Proposition 1 and Corollary 1). In another global game with mixture distributions, Chen et al. (2012) develop a theory of rumors during political regime change. However, they abstract from both contagion and information choice.

Analyzing the private value of information about the macro shock, we show that it increases in the proportion of informed investors (Proposition 2). This strategic complementarity in information choices is similar to Hellwig and Veldkamp (2009), who first studied the optimal information choice in strategic models. They show that the information choices of investors inherit the strategic motive of an un-
derlying beauty contest game. In contrast, we study information acquisition about the common component of regional fundamentals in coordination games of regime change. We relate the incentives to acquire information to distributional characteristics of this common component and the occurrence of a crisis in region 1. For a sufficiently negative third moment of the common component, the incentives to acquire information are higher after a wake-up call, which ensures a unique equilibrium for an intermediate range of the information cost (Proposition 3).

Finally, we consider some policy responses to wake-up call contagion. Specifically, we consider a policymaker that observes the macro shock and wishes to mitigate the incidence of a financial crisis in region 2 after a crisis in region 1. Since revealing the information would be cheap talk in case of a negative macro shock, we focus on increasing the transparency of public information, perhaps via disclosure (Morris and Shin, 2002). We show that the policymaker increases transparency only if the macro shock is favorable. Greater transparency increases the incentives of investors to acquire information about the (favorable) macro shock, which in turn reduces the incidence of contagion.

Transparency in global coordination games has been studied before. In a beauty contest game, Morris and Shin (2002) show that a change in the precision of public information can have an ambiguous effect on welfare. In a game of regime change, the effect of transparency on the incidence of a regime change has been studied by Morris and Shin (1998), Heinemann and Illing (2002), and Bannier and Heinemann (2005). Our paper belongs to the literature that considers the effect of greater transparency on both coordination and information acquisition incentives. In a beauty contest model with private information acquisition, Colombo et al. (2014) find that public information crowds out private information acquisition. In a regime change model, Szkup and Trevino (2012) show that public and private information can also be complements. In contrast, we study information acquisition about a common macro component of regional fundamentals in a regime change model. We find that greater transparency induces information acquisition.

Studying a global coordination game of regime change with a common prior and costly and continuous information choice, Szkup and Trevino (2012) provide conditions sufficient for strategic complementarity in information choices to occur. Ahnert and Kakhbod (2014) obtain strategic complementarity in information choices in a global coordination game of regime change with a common prior, a discrete private information choice and heterogeneous information costs.
The paper proceeds as follows. We describe the model in section 2. We solve for the equilibrium in region 1 in section 3 and the equilibrium in region 2 in section 4, where we also describe the incentives of investors to acquire information. Section 5 contains our contagion results. Section 6 studies some policy responses. Section 7 discusses some extensions and section 8 concludes. Proofs are in the Appendix.

2 Model

We study a sequence of global coordination games of regime change in two regions indexed by $t \in \{1, 2\}$. Each region is inhabited by a different unit continuum of risk-neutral investors indexed by $i \in [0, 1]$. Investors in region $t = 1$ move first and are followed by investors in region $t = 2$.

In each region, investors simultaneously decide whether to attack the regime, $a_{it} = 1$, or not, $a_{it} = 0$. The outcome of the attack depends on both the aggregate attack size, $A_t \equiv \int_0^1 a_{it} di$, and a regional fundamental $\Theta_t \in \mathbb{R}$ that measures the strength of the regime. A regime change occurs if enough investors attack, $A_t > \Theta_t$.

Following Vives (2005), an attacking investor receives a benefit $b_t > 0$ if a regime change occurs and incurs a loss $\ell_t > 0$ otherwise, where $\gamma_t \equiv \frac{\ell_t}{b_t + \ell_t} \in (0, 1)$ captures the relative cost of failure of an investor in region $t$:

$$ u(a_{it} = 1, A_t, \Theta_t) = b_t 1_{\{A_t > \Theta_t\}} - \ell_t 1_{\{A_t \leq \Theta_t\}}. \quad (1) $$

The payoff from not attacking is normalized to zero. Thus, the relative payoff from attacking increases in the attack size $A_t$ and decreases in the fundamental $\Theta_t$. Hence, the attack decisions of investors exhibit global strategic complementarity.

A regime change can be a currency crisis, a bank run, or a sovereign debt crisis. The fundamental can be interpreted as the ability of a monetary authority to defend its currency (Morris and Shin 1998, Corsetti et al. 2004), as the measure of investment profitability (Rochet and Vives 2004, Goldstein and Pauzner 2005, Corsetti et al. 2006) or a sovereign’s taxation power or willingness to repay. Investors are interpreted as currency speculators, as retail or wholesale bank creditors who withdraw funds, or as sovereign debt holders who refuse to roll over.
Our model has two key features. The first key feature is an initially unobserved macro shock. Specifically, the macro shock takes on one of three values:

\[
m = \begin{cases} 
\Delta & \text{w.p. } p \\
-s\Delta & \text{w.p. } q \\
0 & \text{w.p. } 1-p-q,
\end{cases}
\]  

(2)

where \( p \in [0, 1], q \in [0, 1-p], \Delta > 0, \) and \( s > 0. \) We impose \( p = qs \) to ensure a zero mean. The variance of the macro shock is affected by the magnitude of \( \Delta. \) Crucially, \( s \) governs the skewness of the macro shock, where \( s > 1 \) corresponds to negative skewness.

The initial uncertainty about the macro shock is motivated by our applications to financial crises. In the context of currency attacks or sovereign debt crises, this uncertainty about the macro shock reflects the unknown relevance of certain institutional similarities or of real or financial linkages across countries. In the context of bank runs, it reflects the uncertainty about bank portfolios and interbank exposures.

The macro shock is the only link between the two regions. It may induce a positive correlation between regional fundamentals \( \Theta_1 \) and \( \Theta_2. \) Each regional fundamental comprises the macro shock and a regional component, \( \theta_i: \)

\[
\Theta_t = m + \theta_t,
\]

(3)

where each \( \theta_t \) follows an independent normal distribution with mean \( \mu_t \equiv \mu \) and precision \( \alpha_t \equiv \alpha \in (0, \infty) \) that is independent of the macro shock. Region 2 is said to be exposed to region 1 via the common macro shock if \( m \neq 0. \)

Following [Carlsson and van Damme (1993)], there is incomplete information about the fundamental. Each investor receives a noisy private signal \( x_{it} \) before deciding whether to attack [Morris and Shin (2003)]:

\[
x_{it} \equiv \Theta_t + \varepsilon_{it}.
\]

(4)

The idiosyncratic noise \( \varepsilon_{it} \) is identically and independently normally distributed across investors with zero mean and precision \( \beta \in (0, \infty). \) Each noise term is inde-
dependent of both the macro shock and the regional component.

The second key feature is an information stage that precedes the coordination stage. First, investors in region 2 observe whether there is a crisis in region 1. Second, investors in region 2 can acquire costly information about the macro shock. Investors simultaneously decide whether to purchase a signal about the macro shock at cost $c > 0$. The signal is perfectly revealing and publicly available, whereby each investor can purchase the same signal and observes it privately. In terms of wholesale investors or currency speculators, costly information acquisition could be the access to Bloomberg and Datastream terminals or hiring analysts who assess publicly available data. An important channel is how the wake-up call of a crisis in region 1 affects the incentives of investors in region 2 to acquire information.

Table 1 in Appendix A summarizes the timeline of events.

3 Equilibrium in region 1

Let $n_t \in [0, 1]$ be the proportion of investors in region $t$ who are informed about the macro shock. We first analyze the special cases in which all investors in region 1 are informed about $m$, that is $n_1 = 1$. In this special case, the analysis of region 1 is standard (for example, Morris and Shin (2003)). A Bayesian equilibrium is an attack decision $a_{i1}$ for each investor $i$ and an aggregate attack size $A_1$ that satisfy both individual optimality for all investors, $a_{i1}^* = \arg\max_{a_{i1} \in \{0, 1\}} E[u(a_{i1}, A_1, \Theta_1) | x_{i1}] \equiv a(x_{i1})$, and aggregation, $A_1^* = \int_0^1 a(x_{i1}) di$.

Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function (pdf) and cumulative distribution function (cdf) of the standard Gaussian random variable.

Result 1 Morris and Shin (2003) Suppose that $n_1 = 1$. If private information is sufficiently precise, $\beta > \frac{\sigma^2}{2\pi} \in (0, \infty)$, then there exists a unique Bayesian equilibrium in region 1. This equilibrium is characterized by a signal threshold, $x_{1}^*$, and a fundamental threshold, $\Theta_{1}^*$. Investor $i$ attacks whenever $x_{i1} < x_{1}^*$, and a crisis occurs

Abstracting from information acquisition in region 1 does not affect our key insights.
whenever $\Theta_1 < \Theta_1^*$, The fundamental threshold $\Theta_1^*$ is defined by:

$$
\Phi\left(\frac{\alpha}{\sqrt{\alpha + \beta}} [\Theta_1^* - (\mu + m)] - \sqrt{\frac{\beta}{\alpha + \beta}} \Phi^{-1}(\Theta_1^*)\right) = \gamma_1.
$$

and the signal threshold $x_1^*$ is defined by $x_1^* = \Theta_1^* + \frac{1}{\sqrt{\beta}} \Phi^{-1}(\Theta_1^*)$.

When some investors are uninformed about $m$, that is $n_1 < 1$, the analysis is non-standard and requires the use of mixture distributions. The equilibrium is characterized by indifference and critical mass conditions. Different to before, there are now three distinct fundamental thresholds, $\Theta_1^*(n_1, m)$ – one for each macro shock realization – and thus three critical mass conditions. Similarly, there are now four indifference conditions – one for uninformed investors ($U$) and one for informed investors ($I$) for each macro shock realization. We have to distinguish between the signal thresholds, $x_{1U}^*(n_1)$ and $x_{1I}^*(n_1, m)$, for which uninformed and informed investors are indifferent whether to attack. While the threshold of informed investors (subscript $I$) depends on $m$, the threshold of uniformed investors (subscript $U$) is invariant. We derive the corresponding equilibrium conditions in Appendix B.1.

For the special case in which no investor is informed, $n_1 = 0$, the fundamental thresholds for each macro shock realization are identical because the attack behavior of investors is invariant, $\Theta_1^*(0, -s\Delta) = \Theta_1^*(0, 0) = \Theta_1^*(0, \Delta)$. In this case, the equilibrium conditions can be reduced to one equation in one unknown (Lemma 1).

**Lemma 1** Suppose all investors in region 1 are uninformed about the macro shock, $n_1 = 0$. If private information is sufficiently precise, then there exists a unique monotone Bayesian equilibrium in region 1. Each investor attacks if and only if the private signal is below the signal threshold, $x_{1U}^*$. A crisis occurs if and only if the fundamental in region 1 is below the fundamental threshold, $\Theta_1^*(0, m)$. This threshold is a weighted average of the thresholds that prevail if investors were informed:

$$
\min_m \{ \Theta_1^*(1, m) \} < \Theta_1^*(0, m) < \max_m \{ \Theta_1^*(1, m) \}, \forall m
$$

**Proof** See Appendix B.2.
Using the results of Milgrom (1981) and Vives (2005), the best-response function of an individual investor strictly increases in the thresholds used by other investors (Appendix B.1.2). The common requirement of precise private information suffices for uniqueness in monotone equilibrium in case of mixture distributions and heterogeneous priors. For the general case when \( n_1 \in [0, 1] \), both the prior and the posterior of an uninformed investor follow a mixture distribution. Moreover, the priors about the regional fundamental are now heterogeneous across investors when only some informed investors observe the macro shock. Proposition 1 summarizes.

**Proposition 1**  
**Existence of a unique monotone equilibrium in region 1.** If private information is sufficiently precise, then there exists a unique monotone Bayesian equilibrium in region 1 for any proportion of informed investors, \( n_1 \in [0, 1] \). This equilibrium is characterized by signal thresholds for informed and uninformed investors, \( x^*_I(n_1, m) \) and \( x^*_U(n_1) \), and a fundamental threshold, \( \Theta^*_1(n_1, m) \), for each realized macro shock, \( m \in \{-s\Delta, 0, \Delta\} \). Investors attack whenever their private signal is sufficiently low, \( x_{i1} < x^*_U(n_1) \) if uninformed and \( x_{i1} < x^*_I(n_1, m) \) if informed. A crisis occurs whenever the fundamental is sufficiently low, \( \Theta_1 < \Theta^*_1(n_1, m) \).

**Proof**  See Appendix B.3 in which we also derive the thresholds.

**4  Equilibrium in region 2**

To study the equilibrium in region 2, we start by defining the equilibrium concept. Let \( d_i \in \{I, U\} \) denote the information choice of investor \( i \) in region 2 and let \( a_{2I} \equiv a_{i2}(d_i = I) \) and \( a_{2U} \equiv a_{i2}(d_i = U) \) denote the corresponding attack rules.

**Definition 1**  
A pure-strategy monotone perfect Bayesian equilibrium in region 2 comprises an information choice for each investor \( i \in [0, 1] \), \( d^*_i \in \{I, U\} \), an aggregate proportion of informed investors, \( n^*_2 \in [0, 1] \), an attack rule for each investor, \( a^*_{2d}(\cdot) \in [0, 1] \), and an aggregate attack size, \( A^*_2 \in [0, 1] \), such that:

1. At the information stage, investors optimally choose \( d_i \).
2. The proportion of informed investors is consistent with individual information choices, $n^*_2 = \int_0^1 d^*_i \, di$.

3. At the coordination stage, uninformed investors have an optimal attack rule $a^*_2U(\cdot)$. For any given realization of the macro shock $m$, informed investors have an optimal attack rule $a^*_2I(m, \cdot)$.

4. The aggregate attack size is consistent with these attack rules:

$$A^*_2 = n^*_2 \int_0^1 a^*_2I(m, \cdot) di + (1 - n^*_2) \int_0^1 a^*_2U(\cdot) di, \quad \forall m \in \{-s\Delta, 0, \Delta\}.$$  

4.1 Coordination stage

The optimal behavior of investors in region 2 at the coordination stage can be described by extending the results from region 1. One difference arises since all investors in region 2 observe whether a crisis occurred in region 1 before deciding whether to attack at date 2. The information about region 1 is used to re-assess the fundamental of region 2. Let $f_1 \in \{0, 1\}$ indicate whether a crisis occurred in region 1, where $f_1 = 1$ corresponds to a crisis and $f_1 = 0$ corresponds to no crisis.

After observing a crisis in region 1, a wake-up call, investors in region 2 learn that the local fundamental in region 1 must have been low, $\Theta_1 < \Theta^*_1(n_1, m)$. Likewise, the fundamental must have been high after no crisis, $\Theta_1 \geq \Theta^*_1(n_1, m)$. A crisis (no crisis) suggests a less (more) favorable realization of the macro shock is more likely. Investors use the information about region 1 to update their prior about their beliefs about the distribution of the macro shock, using Bayes’ rule:

$$p' \equiv \Pr\{m = \Delta | f_1\} = p \Pr\{f_1 | m = \Delta\} \Gamma^{-1}$$  

$$q' \equiv \Pr\{m = -s\Delta | f_1\} = q \Pr\{f_1 | m = -s\Delta\} \Gamma^{-1},$$

where $\Pr\{f_1 = 1 | m\} = \Pr\{\Theta_1 < \Theta^*_1(n_1, m) | m\}$ for each $m$ and $\Gamma \equiv p \Pr\{f_1 | m = \Delta\} + q \Pr\{f_1 | m = -s\Delta\} + (1 - p - q) \Pr\{f_1 | m = 0\}$.

 Lemma 2 states the evolution of the beliefs about the macro shock.
Lemma 2  **Beliefs about the macro shock.** A crisis in region 1 is associated with less favorable beliefs about the macro shock (wake-up call), while no crisis in region 1 is associated with more favorable beliefs about the macro shock:

\[
\begin{align*}
& p' < p, \quad q' > q \quad \text{if } f_1 = 1 \\
& p' > p, \quad q' < q \quad \text{if } f_1 = 0.
\end{align*}
\]

Moreover, we can state that:

\[
\begin{align*}
& \frac{p'}{1-q'} < \frac{p}{1-q}, \frac{q'}{1-p'} > \frac{q}{1-p} \quad \text{if } f_1 = 1 \text{ and } n_1 \in \{0,1\} \\
& \frac{p'}{1-q'} > \frac{p}{1-q}, \frac{q'}{1-p'} < \frac{q}{1-p} \quad \text{if } f_1 = 0 \text{ and } n_1 \in \{0,1\}.
\end{align*}
\]

The first set of inequalities are an extension of a comparative static in [Morrison and Shin (2003)] and [Vives (2005)]. For the special case of \( n_1 = 1 \), we have \( \frac{d\Theta^*_1(1,m)}{dm} < 0 \). Similarly for the general case, a more favorable information about fundamentals is associated with a lower fundamental threshold. The results follow from Bayesian updating in equations (7) and (8). The second set of inequalities on the right-hand side follow from \( \frac{d}{dm} \left( \text{Pr}\{f_1 = 1|m\} - \text{Pr}\{f_1 = 0|m\} \right) < 0 \). The result is immediate for \( n_1 \in \{0,1\} \) and also hold for the general case, \( n_1 \in [0,1] \), if the thresholds are monotone in \( n_t \). We show this monotonicity in Lemma 3.

Using the updated \( p' \) and \( q' \) as weights, the belief about \( \Theta_2 \) prior to receiving a private signal \( x_{12} \) follows again a mixture distribution. It is an average over the cases of negative, zero and positive macro shocks with weights depending on \( f_1 \):

\[
\Theta_2|f_1 \equiv p' [\Theta_2|m = -s\Delta] + q' [\Theta_2|m = \Delta] + (1 - p' - q') [\Theta_2|m = 0].
\]  

(9)

The analysis for region 2 is identical to that for region 1, with the only difference that the prior beliefs about the macro shock are updated. Corollary 1 extends the result of Proposition 1 to the coordination stage of region 2.

**Corollary 1**  **Existence of unique attack rules in region 2.** For any proportion of informed investors in region 2, \( n_2 \in [0,1] \), there exist unique attack rules for informed investors, \( a^*_2I(m,\cdot) \), and for uniformed investors, \( a^*_2U(\cdot) \).
4.2 Information choice

Turning to the information stage, we study the incentives of investors to acquire
information about the macro shock. First, we describe the value of information and
show that it increases in the proportion of informed investors (strategic complementar-
ity in information choice). Second, we link the value of information and the
optimal information choice of investors to the wake-up call of a crisis in region 1.

4.2.1 The value of information

To characterize the value of information about the macro shock to investors, we
describe how the fundamental thresholds and the signal thresholds depend on the
proportion of informed investors $n_t$, as summarized in Lemma 3. Henceforth, we
maintain three assumptions on $\alpha$, $\beta$, and $1 - p - q$, as summarized below.

Assumption 1 We focus on the case of precise private information, $\beta > \beta < \infty$,
imprecise public information $\alpha < \bar{\alpha} > 0$, and a small probability of a zero macro
shock, $1 - p - q < \eta > 0$.

Assumption 1 provides sufficient conditions for most of the subsequent anal-
ysis. While a sufficiently high relative precision of private information information
is standard in the global games literature, the small probability of a zero macro
shock is used to simplify the exposition.

Lemma 3 Proportion of informed investors and equilibrium thresholds. If As-
sumption 1 holds, then we have these results:

(A) Boundedness. The fundamental thresholds in case of informed investors bound
the fundamental thresholds in case of asymmetrically informed investors:

$$ \Theta_2^+(1, \Delta) \leq \Theta_2^+(n_t, m) \leq \Theta_2^+(1, -s\Delta) \forall m \in \{-s\Delta, 0, \Delta\} \forall n_t \in [0, 1]. \quad (10) $$

(B) Monotonicity in fundamental thresholds. The fundamental threshold in the
case of a negative (positive) macro shock increases (decreases) in the pro-
portion of informed investors. Strict monotonicity is attained if and only if
the fundamental thresholds are strictly bounded, that is \( \forall n_t \in [0, 1) \):

\[
\frac{d\Theta^*_i(n_t, -s\Delta)}{dn_t} = \begin{cases} 
> 0 \text{ if } \Theta^*_i(n_t, -s\Delta) < \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) > \Theta^*_i(1, \Delta) \\
= 0 \text{ if } \Theta^*_i(n_t, -s\Delta) = \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) = \Theta^*_i(1, \Delta), \\
< 0 \text{ if } \Theta^*_i(n_t, -s\Delta) < \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) > \Theta^*_i(1, \Delta) \\
= 0 \text{ if } \Theta^*_i(n_t, -s\Delta) = \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) = \Theta^*_i(1, \Delta).
\end{cases}
\tag{11}
\]

\[
\frac{d\Theta^*_i(n_t, \Delta)}{dn_t} = \begin{cases} 
> 0 \text{ if } \Theta^*_i(n_t, -s\Delta) < \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) > \Theta^*_i(1, \Delta) \\
= 0 \text{ if } \Theta^*_i(n_t, -s\Delta) = \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) = \Theta^*_i(1, \Delta), \\
< 0 \text{ if } \Theta^*_i(n_t, -s\Delta) < \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) > \Theta^*_i(1, \Delta) \\
= 0 \text{ if } \Theta^*_i(n_t, -s\Delta) = \Theta^*_i(1, -s\Delta) \land \Theta^*_i(n_t, \Delta) = \Theta^*_i(1, \Delta).
\end{cases}
\tag{12}
\]

(C) Monotonicity in signal thresholds. As a consequence of the monotonicity in fundamentals thresholds:

\[
\frac{d(x^*_t(n_t, -s\Delta) - x^*_t(n_t, \Delta))}{dn_t} \geq 0, \ \forall n_t \in [0, 1),
\tag{13}
\]

where \( x^*_t(n_t, -s\Delta) - x^*_t(n_t, \Delta) > 0, \ \forall n_t \in [0, 1]. \)

**Proof** See Appendix C.1

These comparative statics follow from the fact that uninformed investors cannot tailor their attack decision to the realized macro shock. Hence, the resulting fundamental and signal thresholds take values in between the case in which all investors are informed. When the proportion of informed investors increases, the thresholds continuously converge to the case in which all investors are informed.

---

![Fundamental thresholds](image_url)

**Figure 1:** Fundamental thresholds are monotonic in the proportion of informed investors, converging to the threshold values when all investors are informed.
Next, we study the value of information about the macro shock. The value of information to an individual investor is the difference in the expected utility between an informed and uninformed investor. These expected utilities are denoted by $EU_I$ and $EU_U$, respectively, and are defined in Appendix C.2. The expected utility of an informed investor takes into account the possible realizations of $m$, since these affect the signal thresholds of an informed investor, $x^*_I(n_t, m)$. By contrast, an uninformed investor cannot tailor the attack strategy and must use the same signal threshold $x^*_U(n_t)$ throughout. Let $\nu \equiv EU_I - EU_U$ be the value of information conditional on the proportion of informed investors and the information set in region $t$:

$$v(n_t) = -p_t \left( \int_{-\infty}^{\Theta_I^*(n_t, -sA)} b_t \int_{x^*_I(n_t)}^{\Theta_I^*(n_t, -sA)} g(x_{it}|\Theta)dx_{it}f(\Theta_t|\Delta)d\Theta_t \right) + \int_{\Theta_U(n_t, -sA)}^{\Theta_I^*(n_t, -sA)} b_t \int_{x^*_I(n_t)}^{\Theta_I^*(n_t, -sA)} g(x_{it}|\Theta)dx_{it}f(\Theta_t|\Delta - sA)d\Theta_t$$

$$+ \left(1 - p_t - q_t\right) \left( -\int_{-\infty}^{\Theta_I^*(n_t, -sA)} b_t \int_{x^*_I(n_t)}^{\Theta_I^*(n_t, -sA)} g(x_{it}|\Theta)dx_{it}f(\Theta_t|0)d\Theta_t \right)$$

where $p_1 = p$, $q_1 = q$, and $p_2 = p'$, $q_2 = q'$. The distribution of the fundamental conditional on the realized macro shock, $f(\Theta_t|m)$, is normal with mean $\mu + m$ and precision $\alpha$. The distribution of the private signal conditional on the fundamental, $g(x_t|\Theta_t)$, is normal with mean $\Theta_t$ and precision $\beta$.

To build intuition, suppose that $1 - p - q \to 0$. Given $\Theta_I^*(1, -sA) > \Theta_I^*(1, A)$ we have that $x^*_I(n_t, -sA) > x^*_I(n_t) > x^*_I(n_t, A)$. Thus, the marginal benefit of increasing $x^*_I(n_t, -sA)$ above $x^*_I(n_t)$ is:

$$p_t \left( b_t \int_{-\infty}^{\Theta_I^*(n_t, -sA)} g(x_{it}|\Theta_t)f(\Theta_t)d\Theta_t \right) > 0,$$

while the marginal benefit of increasing $x^*_I(n_t, A)$ above $x^*_I(n_t)$ is:

$$q_t \left( b_t \int_{-\infty}^{\Theta_I^*(n_t, A)} g(x_{it}|\Theta_t)f(\Theta_t|\Delta)d\Theta_t \right) < 0.$$
the expressions in equations (15) and (16) have two components. The first component in each equation represents the marginal benefit of attacking when a crisis occurs. Equivalently, this is the marginal loss from not attacking when a crisis occurs (type-I error). The second component in each equation is negative and represents the marginal cost of attacking when no crisis occurs (type-II error).

Lemma 3 together with Proposition 1 and Corollary 1 imply the following. The marginal benefit of increasing \( x_{tI}^* (n_t, -s\Delta) \) above \( x_{tU}^* (n_t) \) is positive because the type-I error is relatively more costly than the type-II error. By contrast, the marginal benefit of decreasing \( x_{tI}^* (n_t, \Delta) \) below \( x_{tU}^* (n_t) \) is positive because the type-II error is more costly. In sum, informed investors attack more aggressively upon learning that \( m = -s\Delta \) and less aggressively upon learning \( m = \Delta \). The value of information is governed by the relationship between the type-I and type-II errors.

**Proposition 2 Strategic complementarity in information choices.** If Assumption 1 holds, the value of information increases in the proportion of informed investors:

\[
\frac{dv(n_t)}{dn_t} \geq 0, \tag{17}
\]

with strict inequality for small values of \( n_t \).

**Proof** See Appendix C.2.

When the signal threshold of informed and uninformed investors differ, the value of information is positive because the difference in signal thresholds increases in the proportion of informed investors (Lemma 3). A strategic complementarity in information choices arises, as in [Hellwig and Veldkamp (2009)]. Formally, the signal thresholds \( x_{tI}^* (n_t, \Delta) \) and \( x_{tI}^* (n_t, -s\Delta) \) diverge as \( n_t \) increases (Figure 2).

If \( 1 - p - q \) is sufficiently small, this effect translates one-for-one into an increase in the value of information because the role of the third threshold \( x_{tI}^* (n_t, 0) \) becomes negligible. As a result, the individual attack decision of an informed investor is more strongly adjusted, the larger the proportion of informed investors, which in turn increases the value of information.
4.2.2 Endogenous information and crises

Next, we link the value of information in region 2 to the occurrence of a crisis in region 1. We study how such a wake-up call affects the information choice of investors in region 2.

Lemma 4 Wake-up call and the value of information. If Assumption 4 holds, crises are a rare events, $\mu > \mu$, and the macro shock is sufficiently negatively skewed, $s > s$, then the value of information is higher after a crisis in region 1:

$$v(n_2 = 1, f_1 = 1) > v(n_2 = 0, f_1 = 1) > v(n_2 = 1, f_1 = 0) > v(n_2 = 0, f_1 = 0).$$ (18)

Proof See Appendix C.3

If crises are rare events and the macro shock is sufficiently negatively skewed, there is a strong Bayesian updating channel after a crisis in region 1. While only a remote possibility when no crisis is observed in region 1, there is a substantially elevated risk of a negative macro shock after a crisis. Intuitively, the elevated risk of a strongly negative macro shock generates high incentives of investors to learn about the exposure. This explains the second inequality in (18). The first and third inequality directly arise from the strategic complementarity in information choices. Equipped with Lemma 4, we analyze the equilibrium in region 2.

Figure 2: Signal thresholds are monotonic in the proportion of informed investors.
Proposition 3 Existence of a unique monotone equilibrium in region 2. Suppose the conditions of Lemma 4 hold. If the information cost takes an intermediate value, \( v(0,1) > c > v(1,0) \), then there exists a unique monotone perfect Bayesian equilibrium in region 2. At the information stage, investors in region 2 acquire information about the macro shock only after a crisis in region 1 occurred, \( n^*_2(f_1) = f_1 \). At the coordination stage, this equilibrium is characterized by threshold strategies. If no crisis occurred, investors attack whenever their private signal is sufficiently low, \( x_{2U}(0) \), and a crisis occurs in region 2 whenever the fundamental is sufficiently low, \( \Theta_2 < \Theta_2^*(0,m) \), for each realized macro shock, \( m \in \{-s\Delta, 0, \Delta\} \). After the wake-up call of a crisis in region 1, investors attack whenever their private signal is sufficiently low, \( x_{2L}(1,m) \), and a crisis occurs in region 2 whenever the fundamental is sufficiently low, \( \Theta_2 < \Theta_2^*(1,m) \).

For an intermediate range of costs, the equilibrium is in dominant actions at the information stage. Each investor has an incentive to acquire information after a wake-up call, and no incentive to acquire information after no crisis in region 1, irrespective of the information choices of other investors. Unique equilibria characterized by \( n^*_2 = 0 \) or \( n^*_2 = 1 \) also exist if costs are high, \( c \geq v(1,1) \), or low, \( c \leq v(0,0) \), respectively. For other information cost ranges, there exist multiple equilibria due to strategic complementarity, as studied by [Hellwig and Veldkamp, 2009] who also discuss how the equilibrium set may be reduced. Figure 3 illustrates.

The result on the differential incentives to acquire information about the macro shock is consistent with empirical evidence. In a study on the U.S. stock market, Vlastakis and Markellos (2012) find evidence for a positive association between the demand for market information (that is, information about the aggregate component \( m \)) with measures of volatility that, in the context of our model, can be related to the occurrence of a crisis in region 1. Furthermore, the authors find that the demand for market information is relatively important vis à vis the demand for idiosyncratic information (that is, information about \( \theta_2 \)).
5 Contagion

In this section we derive two results on contagion. First, we assume that information costs are high such that information acquisition never occurs, \( c > v(1, 1) \), and derive a result on information contagion based on Bayesian updating. Second, for intermediate costs, \( v(0, 1) > c > v(1, 0) \), we derive the result of wake-up call contagion. Investors acquire information only after a wake-up call and contagion occurs even if investors learn that the macro shock is zero. Thereby, we isolate the wake-up call component of contagion. That is, contagion occurs despite the favorable news of no exposure to the crisis in region 1. Notably, these contagion results do not hinge upon a common investor base or balance sheet links across regions.

Lemma 5 **Information contagion.** Under the sufficient conditions of Lemma 4 but a high information cost, \( c > v(1, 1) \), there exists a unique monotone perfect Bayesian equilibrium in region 2 in which no investor acquires information, \( n_2^* = 0 \). A crisis in region 2 is more likely after a crisis in region 1 than after no crisis:

\[
\Pr\{\Theta_2 < \Theta_2^*(f_1 = 1)\} > \Pr\{\Theta_2 < \Theta_2^*(f_1 = 0)\}.
\]

(19)

**Proof** See Appendix D.1

Lemma 5 compares the probability of a crisis in region 2 conditional on whether or not a crisis occurred in region 1. For a sufficiently high information cost, investors in region 2 choose not to acquire information irrespective of the occurrence of a crisis in region 1. In this case, a crisis in region 1 is unfavorable news about the fundamental in region 1. Since the macro shock is a common component of both regional fundamentals, this crisis is also unfavorable news about the fundamental in region 2. As a result, the re-assessment of the local fundamental \( \Theta_2 \) via Bayesian updating increases the probability of a crisis in region 2.

---

This result on information contagion is related to the existing literature on contagion due to ex-post correlated fundamentals, such as Acharya and Yorulmazer (2008) and Allen et al. (2012). Acharya and Yorulmazer (2008) show that the funding cost of one bank increases after bad news about another bank when the banks’ loan portfolio returns have a common factor. To avoid information contagion ex post, banks herd their investment ex ante. Allen et al. (2012) compare the impact of information contagion on systemic risk across asset structures. Adverse news about the solvency of the banking system leads to runs on multiple banks.
Next, we isolate the wake-up call component of contagion, which goes beyond information contagion. For an intermediate information cost, investors choose to acquire information about the macro shock only after a wake-up call. We show that contagion occurs even if investors learn that the macro shock is zero and region 2 has therefore no exposure to (the crisis in) region 1.

**Proposition 4 Wake-up call contagion.** Consider the sufficient conditions of Proposition 3. A crisis in region 2 is more likely after a crisis in region 1, when all investors acquire information and learn that the macro shock is zero, than after no crisis in region 1, when investors choose not to acquire information:

\[ \Pr\{\Theta_2 < \Theta_2^*(f_1 = 1, m = 0)\} > \Pr\{\Theta_2 < \Theta_2^*(f_1 = 0)\}. \]  

(20)

**Proof** See Appendix D.2

The right-hand side of inequality (20) is the probability of a crisis in region 2 after no crisis in region 1. As shown in section 4.2, investors choose not to acquire information about the macro shock without a wake-up call. Thus, the macro shock is unobserved and the conditional probability allows for any realization \( m \in \{-s\Delta, 0, \Delta\} \). By contrast, the left-hand side of inequality (20) is the probability of a crisis in region 2 after a crisis in region 1 and a zero macro shock, \( m = 0 \). As shown in section 4.2, all investors choose to acquire information about the macro shock after a wake-up call and learn about the zero exposure to region 1.

Inequality (20) rests on the Bayesian updating about the distribution of the macro shock. Learning that the macro shock is zero on the left-hand side, the crisis in region 1 is uninformative about the fundamental in region 2. In contrast, no crisis in region 1 implies a more favorable view about the fundamental in region 2 due to the unobserved common macro shock. This effect tends to lower the right-hand side. While updating is somewhat mechanical, the result of wake-up call contagion arises endogenously because, for intermediate information costs, investors only choose to acquire information after the wake-up call.
6 Transparency

How could a policymaker mitigate the incidence of contagion – a crisis in region 2 after a crisis in region 1? We focus on $f_1 = 1$ and suppose that the policymaker has superior information about the macro shock by observing it at the beginning of $t = 2$. Direct communication via revealing the macro shock is not credible, since the policymaker has an incentive to misrepresent the macro shock after the low realization, $m = -s\Delta$. Therefore, we focus on the manipulation of transparency by the policymaker, for example via disclosure, and ex-post verification by the investors through information acquisition. As in Morris and Shin (2002), the policymaker can costlessly increase the quality of public information about the local fundamental in region 2, $\alpha_2$. Crucially, increases in transparency affect the incentives of investors in region 2 to acquire information about the macro shock.

**Proposition 5 Transparency and information acquisition.** Suppose Assumption 7 holds. An increase in transparency, $\alpha_2$, increases the incentives to acquire information, $v(1, f_1)$, irrespective of $f_1$.

**Proof** See Appendix D.3

Proposition 5 states that greater transparency increases the incentives to acquire information. This result is due to the larger benefit of tailoring the signal thresholds to the realized macro shock. The distance between the signal thresholds for informed investors increases in $\alpha_2$ given Assumption 7. In particular, a sufficiently high values of $s$ and $\mu$ imply that $\frac{d\alpha_2^*(1, \Delta)}{d\alpha_2} < 0$ and $\frac{d\alpha_2^*(1, -s\Delta)}{d\alpha_2} > 0$. The differential impact of greater transparency is an extension of a comparative static result in Metz (2002) in a standard model with one signal threshold only. Intuitively, an increase in transparency is associated with less aggressive attacks against the regime if the prior about the fundamentals is strong ($m = \Delta$), and with more aggressive attacks if the prior is weak ($m = -s\Delta$).

The impact of transparency on information acquisition in coordination games has been studied before. In the context of a beauty contest model with private information acquisition, Colombo et al. (2014) find a crowding-out effect of public
information. In particular, the incentives to acquire more precise private information decrease in the precision of public information. In contrast, Szkup and Trevino (2012) study private information acquisition in a regime change model and show that public and private information can also be complements. We also study a global coordination game of regime change. However, we study the information acquisition about the macro component of regional fundamentals that can be learned by other investors, not an improvement in the precision of private information. We find that greater transparency induces information acquisition, giving rise to a complementary relationship. Next, we consider policy implications of this result.

The policymaker lacks credibility and cannot commit to a certain information policy ex-ante. Through our analysis of the policy game, we restrict attention to equilibria in pure strategies and focus on the most interesting case of a policy maker who faces a crisis and observes favorable news.

**Corollary 2 Policy response to crisis in region 1.** Consider a crisis in region 1. If Assumption 1 holds and the policymaker observes \( m \geq 0 \), then its optimal response is to increase transparency. Greater transparency facilitates information acquisition by investors when it may not occur otherwise, \( v(1, 1, \alpha_{low}^2) < c < v(1, 1, \alpha_{high}^2) \).

After a crisis in region 1, investors assign a high probability to a negative macro shock. If the policymaker observes favorable news about the macro shock, \( m \geq 0 \), it has an incentive to induce information acquisition in order to reduce the incidence of crises. Therefore, the policymaker increases transparency, creating a range of information costs, \( [v(1, 1, \alpha_{low}^2), v(1, 1, \alpha_{high}^2)] \), for which information acquisition does not take place when transparency is low but may take place when transparency is high. Specifically, greater transparency increases the value of information when all investors are informed (Proposition 5), so the incidence of a crisis in region 2 can be mitigated in case of favorable news about the macro shock.

We briefly discuss the other cases. First, since the policymaker increases transparency after favorable news, investors infer that the macro shock is negative (\( m < 0 \)) if they observe no increase in transparency. However, the case of a negative macro shock is less interesting because in this case the probability of crisis is high independent of the information policy (‘doomed policymaker’). Second, the case
without a crisis in region 1 is also less interesting because investors assign a tiny probability to the negative macro shock.

7 Discussion

We discuss two extensions and an alternative modeling approach. First, we have so far considered the case of a perfect signal about the macro shock. A generalization to noisy signals is possible but would not affect the key insights of the paper.

Second, an important channel of our paper is how a wake-up call affects the incentives of investors in region 2 to acquire information about the macro shock. An additional channel of interest could be private information acquisition (Szkup and Trevino 2012; Ahnert and Kakhbod 2014), whereby investors improve the precision of their private information at a cost after the wake-up call. In case of a continuous private signal precision choice with convex costs, the marginal benefit of private information decreases (increase) in the public signal if the prior of the fundamental in region 2 is strong (weak). Together with Assumption 1, these results suggest that learning about a favorable macro shock increases the incentives to acquire private information and, in turn, increases the incidence of contagion. Likewise, learning about a negative macro shock decreases the incentives to acquire private information and, in turn, increases the incidence of contagion, which is due to the weak prior after learning \( m = -s\Delta \). As a result, the effect of wake-up call contagion is larger when private information choice is also allowed (Ahnert and Bertsch 2015).

Third, one could consider an alternative model setup, where learning is not about the realization of the macro shock but about whether region 2 is exposed to the macro shock itself. In this setup, two macro shock realizations suffice, so \( 1 - p - q = 0 \), where the scenario of no exposure to the macro shock is equivalent to \( m = 0 \). As before, both observing a crisis in region 1 and learning about an exposure to the macro shock suggest that the fundamentals in region 2 are likely to be affected by a negative macro shock that also contributed to the crisis in region 1. Conversely, learning about no exposure to the macro shock after observing a crisis in region 1 is favorable information for the local fundamentals in region 2. Still, no crisis in region 1 implies a more favorable view about the fundamental in region 23.
2 when an exposure to the macro shock has positive weight due to the Bayesian updating channel described in section 5. Hence, the wake-up call component of contagion can be isolated in the same way in this alternative model setup. For the incentives to acquire information about the macro shock in the alternative setup, the underlying mechanics and intuition are similar to those discussed in section 4.2.2.

8 Conclusion

We propose a theory of contagion to explain how wake-up calls transmit financial crises. We study global coordination games of regime change in two regions that may only be linked via an initially unobserved and negatively skewed macro shock. A crisis in region 1 is a wake-up call for investors in region 2 and induces them to re-assess the local fundamental in region 2. Because of the negatively skewed macro shock, investors have an incentive to acquire information only after a wake-up call. The probability of a crisis in region 2 is higher after crisis in region 1 than after no crisis, even if investors learn that the macro shock is zero and, hence, that there is no exposure to the crisis in region 1. In short, we isolate the wake-up call component of contagion by abstracting from ex-post exposure to the crisis region, common lenders effects and balance sheet links.

Our contribution is to link the optimal information choices of investors to the notion of wake-up calls. We describe how the incentives to acquire information about the macro shock depend on distributional characteristics of the regional fundamental and the macro shock. Information choices are strategic complements and we show how contagion occurs even if investors learn that the macro shock is zero. To mitigate contagion, a policymaker with favorable news about her region induces information acquisition by increasing transparency after a wake-up call.

The wake-up call theory of contagion has several applications. Currency speculators observe an exchange rate crisis elsewhere and are uncertain about the magnitude of trade and financial links. Uninsured bank creditors observe a run elsewhere and are uncertain about interbank linkages. Sovereign debt holders observe a default elsewhere and are uncertain about the resources and commitment of multilateral bail-out funds or the international lender of last resort.
References


A  Timeline

Date 1:  
• The macro shock $m$ is realized but unobserved.  
• The regional component $\theta_1$ is realized but unobserved.  

Coordination stage in region 1  
• Investors receive private information $x_{i1}$ about the fundamental $\Theta_1$.  
• Investors simultaneously decide whether to attack, $a_{i1}$.  
• Payoffs to investors in region 1.  

Date 2:  Information stage in region 2  
• The regional component $\theta_2$ is realized but unobserved.  
• Investors observe whether there was a crisis in region 1.  
• Investors re-assess the local fundamental $\Theta_2$.  
• Investors in region 2 decide simultaneously whether to purchase a signal about $m$ at a cost $c > 0$.  

Coordination stage  
• Investors receive private information $x_{i2}$ about the fundamental $\Theta_2$.  
• Investors simultaneously decide whether to attack, $a_{i2}$.  
• Payoffs to investors in region 2.  

Table 1: Timeline of events.

B  Equilibrium

B.1 Deriving the equilibrium in region 1 for the general case

Result I characterizes the equilibrium of the coordination stage in region 1 for the special case when all investors are informed, $n_1 = 1$. In this section we derive the equilibrium conditions for the general case in which some investors are uninformed, $n_1 \in [0, 1)$. In Appendix B.2 and B.3 we subsequently prove the existence of a unique monotone equilibrium, first, for the special case when $n_1 = 0$ and, thereafter, for the general case when $n_1 \in [0, 1]$.  

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Some investors are uninformed

All investors receive a private signal $x_{i1}$ about the regional fundamental. Informed investors also observe the realized macro shock, while uninformed investors use their private signal to update their belief about the macro shock.

Bayesian updating We show that the relationship between the posterior probabilities and the private signal, $x_{i1}$, is non-monotone. Uninformed investors use Bayes’ rule to form a belief about the macro shock, where $\hat{p} \equiv \Pr\{m = \Delta | x_{i1}\}$, and $\hat{q} \equiv \Pr\{m = -s\Delta | x_{i1}\}$:

$$\hat{p} = \frac{p \Pr\{x_{i1} | m = \Delta\} + q \Pr\{x_{i1} | m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1} | m = 0\}}{p \Pr\{x_{i1} | m = \Delta\} + q \Pr\{x_{i1} | m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1} | m = 0\}}. \quad (21)$$

$$\hat{q} = \frac{q \Pr\{x_{i1} | m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1} | m = 0\}}{p \Pr\{x_{i1} | m = \Delta\} + q \Pr\{x_{i1} | m = -s\Delta\} + (1 - p - q) \Pr\{x_{i1} | m = 0\}}. \quad (22)$$

For each $m$, $\Pr\{x_{i1} | m\}$ can be expressed as follows:

$$\Pr\{x_{i1} | m\} = \frac{1}{\sqrt{\text{Var}[x_{i1} | m]}} \phi\left(\frac{x_{i1} - \mathbb{E}[x_{i1} | m]}{\sqrt{\text{Var}[x_{i1} | m]}}\right) = \left(\frac{1}{\alpha} + 1\right)^{-\frac{1}{2}} \phi\left(\frac{x_{i1} - (\mu + m)}{\sqrt{\frac{1}{\alpha} + 1}}\right)$$

where we use $p = q s$ to obtain:

$$\frac{d\hat{p}}{dx_{i1}} > 0, \quad \frac{d\hat{q}}{dx_{i1}} < 0, \quad d\left(1 - \hat{p} - \hat{q}\right) \left[x_{i1} - \mu + \frac{1 - s}{2}\Delta\right] \leq 0, \quad (23)$$

with strict inequality if $x_{i1} \neq \mu + \frac{1 - s}{2}\Delta$. An investor in region 1 places more weight on the probability of a positive (negative) macro shock after a higher (lower) private signal. Also, the posterior probability for a zero macro shock increases in the private signal if the signal is sufficiently high. The bound on the private signal, $x_1(s, \Delta) = \mu + \frac{1 - s}{2}\Delta$ is below $\mu$ if the macro shock is negatively skewed ($s > 1$).

Equilibrium conditions For the general case of $n_1 \in [0, 1]$, we derive a system of equations that comprise the critical mass and indifference conditions for region 1. The critical mass conditions state that the proportion of attacking investors $A^*_1(m)$ equals the fundamental threshold $\Theta^*_1(m)$ for each realized $m \in \{-s\Delta, 0, \Delta\}$:

$$\Theta^*_1(m) = n_1 \Phi\left(\sqrt{\beta}\left[x^*_1(m) - \Theta^*_1(m)\right]\right) + (1 - n_1) \Phi\left(\sqrt{\beta}\left[x^*_1U - \Theta^*_1(m)\right]\right), \quad (24)$$
where the short-hands are $\Theta_i^*(m) \equiv \Theta_i^*(n_1, m)$, $x_{1I}^*(m) \equiv x_{1I}^*(n_1, m)$, and $x_{1U}^* \equiv x_{1U}^*(n_1)$ for the fundamental threshold and the signal thresholds of informed and uninformed investors, respectively. The first indifference condition states that an uninformed investor with threshold signal $x_{i1} = x_{1U}^*$ is indifferent whether to attack:

$$
\hat{p}^* \Psi(\Theta_i^*(\Delta), x_{1U}^* \Delta) + \hat{q}^* \Psi(\Theta_i^*(-s\Delta), x_{1U}^* (-s\Delta))
$$

$$
\equiv (1 - \hat{p}^* - \hat{q}^* ) \Psi(\Theta_i^*(0), x_{1U}^*, 0) = \gamma_i
$$

(25)

where $\hat{p}^* = \hat{p}(x_{1U}^*)$ and $\hat{q}^* = \hat{q}(x_{1U}^*)$, for $d \in \{I, U\}$ and $m \in \{-s\Delta, 0, \Delta\}$:

$$
\Psi(\Theta_i^*(m), x_{1d}^*, m) \equiv \Phi \left( \Theta_i^* \sqrt{\alpha + \beta} - \frac{\alpha(m) + \beta x_{1d}^*}{\sqrt{\alpha + \beta}} \right).
$$

(26)

Three additional indifference conditions, one for each realized macro shock, state that an informed investor is indifferent between attacking or not upon receiving the threshold signal $x_{i1} = x_{1I}^*(m)$:

$$
\Psi(\Theta_i^*(n_1, m), x_{1I}^*(m), m) = \gamma_i \ \forall \ m \in \{-s\Delta, 0, \Delta\}.
$$

(27)

We have seven equations in seven unknowns. In the simplest case with $n_1 = 1$ (Result 1), we had two thresholds $x_1^*$ and $\Theta_1^*$ for each $m$. There, the objective was to establish aggregate behavior by inserting the critical mass condition, which states $x_1^*$ in terms of $\Theta_1^*$, into the indifference condition. This yields one equation implicit in $\Theta_1^*$. We pursue a similar strategy here, solving this system of equations in order to express the equilibrium in terms of $\Theta_1^*(-s\Delta)$, $\Theta_1^*(0)$ and $\Theta_1^*(\Delta)$ only.

We also use the following insight. Since uninformed investors do not observe the macro shock realization, the signal threshold must be identical across these realizations, $x_{1U}^*(-s\Delta) = x_{1U}^*(0) = x_{1U}^*(\Delta)$. In the following steps, we derive this threshold for each realization of $m$ by using the fundamental threshold $\Theta_i^*(m)$ and equalize both expressions. First, we use the critical mass conditions in equation (24) for $\Theta_i^*(m)$ to express $x_{1U}^*$ as a function of each $\Theta_i^*(m)$ and $x_{1I}^*(m)$. Second, we use the indifference condition of informed investors for each $m$, equation (27), to obtain $x_{1I}^*(m)$ as a function of $\Theta_i^*(m)$. Thus, $\forall m$:

$$
x_{1U}^*(m) = \Theta_i^*(m) + \frac{\Phi^{-1} \left( \Theta_i^*(m) - n_1 \Phi \left( \frac{\alpha(\Theta_i^*(m) - (\mu + m)) - \sqrt{\alpha + \beta}}{\sqrt{\alpha + \beta}} \Phi^{-1}(\gamma_i) \right) \right)}{1 - n_1 \Phi^{-1}(\gamma_i)}.
$$

(28)
Hence, for \( m \in \{-s\Delta, 0, \Delta\} \), there exists a \( \beta_1 \in (0, \infty) \) such that for all \( \beta > \beta_1 \):

\[
\frac{dx^*_U(m)}{d\Theta_1^*(m)} > 0. \tag{29}
\]

Since the signal threshold is the same for an uninformed investor, subtracting equation (28) evaluated at \( m = 0 \) from the same equation evaluated at \( m = -s\Delta \) or at \( m = \Delta \) must yield zero. This yields the first two pair-wise implicit relationships between \( \Theta_1^*(-s\Delta), \Theta_1^*(0) \) and \( \Theta_2^*(\Delta) \):

\[
K_1(n_1, \Theta_1^*(-s\Delta), \Theta_1^*(0)) \equiv x_1^*_U(0) - x_1^*-s\Delta = 0 \tag{30}
\]

\[
K_2(n_1, \Theta_1^*(0), \Theta_1^*(\Delta)) \equiv x_1^*_U(0) - x_1^*\Delta = 0. \tag{31}
\]

Now, we construct the third implicit relationship between the three aggregate thresholds by inserting equation (28) evaluated at each \( m \) in \( \Psi'(\Theta_1^*(n_1,m), x_{1U}^*(m), m) \), respectively, and in \( \hat{p} \) and \( \hat{q} \) as used in \( J(n_1, \Theta_1^*(-s\Delta), \Theta_1^*(0), \Theta_1^*(\Delta), x_{1U}^*) \). Combining the expressions yields:

\[
L(n_1, \Theta_1^*(-s\Delta), \Theta_1^*(0), \Theta_1^*(\Delta)) \equiv J(n_1, \Theta_1^*(-s\Delta), \Theta_1^*(0), \Theta_1^*(\Delta)) = \gamma_1. \tag{32}
\]

### B.1.2 Monotone equilibria

If all investors are uninformed, \( n_1 = 0 \), the system of equations derived in Appendix section B.1.1 simplifies. Specifically, there is only one fundamental threshold and the system can be reduced to one equation in one unknown, where \( \Theta_1^*(0, -s\Delta) = \Theta_1^*(0, 0) = \Theta_1^*(0, \Delta) \) in equation (25). Using the results of Milgrom (1981) and Vives (2005), we show that the best-response function of an individual investor strictly increases in the threshold used by other investors. Therefore, our focus on monotone equilibria is valid. Next, we determine conditions sufficient for a unique monotone Bayesian equilibrium in Lemma 1.

In contrast to the standard analysis of region 1, \( J(0, \Theta_1) \) is harder to characterize. The weights of the mixture distribution and the posterior beliefs about the correlation now depend on the threshold signal \( x_{1U}^* \). Therefore, the question arises whether or not our focus on monotone equilibria is justified, in light of the global non-monotonicity of \((1 - \hat{p}(x_{1U}^*) - \hat{q}(x_{1U}^*)) \) in \( x_{1U}^* \) and, hence, in \( \Theta_1^*(0, m) \), as established above. Fortunately, the best-response function of an individual investor \( i \)
is proven to be strictly increasing in the threshold used by other investors:

$$r' = \frac{d\Pr(\Theta_1 < \hat{\Theta}_1(\hat{x}_i)|x_i)}{d\hat{x}_i} > 0,$$

(33)

where $\hat{x}_i$ is the critical threshold of the private signal used by player $i$, $\hat{x}_1$ is the threshold used by all other investors, and $\hat{\Theta}_1(\hat{x}_1)$ is the critical threshold of the fundamental in region 1 when $n = 0$. This is because $\Pr(\Theta_1 < \Theta_1'|x_i)$ is monotonically decreasing in $x_i$, using a result of [Milgrom (1981)](see below). Furthermore, given all other investors use a threshold strategy, $\Pr(\Theta_1 < \hat{\Theta}_1(\hat{x}_1)|x_i)$ increases in $\hat{x}_1$ (again see below). Following [Vives (2005)], the best response of player $i$ is to use a threshold strategy with attack threshold $\hat{x}_i$, where $\Pr(\Theta_1 < \hat{\Theta}_1(\hat{x}_1)|\hat{x}_i) = \gamma_1$, implying $r' > 0$. Therefore, our focus on monotone equilibria is valid.

The conditional density function $f(x|\Theta)$ is normal with mean $\Theta$ and satisfies the monotone likelihood ratio property: for all $x_i > x_j$ and $\Theta' > \Theta$, we have:

$$\frac{f(x_i|\Theta')}{f(x_i|\Theta)} \geq \frac{f(x_j|\Theta')}{f(x_j|\Theta)} \iff \frac{\phi\left(\sqrt{\beta} (x_i - \Theta')\right)}{\phi\left(\sqrt{\beta} (x_j - \Theta')\right)} \geq \frac{\phi\left(\sqrt{\beta} (x_i - \Theta)\right)}{\phi\left(\sqrt{\beta} (x_j - \Theta)\right)}.$$

(34)

Using Proposition 1 of [Milgrom (1981)], we conclude that $\Pr(\Theta_1 \leq \Theta_1'|x_i)$ monotonically decreases in $x_i$. Hence, $\frac{d\hat{\Theta}_1(\hat{x}_i)}{d\hat{x}_i} > 0$. Equation (25) then implies:

$$0 \leq \frac{d\hat{\Theta}_1(\hat{x}_i)}{d\hat{x}_i} \leq \left(1 + \sqrt{\frac{2\pi}{\beta}}\right)^{-1}.$$

(35)

### B.2 Proof of Lemma 1

The proof consists of three steps. First, we show that $J(0, \Theta_1(0,m), x_{1U}(m)) \equiv J(0, \Theta_1) \to 1 > \gamma_1$ as $\Theta_1 \to 0$, and $J(0, \Theta_1) \to 0 < \gamma_1$ as $\Theta_1 \to 1$. Second, we show that $\frac{dJ(0,\Theta_1)}{d\Theta_1} < 0$ for some sufficiently high but finite values of $\beta$, such that $J$ strictly decreases in $\Theta_1$. We denote this lower bound as $\beta_1$. Therefore, if $\Theta_1'$ exists, it is unique. Third, by continuity, there exists a $\Theta_1'(0,m)$ that is characterized by (6).

**Step 1 (limiting behavior):** Let $\Psi(\Theta_1(0,m), x_{1L}(m), m) \equiv \Psi(\Theta_1(0,m), m)$. Observe that $J(0, \Theta_1(0,m), x_{1U}(m))$ is a weighted average of the $\Psi(\Theta_1(0,m), m)$'s
evaluated at the different levels of $m$. As $\Theta_1 \to 0$, then $\Psi(\Theta_1(0,m), m) \to 1$ for any $m \in \{-s\Delta, 0, \Delta\}$, so $J(0, \Theta_1) \to 1 > \gamma_1$. Likewise, as $\Theta_1 \to 1$, then $\Psi(\Theta_1(0,m), m) \to 0$ for any $m \in \{-s\Delta, 0, \Delta\}$, so $J(0, \Theta_1) \to 0 < \gamma_1$.

**Step 2 (strictly negative slope):** Using the indifference condition of uninformed investors to substitute $x_{1U}^*$ in equation (25), the total derivative of $J$ is:

$$
\frac{dJ(0, \Theta_1)}{d\Theta_1} = \dot{p}(x_{1U}(\Theta_1)) \frac{d\Psi(\Theta_1, \Delta)}{d\Theta_1} + \dot{q}(x_{1U}(\Theta_1)) \frac{d\Psi(\Theta_1, -s\Delta)}{d\Theta_1} + (1 - \dot{p}(x_{1U}(\Theta_1)) - \dot{q}(x_{1U}(\Theta_1))) \frac{d\Psi(\Theta_1, 0)}{d\Theta_1}
$$

$$
+ \frac{d\dot{p}(x_{1U}(\Theta_1))}{dx_{1U}(\Theta_1)} \frac{dx_{1U}(\Theta_1)}{d\Theta_1} [\Psi(\Theta_1, \Delta) - \Psi(\Theta_1, 0)]
$$

$$
+ \frac{d\dot{q}(x_{1U}(\Theta_1))}{dx_{1U}(\Theta_1)} \frac{dx_{1U}(\Theta_1)}{d\Theta_1} [\Psi(\Theta_1, -s\Delta) - \Psi(\Theta_1, 0)].
$$

(36)

The proof proceeds by inspecting the individual terms of equation (36). We know from our analysis of the case of informed investors that $\frac{d\Psi(\Theta_1, m)}{d\Theta_1} < 0$ if $\beta > \frac{\alpha^2}{2\pi}$ for all $m$. Thus, the first three components of the sum are negative and finite for sufficiently high but finite private noise.

The sign of the two terms in square brackets in the last two summands in (36) is negative and positive, respectively: $\Psi(\Theta_1^*(0, \Delta), \Delta) \leq \Psi(\Theta_1^*(0, 0), 0)$ given $\Theta_1^*(1, \Delta) \leq \Theta_1^*(1, 0)$ and $\Psi(\Theta_1^*(0, -s\Delta), \Delta) \geq \Psi(\Theta_1^*(0, 0), 0)$ given $\Theta_1^*(1, -s\Delta) \geq \Theta_1^*(1, 0)$, where $\Theta_1^*(0, \Delta) = \Theta_1^*(0, 0) = \Theta_1^*(0, -s\Delta)$. However, the difference vanishes in the limit when $\beta \to \infty$.

The last terms to consider are $\frac{d\dot{p}(x_{1U}(\Theta_1))}{dx_{1U}(\Theta_1)} \frac{dx_{1U}}{d\Theta_1}$ and $\frac{d\dot{q}(x_{1U}(\Theta_1))}{dx_{1U}(\Theta_1)} \frac{dx_{1U}}{d\Theta_1}$. Given the previous sufficient conditions on the relative precision of the private signal:

$$
0 < \frac{dx_{1U}}{d\Theta_1} = 1 + \frac{1}{\sqrt{\beta \phi(\Phi^{-1}(\Theta_1))}} < 1 + \frac{\sqrt{2\pi}}{\alpha}.
$$

The derivative is finite for $\beta \to \infty$. Taken together with the zero limit of the first factor of the third and forth term, this terms vanish in the limit. As a result, by continuity, there must exist a finite level of precision $\beta > \beta_1 \in (0, \infty)$ such that $\frac{dJ(0, \Theta_1)}{d\Theta_1} < 0$ for all $\beta > \beta_1$.

**Step 3 (existence and characterization):** By continuity, there exists a $\Theta_1^*(0, m)$ that solves $J(0, \Theta_1) = \gamma_1$. It is characterized by inequality (6). To see this, recall
that $J(0, \Theta_1(0, m), x_U(m))$ is a weighted average of the $\Psi(\Theta_1(0, m), m)$’s evaluated at the different levels of $m$. Given the strict difference in the $\Theta_1^\ast(1, m)$’s for different levels of $m$ and due to $\frac{d\Psi(\Theta_1, m)}{d\Theta_1}, \frac{dJ(0, \Theta_1)}{d\Theta_1} < 0$, $\Theta_1^\ast(0, m)$ is a weighted average of the $\Theta_1^\ast(1, m)$’s for different levels of $m$. Inequality (6) follows. This concludes the third step of the proof and therefore the overall proof of Lemma 1.

### B.3 Proof of Proposition 1

The equilibrium conditions can be expressed as a system of three equations in three unknowns, that is the fundamental thresholds $(\Theta_1^\ast(-s\Delta), \Theta_1^\ast(0), \Theta_1^\ast(\Delta))$. The first and the second equation depend only on two thresholds, $K_1(\Theta_1^\ast(-s\Delta), \Theta_1^\ast(0)) = 0$ and $K_2(\Theta_1^\ast(0), \Theta_1^\ast(\Delta)) = 0$, while the third equation depends on all three thresholds, $L(\Theta_1^\ast(-s\Delta), \Theta_1^\ast(0), \Theta_1^\ast(\Delta)) = \gamma_1$.

In a first step, we analyze the relationship between $\Theta_1(-s\Delta)$ and $\Theta_1(0)$, as governed by $K_1$. Using equations (29) and (30), we obtain $\frac{\partial K_1}{\partial \Theta_1(0)} > 0$, $\frac{\partial K_1}{\partial \Theta_1(-s\Delta)} < 0$, and $\frac{\partial K_1}{\partial \Theta_1(\Delta)} = 0$. Hence, $\frac{d\Theta_1(0)}{d\Theta_1(-s\Delta)} > 0$ by the implicit function theorem. Likewise, we analyze the relationship between $\Theta_1^\ast(0)$ and $\Theta_1^\ast(\Delta)$, as governed by $K_2$. Using equations (29) and (31), we obtain $\frac{\partial K_2}{\partial \Theta_1(0)} > 0$, $\frac{\partial K_2}{\partial \Theta_1(-s\Delta)} = 0$, and $\frac{\partial K_2}{\partial \Theta_1(\Delta)} < 0$. Hence, $\frac{d\Theta_1(0)}{d\Theta_1(\Delta)} > 0$ by the implicit function theorem. These results do not require a bound on the precision of private information.

In a second step, we analyze the relationship between all three fundamental thresholds, as governed by $L$. We know from our analysis of the case of informed investors that $\frac{d\Psi(\Theta_1^\ast, m)}{d\Theta_1} < 0$ for all $m$ if $\beta > \frac{\alpha^2}{2\pi}$. Analogous to the argument in the proof of Lemma 1 there exists a sufficiently high but finite value of the private precision such that $\frac{dL}{d\Theta_1(m)} < 0$ for all $m$. Hence, in the limit $\frac{d\Theta_1(0)}{d\Theta_1(-s\Delta)} < 0$ for a given $\Theta_1(\Delta)$, $\frac{d\Theta_1(0)}{d\Theta_1(-s\Delta)} < 0$ for a given $\Theta_1(-s\Delta)$, and $\frac{d\Theta_1(\Delta)}{d\Theta_1(-s\Delta)} < 0$ for a given $\Theta_1(0)$. By continuity, there exists a finite precision of private information, $\beta_2 \in (0, \infty)$, that guarantees the inequality if $\beta > \beta_2$.

In a third step, we establish uniqueness conditional on existence. Thus suppose for now that an equilibrium exists. Then, due to the monotonicity and the opposite signs of the respective derivatives, we have that there is a single crossing of $K_1$ and $L$ in the $\Theta_1(-s\Delta), \Theta_1(0)$ space and a single crossing of $K_2$ and $L$ in
the \( (\Theta_1(\Delta), \Theta_1(0)) \) space, as shown in Figure B.3. Observe that this is a “partial equilibrium” argument since the third threshold is taken as given. We now move to a “general equilibrium” argument. The argument builds on a second feature of the system, the opposite signs of the respective derivatives are not only a sufficient condition for a single crossings in the two panels of Figure B.3 but they also imply that \( \Theta_1(-s\Delta) \) and \( \Theta_1(0) \) are each decreasing in \( \Theta_1(\Delta) \) (left panel), where an increase in \( \Theta_1(\Delta) \) shifts the \( L \) curve inwards. Likewise, \( \Theta_1(-s\Delta) \) and \( \Theta_1(0) \) are each decreasing in \( \Theta_1(0) \) (right panel). Hence, starting from a general equilibrium, any modification of \( \Theta_1(\Delta) \) and \( \Theta_1(-s\Delta) \) must lead to a violation of the system of equations. Hence, given \( \frac{\partial L}{\partial \Theta_1(\Delta)} < 0 \) and \( \frac{\partial L}{\partial \Theta_1(-s\Delta)} < 0 \), the combination of fundamental thresholds \( (\Theta_1^*(0), \Theta_1^*(-s\Delta)) \) that satisfies \( K_1 \) and \( L \) in the \( (\Theta_1(-s\Delta), \Theta_1(0)) \) space and \( K_2 \) and \( L \) in the \( (\Theta_1(\Delta), \Theta_1(0)) \) space is unique.

![Figure 4: Single crossing.](image)

In a fourth step, we establish the existence of a combination of fundamental thresholds. We outline the proof of this step but relegate a more formal argument to the Technical Appendix. Existence can be shown by proving the following sequence of points: (i) for the highest permissible value of \( \Theta_1(-s\Delta) \), the value of \( \Theta_1(0) \) prescribed by \( K_1 \) is strictly larger than the value of \( \Theta_1(0) \) prescribed by \( L \); (ii) for the lowest permissible value of \( \Theta_1(-s\Delta) \), the value of \( \Theta_1(0) \) prescribed by \( K_1 \) is strictly smaller than the value of \( \Theta_1(0) \) prescribed by \( L \); (iii) for the highest permissible value of \( \Theta_1(\Delta) \), the value of \( \Theta_1(0) \) prescribed by \( K_2 \) is strictly larger than the value of \( \Theta_1(0) \) prescribed by \( L \); (iv) for the lowest permissible value of \( \Theta_1(\Delta) \), the value of \( \Theta_1(0) \) prescribed by \( K_2 \) is strictly smaller than the value of \( \Theta_1(0) \) prescribed by \( L \); (v) for the lowest (highest) permissible value of \( \Theta_1(-s\Delta) \), also \( \Theta_1(0) \) must be at its lowest (highest) permissible value from \( K_1 \) and, hence,
also $\Theta_1(\Delta)$ must be at its lowest (highest) permissible value from $K_2$, leading to a violation of $L$ in both the $(\Theta_1(-s\Delta), \Theta_1(0))$ space and the $(\Theta_1(\Delta), \Theta_1(0))$ space; (vi) a successive increase (decrease) in $\Theta_1(0)$ shifts $L$ continuously inwards (out-wards) in both spaces until a fixed point is reached. This completes the proof.

\section{Information}

\subsection{Proof of Lemma 3}

The idea of the proof of Lemma 3 is as follows. We analyse the system of equilibrium conditions derived in Appendix B. Next, the analysis of $\Theta_t^\ell(n_t, -s\Delta)$ and $\Theta_t^\ell(n_t, \Delta)$ greatly simplifies if the ex-ante probability of a zero macro shock is small, that is $1 - p - q \to 0$. The monotonicity of these thresholds in the proportion of informed investors holds under Assumption 1. This is proven by applying the implicit function theorem and the envelope theorem, taking into account all possible orderings of fundamental thresholds. We also establish the boundedness of these thresholds. By continuity, these results extends to the case in which $1 - p - q$ is sufficiently small but positive. We offer a formal argument in the Technical Appendix.

\subsection{Proof of Proposition 2}

Given $\beta < \beta < \infty$, $0 < \alpha < \overline{\alpha}$, and $1 - p - q$ is sufficiently small, the results of Lemma 3 apply. The proof builds on equation (14). Equation (14) is constructed from $EU_I$ and $EU_U$. The expected utility of an informed investor writes:

\begin{align*}
\mathbb{E}[u(d_i = I, n_t)] &\equiv EU_I - c \\
&= -c + p_t \left( \int_{-\infty}^{\Theta_t^\ell(n_t, \Delta)} b_t \int_{x_{it} \leq \xi_t^\ell(n_t, \Delta)} g(x_{it} | \Theta) dx_{it} f(\Theta_t | \Delta) d\Theta_t - \int_{\Theta_t^\ell(n_t, \Delta)}^{\Theta_t^\ell(n_t, -s\Delta)} \int_{x_{it} \leq \xi_t^\ell(n_t, -s\Delta)} g(x_{it} | \Theta) dx_{it} f(\Theta_t | -s\Delta) d\Theta_t \right) + \int_{\Theta_t^\ell(n_t, -s\Delta)}^{\Theta_t^\ell(n_t, -s\Delta)} \int_{x_{it} \leq \xi_t^\ell(n_t, -s\Delta)} g(x_{it} | \Theta) dx_{it} f(\Theta_t | -s\Delta) d\Theta_t + \int_{\Theta_t^\ell(n_t, -s\Delta)}^{\Theta_t^\ell(n_t, 0)} \int_{x_{it} \leq \xi_t^\ell(n_t, 0)} g(x_{it} | \Theta) dx_{it} f(\Theta_t | 0) d\Theta_t) \right) + (1 - p_t - q_t) \left( \int_{\Theta_t^\ell(n_t, 0)}^{\Theta_t^\ell(n_t, -s\Delta)} \int_{x_{it} \leq \xi_t^\ell(n_t, -s\Delta)} g(x_{it} | \Theta) dx_{it} f(\Theta_t | -s\Delta) d\Theta_t \right) \right);
\end{align*}

36
By contrast, the expected utility of an uninformed investor writes:

\[
\mathbb{E}[u(d_t = U, n_t)] = \mathbb{E}[u]
\]

\[
= p_t \left( \int_{-\infty}^{\Theta_t(n_t, \Delta)} b_t \int_{\Theta_t(n_t, \Delta)} x_{vu}^t(n_t) g(x_{vu} | \Theta) dx_{vu} f(\Theta | \Delta) d\Theta - \int_{\Theta_t(n_t, \Delta)}^{+\infty} b_t \int_{\Theta_t(n_t, \Delta)} x_{vu}^t(n_t) g(x_{vu} | \Theta) dx_{vu} f(\Theta | \Delta) d\Theta + \right)
\]

\[
q_t \left( \int_{-\infty}^{\Theta_t(n_t, -s\Delta)} b_t \int_{\Theta_t(n_t, -s\Delta)} x_{vu}^t(n_t) g(x_{vu} | \Theta) dx_{vu} f(\Theta | -s\Delta) d\Theta - \int_{\Theta_t(n_t, -s\Delta)}^{+\infty} b_t \int_{\Theta_t(n_t, -s\Delta)} x_{vu}^t(n_t) g(x_{vu} | \Theta) dx_{vu} f(\Theta | -s\Delta) d\Theta + \right)
\]

\[
(1 - p_t - q_t) \left( \int_{-\infty}^{\Theta_t(n_t, \Delta)} b_t \int_{\Theta_t(n_t, \Delta)} x_{vu}^t(n_t) g(x_{vu} | \Theta) dx_{vu} f(\Theta | 0) d\Theta - \right)
\]

\[
= \int_{-\infty}^{\Theta_t(n_t, \Delta)} b_t \int_{\Theta_t(n_t, \Delta)} x_{vu}^t(n_t) g(x_{vu} | \Theta) dx_{vu} f(\Theta | \Delta) d\Theta
\]

Under the sufficient conditions of Lemma 3 we have that \(\Theta_t(n_t, -s\Delta) < \Theta_t(n_t, \Delta)\) and \(x_{vu}^t(n_t, -s\Delta) > x_{vu}^t(n_t, 0) > x_{vu}^t(n_t, \Delta)\). We will prove that \(\frac{dv(n_t)}{dn_t} \geq 0\) and \(v(n_t) > 0\) \(\forall n_t \in (0, 1]\).

Suppose that \(1 - q - p \to 0\), then the last term of equations (37) and (38) vanishes. Given that \(\Theta_t(n_t, -s\Delta) > \Theta_t(n_t, \Delta)\) the first two summands of equation (14) are strictly positive and, hence, \(v(n_t) > 0\) \(\forall n_t \in (0, 1]\). Furthermore, given Lemma 3 an increase in the proportion of informed investors is associated with a (weak) increase in both \(\Theta_t(n_t, -s\Delta)\) and \(x_{vu}^t(n_t, -s\Delta)\) as well as a (weak) decrease in both \(\Theta_t(n_t, \Delta)\) and \(x_{vu}^t(n_t, \Delta)\). For a given \(x_{vu}^t\), an increase in \(n_t\) leads to a relative increase of the (positive) loss component in the first summand of equation (14) and a relative increase of the benefit component in the second summand. By continuity and monotonicity, any general equilibrium adjustment of \(x_{vu}^t(n_t)\) with \(n_t\) cannot fully off-set the previous effects. For this reason, the left-hand side of equation (14) increases in \(n_t\). Thus, \(\frac{dv(n_t)}{dn_t} \geq 0\). By continuity, the results continue to hold if \(1 - p - q\) is sufficiently small, that is if \(1 - p - q < \eta\). This concludes the proof.

C.3 Proof of Lemma 4

Given assumption 1 the results of Proposition 2 apply. Therefore, the first and the third inequality follow. The proof of the second inequality builds on equation (14) and consists of five steps.

Step 1: Suppose that \(1 - p - q \to 0\) and evaluate the value of information at
First, observe that the first term in brackets is only affected by $s$ through $x_{2U}^*(1)$. Second, observe that the second term in brackets is growing strictly larger in $s$ for a given $x_{2U}^*(0)$, as $x_{2U}^*(1, -s\Delta)$ grows in $s$ because of equation (27). Third, observe that $x_{2U}^*(1) \to x_{2U}^*(1, \Delta)$ as $s \to \infty$. Given that the term in the second bracket is finite and multiplied by $q = \frac{p}{s}$. We have that $v(1) > v(0) \to 0$ for $s \to \infty$, where the inequality is due to the result in Proposition 2.

**Step 2:** Next, consider the value of information in region 2 and suppose that $f_1 = 0$, where $q' \to 0$ as $s \to \infty$. Hence, $v(1, f_1 = 0) > v(0, f_1 = 0) \to 0$ for $s \to \infty$.

**Step 3:** Now, suppose that $f_1 = 1$ and observe that:
\[
\frac{\partial}{\partial s} \left( \frac{q'}{p'} s \Pr\{f_1 = 1, m = -s\Delta\} \Pr\{f_1 = 1|m = \Delta\} \right) > 0.
\]
Moreover:
\[
q' = \frac{p}{s} \Pr\{f_1 = 1|m = -s\Delta\} + p \Pr\{f_1 = 1|m = \Delta\} + (1 - p - \frac{p}{s}) \Pr\{f_1 = 1|m = 0\},
\]
is increasing in $\mu$, with $q' \to 1$ if $\mu \to \infty$. The results flip if $f_1 = 0$.

**Step 4:** Observe that, for a given $\mu$ and $s > 1$, the event of a negative macro shock is never considered to be the most probable state of the world provided that $s$ is sufficiently high. This is because $q' < p'$ holds or $s$ sufficiently high:
\[
s \geq \frac{\Pr\{f_1|m = -s\Delta\}}{\Pr\{f_1|m = \Delta\}}.
\]

**Step 5:** Given the comparative statics in step 3, we have that $[q'|f_1 = 1] >> 0$ for large values of $s$ and $\mu$. From step 4, the first and the second summand of $[v(1, f_1 = 1]$ must be strictly positive and away from zero if $s \to \infty$, since now $x_{2U}^* \to x_{2U}^*(1, \Delta)$ and $x_{2U}^* \to x_{2U}^*(1, -s\Delta)$ as $s \to \infty$. By continuity, the result also holds for
large, but finite, values of μ and s, as well as for sufficiently small \(1 - p - q\). Hence, equation (18) follows provided that Assumption 1 holds and μ and s are sufficiently high. This concludes the proof.

D  Contagion

D.1  Proof of Lemma 5

The proof consists of two steps that deal with the impact on the fundamental thresholds and conditional distribution. First, we consider the fundamental thresholds in regions 2 after observing the outcome in region 1. From Lemma 2, we have that \(f_1 = 1\) coincides with \(p' < p\), \(q' > q\), and \(\frac{p'}{1-q'} < \frac{p}{1-q}\), while the reverse inequalities hold if \(f_1 = 0\). Hence, observing \(f_1 = 1\) induces a lower weight on the first summand and a higher weight on the second summand of \(J\) in equation (25), while the effect of the third summand is ambiguous. Still, from \(\frac{q'}{1-p'} > \frac{q}{1-p}\) it follows that the relative increase of the weight on the second summand must be higher when compared to the potential increase of the weight on the third summand.

Hence, using the comparative static result underlying Lemma 2, the \(\Theta_2^*(0, m)\) that solves the version of equation (25) for region 2 and \(n_2 = 0\) must be higher after observing a crisis in region 1 due to more aggressive attacks after unfavorable public information. A higher fundamental threshold is, ceteris paribus, associated with a higher conditional probability of a crisis in region 2.

Second, the distribution of the unobserved macro shock is updated after observing the outcome in region 1. Specially, the distribution of the fundamental of region 2 conditional on a crisis in region 1 (\(f_1 = 1\)) is less favorable than the distribution of the fundamental of region 2 conditional on no crisis in region 1 (\(f_1 = 0\)). The second effect strengthens the first effect. This concludes the proof.

D.2  Proof of Proposition 4

The proof consists of four steps.

First, suppose that \(s \to \infty\). Not observing a crisis in region 1 implies that
\[ \frac{d}{dq} \to 0 \text{ as } q' \text{ goes to zero faster than } q. \] To see this, observe that \( \Pr\{f_1 = 0|m = -s\Delta\} \to 0 \text{ if } s \to \infty, \) since a \( \Theta_2 \) drawn from a distribution with a highly negative mean, \( \mu - s\Delta, \) is increasingly unlikely to have a sufficiently high realization such that \( f_1 = 0 \) occurs. At the same time, \( \frac{1 - p' - q'}{1 - p - q} \to 0 \) if \( s \to \infty \) and \( f_1 = 0. \)

Second, the right-hand side of inequality (20) has a fundamental threshold that is lower than the fundamental threshold on the left-hand side. To see this, we again use the comparative static result underlying Lemma 2. Observing \( f_1 = 0 \) implies that the second summand of \( J \) in equation (25) goes to zero if \( s \to \infty. \) Hence, \( \Theta_2^*(n_2 = 0, f_1 = 0) < \Theta_2^*(n_2 = 1, f_1 = 1, m = 0). \)

Third, given \( s \to \infty, \) the \( \Theta's \) on the right-hand side of inequality (20) are drawn from equally favorable or, with a positive probability \( (\frac{p'}{p} \not\to 0) \) that is away from zero, from a more favorable distribution if \( f_1 = 0. \) Taken together, the likelihood of a crisis in region 2 is lower if \( f_1 = 0 \) and \( s \to \infty. \)

Fourth, by continuity, the result can be generalized to hold for a sufficiently high, but finite, value of \( s, \) say \( s > s. \) This concludes the proof.

D.3 Proof of Proposition 5

The proof builds on equation (14) and consists of three steps.

Step 1: Following [Metz (2002)], we analyze how the precision of public information affects the fundamental thresholds. That is, we derive the following comparative static in case of \( n_2 = 1: \)

\[
\frac{d\Theta_2^*(n_2 = 1)}{d\alpha_2} = \begin{cases} 
< 0 & \text{if } \Theta_2^* < \mu + m + \frac{1}{2\alpha_2 + \beta} \Phi^{-1}(\gamma_2) \\
\geq 0 & \text{otherwise}
\end{cases}
\] (40)

Recall that the signal and fundamental thresholds are in a one-to-one relationship: \( x_2^* = \Theta_2^* + \beta^{-\frac{1}{2}}\Phi^{-1}(\Theta_2^*). \) For sufficiently high values of \( \mu \) and \( s \) (Assumption 1), we obtain \( \frac{dx_2^*(s\Delta)}{d\alpha_2} < 0, \frac{dx_2^*(-s\Delta)}{d\alpha_2} < 0, \frac{d\Theta_2^*(s\Delta)}{d\alpha_2} > 0, \text{ and } \frac{d\Theta_2^*(-s\Delta)}{d\alpha_2} > 0. \)

Step 2: Suppose that \( 1 - p - q \to 0 \) and evaluate the value of information at \( n_2 = 1 \) in equation (39). All terms are affected by \( \alpha_2 \) through the conditional dis-
tribution \( f(\Theta_2|m) \), which becomes more concentrated as \( \alpha_2 \) increases. Moreover, greater transparency affects the signal and fundamental thresholds.

We now look at these terms in greater detail. First, for a given \( x^*_U(0) \), the loss term in the first bracket grows relatively larger in \( \alpha_2 \) due to the reduction in \( \Theta_2^*(1,\Delta) \). This effect is further strengthened by a more concentrated \( f(\Theta_2|\Delta) \), since \( \Theta_2^*(1,\Delta) < \mu + \Delta \) for a sufficiently high \( \mu \). Second, for a given \( x^*_U(0) \), the benefit term in the second bracket grows relatively larger in \( \alpha_2 \) due to the increase in \( \Theta_2^*(1,-s\Delta) \). This effect is further strengthened by a more concentrated \( f(\Theta_2|-s\Delta) \), since \( \Theta_2^*(1,-s\Delta) > \mu - s\Delta \) if \( s \) is sufficiently high. As a result, for a given \( x^*_U(0) \), the value of information \( v(1,f_1) \) increases in \( \alpha_2 \) for sufficiently high values of \( \mu \) and \( s \), as guaranteed by Assumption [1].

**Step 3:** By continuity and monotonicity, any general equilibrium adjustment of \( x^*_U \) with \( \alpha_2 \) cannot fully off-set the previous effects. Thus \( \frac{dv(1)}{d\alpha_2} > 0 \) for sufficiently high values of \( \mu \) and \( s \). By continuity, the result continues to hold if \( 1 - p - q \) is sufficiently small, that is if \( 1 - p - q < \eta \). This concludes the proof.
E Technical Appendix (for online publication)

E.1 Proof of Proposition 1: formal argument for points (i)-(iv)

Before addressing formally points (i)-(iv) of the Proof of Proposition 1, we start by analyzing the following auxiliary step. For any \( \Theta_1(m) \geq \Theta_1^*(1,m) \), it can be shown that:

\[
\frac{\partial}{\partial n_1} \Phi^{-1}
\left(
\frac{\Theta_1(m) - n_1 \Phi\left(\frac{\Theta_1(m) - (\mu + m) - \sqrt{\alpha + \beta} \Phi^{-1}(\gamma_1)}{\sqrt{\beta}}\right)}{1 - n_1}
\right) \geq 0
\]

because \( J(1, \Theta_1) \leq \gamma_1 \) for any \( m \). Note that both the previous expression and the partial derivative hold with strict inequality if \( \Theta_1(m) > \Theta_1^*(1,m) \).

Inspecting the inside of the inverse of the cdf, \( \Phi^{-1} \), we define the highest permissible values of \( \Theta_1(m) \) that are labeled \( \Theta_1(n_1, m) \) for all \( m \):

\[
\Theta_1(n_1, m) = n_1 \Phi\left(\frac{\Theta_1(n_1, m) - (\mu + m) - \sqrt{\alpha + \beta} \Phi^{-1}(\gamma_1)}{\sqrt{\beta}}\right)
\]

Hence, \( 1 \geq \Theta_1(n_1, m) \geq \Theta_1^*(1,m) \) \( \forall m \), where the first (second) inequality binds if and only if \( n = 0 \) \( (n = 1) \).

We now prove points (i) and (iii). Evaluate \( K_1 \) and \( K_2 \) at the highest permissible value, \( \Theta_2(0) = \Theta_1(n_1,0) \), which yields \( \Theta_1(n_1,-s\Delta) \) and \( \Theta_1(n_1,\Delta) \), respectively. Likewise, evaluate \( L \) at the highest permissible values, \( \Theta_1(n_1,0) \) and \( \Theta_1(n_1,-s\Delta) \), which yields \( \Theta_1(\Delta) < \Theta_1(n_1,\Delta) \). Similarly, evaluate \( L \) at \( \Theta_1(n_1,0) \) and \( \Theta_1(n_1,\Delta) \), which yields \( \Theta_1(-s\Delta) < \Theta_1(n_1,-s\Delta) \). This proves points (i) and (iii).

Next, we now proceed with points (ii) and (iv). We can similarly define the lowest permissible value of \( \Theta_1(m) \), which is labeled \( \Theta_1(n_1,m) \) for all \( m \). Now, \( 0 \leq \Theta_1(1,m) \leq \Theta_1^*(1,m) \) \( \forall m \), where the first (second) inequality binds if and only if \( n = 0 \) \( (n = 1) \). Evaluate \( K_1 \) and \( K_2 \) at the lowest permissible value, \( \Theta_1(0) = \Theta_1(n_1,0) \), which yields \( \Theta_1(n_1,-s\Delta) \) and \( \Theta_1(n_1,\Delta) \), respectively. Likewise, evaluate \( L \) at the lowest permissible values, \( \Theta_1(n_1,0) \) and \( \Theta_1(n_1,-s\Delta) \), which yields \( \Theta_1(\Delta) > \Theta_1(n_1,\Delta) \). Similarly, evaluate \( L \) at \( \Theta_1(n_1,0) \) and \( \Theta_1(n_1,\Delta) \), which yields \( \Theta_1(-s\Delta) > \Theta_1(n_1,-s\Delta) \). This proves points (ii) and (iv).
E.2 Formal argument for the Proof of Lemma 3

We formally prove the results of Lemma 3 in turn. A general observation is that the updated belief on the probability of positive macro shock becomes degenerate: \( \hat{p} \to p \) for \( \alpha \to 0 \). Results (A) and (B) are closely linked, so we start by proving them below. As will become clear, it is useful to consider a modified system of equations where either \( K_1 \) or \( K_2 \) are used alongside \( K_3 \):

\[
K_3(n_t, \Theta^*_t(-s\Delta), \Theta^*_t(\Delta)) \equiv x^*_U(-s\Delta) - x^*_U(\Delta) = 0. \tag{43}
\]

**Results (A) and (B).** This prove has three steps.

*Step 1:* We show in the first step that for \( 1 - p - q \to 0 \) the fundamental thresholds \( \Theta^*_t(-s\Delta) \) and \( \Theta^*_t(\Delta) \) in the case of asymmetrically informed investors lie either both within these bounds or outside of them. As a consequence of \( \hat{p} \to p \), condition \( L(n_t, \Theta^*_t(-s\Delta), \Theta^*_t(\Delta)) = 0 \) prescribes that, for any \( n_t \), the thresholds \( \Theta^*_t(\Delta) \) and \( \Theta^*_t(-s\Delta) \) are either simultaneously within or outside of the two bounds given by the fundamental thresholds if all investors are informed, \( \Theta^*_t(1, \Delta) \) and \( \Theta^*_t(1, -s\Delta) \). This is proven by contradiction. First, suppose that \( \Theta^*_t(\Delta) < \Theta^*_t(1, \Delta) \) and \( \Theta^*_t(-s\Delta) < \Theta^*_t(1, -s\Delta) \). This leads to a violation of \( L(\cdot) = 0 \) because \( J(\cdot) > \gamma \forall n_t \) if \( \alpha \to 0 \). Second, suppose that \( \Theta^*_t(\Delta) > \Theta^*_t(1, \Delta) \) and \( \Theta^*_t(-s\Delta) > \Theta^*_t(1, -s\Delta) \). Again, leading to a violation because \( J(\cdot) < \gamma \forall n_t \) if \( \alpha \to 0 \). By continuity, the results continue to hold provided that \( 1 - p - q \) is sufficiently small.

*Step 2:* We now derive the derivatives of the fundamental thresholds with respect to the proportion of informed investors, \( \frac{d\Theta^*_t(m)}{dn_t} \).

\[
\frac{d\Theta^*_t(n_t, -s\Delta)}{dn_t} = \begin{vmatrix}
-\frac{\partial K_{1,2}}{\partial n_t} & \frac{\partial K_{1,2}}{\partial \Theta_1(n_t, 0)} & \frac{\partial K_{1,2}}{\partial \Theta_1(n_t, \Delta)} \\
-\frac{\partial K_1}{\partial n_t} & \frac{\partial K_1}{\partial \Theta_1(n_t, 0)} & \frac{\partial K_1}{\partial \Theta_1(n_t, \Delta)} \\
-\frac{\partial L}{\partial n_t} & \frac{\partial L}{\partial \Theta_1(n_t, 0)} & \frac{\partial L}{\partial \Theta_1(n_t, \Delta)} \\
\end{vmatrix} \equiv \frac{\text{[M]}_1}{\text{[M]}} \tag{44}
\]
where $|M| \equiv \det(M)$. We also find that:

$$
\frac{d\Theta^*_t(n,0)}{dn_t} = \frac{\partial K_{1,2}}{\partial \Theta_t(n,-s\Delta)} \frac{\partial K_{1,2}}{\partial \Theta_t(n,0)} \frac{\partial K_{1,2}}{\partial \Theta_t(n,-s\Delta)} = \frac{|M_2|}{|M|} \quad (45)
$$

and:

$$
\frac{d\Theta^*_t(n,\Delta)}{dn_t} = \frac{\partial K_{1,2}}{\partial \Theta_t(n,-s\Delta)} \frac{\partial K_{1,2}}{\partial \Theta_t(n,0)} \frac{\partial K_{1,2}}{\partial \Theta_t(n,-s\Delta)} = \frac{|M_3|}{|M|} \quad (46)
$$

To find $|M|$, recall from the proof of Proposition 1 that $\frac{\partial K_1}{\partial \Theta_t(0)} > 0$, while $\frac{\partial K_1}{\partial \Theta_t(0)} = 0$, while $\frac{\partial K_2}{\partial \Theta_t(0)} > 0$, $\frac{\partial K_2}{\partial \Theta_t(0)} = 0$, and $\frac{\partial K_2}{\partial \Theta_t(0)} < 0$. Furthermore, $\frac{\partial K_3}{\partial \Theta_t(0)} = 0$, $\frac{\partial K_3}{\partial \Theta_t(0)} < 0$, and $\frac{\partial K_3}{\partial \Theta_t(0)} > 0$. Finally, $\frac{\partial L}{\partial \Theta_t(m)} < 0 \forall m$ for a sufficiently high but finite value of $\beta$. As a result, $|M| > 0$ for a sufficiently high but finite value of $\beta$, irrespective of which of the two systems is used.

The proof proceeds by analyzing $|M_1|$, $|M_2|$, and $|M_3|$. To do this, we first examine the derivatives $\frac{\partial K_1}{\partial n_t}$, $\frac{\partial K_2}{\partial n_t}$, and $\frac{\partial L}{\partial n_t}$. Thereafter, we combine the results to obtain the signs of the determinants.

$$
\frac{\partial K_1}{\partial n_t} = \frac{\partial x_{iU}(0)}{\partial n_t} - \frac{\partial x_{iU}(-s\Delta)}{\partial n_t} \quad (47)
$$

$$
= \sqrt{\frac{1}{\beta}} \Theta_t(0) - \Phi\left(\frac{\alpha(\Theta_t(0)-\mu) - \sqrt{\alpha+\beta} \Phi^{-1}(\gamma)}{\sqrt{\beta}}\right)
$$

$$
- \sqrt{\frac{1}{\beta}} \Theta_t(-s\Delta) - \Phi\left(\frac{\alpha(\Theta_t(-s\Delta)-(\mu+s\Delta) - \sqrt{\alpha+\beta} \Phi^{-1}(\gamma))}{\sqrt{\beta}}\right)
$$

$$
\frac{\partial K_2}{\partial n_t} = \frac{\partial x_{iU}(0)}{\partial n_t} - \frac{\partial x_{iU}(\Delta)}{\partial n_t} \quad (48)
$$

$$
= \frac{\partial x_{iU}(0)}{\partial n_t} - \sqrt{\frac{1}{\beta}} \Theta_t(\Delta) - \Phi\left(\frac{\alpha(\Theta_t(\Delta)-(\mu+\Delta) - \sqrt{\alpha+\beta} \Phi^{-1}(\gamma))}{\sqrt{\beta}}\right)
$$

$$
(1 - n_t)^{2} \phi(\Phi^{-1}(\gamma))
$$

44
\[ \frac{\partial K_3}{\partial n_t} = \frac{\partial x_{U}(s\Delta)}{\partial n_t} - \frac{\partial x_{U}(\Delta)}{\partial n_t} \] (49)

To evaluate this partial derivatives, we can use the optimality condition in the case of symmetrically informed investors, \( n_t = 1 \). That is, \( \Theta_t^*(1,m) \) is defined as the solution to \( F_t(\Theta_t^*(1,m),m) = 0 \), where uniqueness requires that \( F_t \) is strictly decreasing in the first argument. This implies:

\[ \Theta_t(m) - \Phi\left( \frac{\alpha(\Theta_t(m) - (\mu + m)) - \sqrt{\alpha + \beta \Phi^{-1}(\gamma)}}{\sqrt{\beta}} \right) \leq 0 \text{ if } \Theta_t(m) \leq \Theta_t(1,m). \]

Four cases are considered in turn. Case 1: \( \Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(n_t,-s\Delta) \leq \Theta_t^*(1,-s\Delta) \). Case 2: \( \Theta_t^*(1,\Delta) \leq \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(n_t,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta) \). Case 3: \( \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(1,0) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta) \). Case 4: \( \Theta_t^*(n_t,\Delta) \leq \Theta_t^*(1,\Delta) \leq \Theta_t^*(0,m) \leq \Theta_t^*(1,0) \leq \Theta_t^*(1,-s\Delta) \leq \Theta_t^*(n_t,-s\Delta) \).

Case 1: Using the system of equations with \( K_1 \) and \( K_3 \) we obtain \( \frac{\partial K_1}{\partial n} > 0 \forall n_t \in [0,1) \) and \( \frac{\partial K_3}{\partial n} < 0 \forall n_t \in [0,1) \).

Case 2: Using the system of equations with \( K_2 \) and \( K_3 \) we obtain \( \frac{\partial K_2}{\partial n} < 0 \forall n_t \in [0,1) \) and \( \frac{\partial K_3}{\partial n} < 0 \forall n_t \in [0,1) \).

Case 3: Using the system of equations with \( K_1 \) and \( K_3 \) we obtain \( \frac{\partial K_1}{\partial n} > 0 \forall n_t \in [0,1) \) and \( \frac{\partial K_3}{\partial n} > 0 \forall n_t \in [0,1) \).

Case 4: Using the system of equations with \( K_2 \) and \( K_3 \) we obtain \( \frac{\partial K_2}{\partial n} < 0 \forall n_t \in [0,1) \) and \( \frac{\partial K_3}{\partial n} > 0 \forall n_t \in [0,1) \).

After having found the partial derivative for first two equilibrium conditions \( (K_{1,2}) \), we turn to the other equilibrium condition \( (L) \). Here, we can invoke the envelope theorem in order to obtain \( \frac{\partial L}{\partial n} = 0 \). The idea is the following. Since \( L \) represents the indifference condition of an uninformed investor, the proportion of informed investors enters only indirectly via \( x_{U}^* \) and we can write:

\[ \frac{\partial L}{\partial n} = \frac{\partial J}{\partial x_{U}^*} \frac{\partial x_{U}^*}{\partial n} + \frac{\partial J}{\partial n} = 0. \] (50)
Since $x_{U}^{*}$ is the optimal signal threshold of an uninformed investor, it satisfies $J(\cdot, x_{U}^{*}) = \gamma_{t}$. Thus, we must have $\frac{dJ}{dx_{U}^{*}} = 0$, which corresponds to a first-order optimality condition. (This implicitly uses the result that the equilibrium is unique.)

To conclude, we have for all cases that $|M| > 0$. It shows that $|M_1| > 0$ for case 1 and $|M_3| < 0$ for case 2, while $|M_1| < 0$ for case 1 and $|M_3| > 0$ for case 2. Furthermore, for the probability of $m = 0$, i.e. $1 - p - q$, sufficiently small we have that $|M_1| > 0$ also for case 2 and $|M_3| < 0$ also for case 1, while $|M_1| < 0$ also for case 2 and $|M_3| > 0$ also for case 1. Hence, provided that $1 - p - q$ sufficiently small, we find $\forall n_t \in [0, 1)$:

$$
\frac{d\Theta_t^*(n_t, -s\Delta)}{dn_t} = \begin{cases} 
 0 \text{ if } \Theta_t^*(n_t, -s\Delta) < \Theta_t^*(1, -s\Delta) \wedge \Theta_t^*(n_t, \Delta) > \Theta_t^*(1, \Delta) \\
 0 \text{ if } \Theta_t^*(n_t, -s\Delta) > \Theta_t^*(1, -s\Delta) \wedge \Theta_t^*(n_t, \Delta) < \Theta_t^*(1, \Delta) 
\end{cases}
$$

and $\forall n_t \in [0, 1)$:

$$
\frac{d\Theta_t^*(n_t, \Delta)}{dn_t} = \begin{cases} 
 < 0 \text{ if } \Theta_t^*(n_t, -s\Delta) < \Theta_t^*(1, -s\Delta) \wedge \Theta_t^*(n_t, \Delta) > \Theta_t^*(1, \Delta) \\
 > 0 \text{ if } \Theta_t^*(n_t, -s\Delta) > \Theta_t^*(1, -s\Delta) \wedge \Theta_t^*(n_t, \Delta) < \Theta_t^*(1, \Delta) \\
 0 \text{ if } \Theta_t^*(n_t, -s\Delta) = \Theta_t^*(1, -s\Delta) \wedge \Theta_t^*(n_t, \Delta) = \Theta_t^*(1, \Delta) 
\end{cases}
$$

**Step 3:** In this final step, we combine the results from the previous two steps to show both boundedness and monotonicity. In particular, we use the result that the derivative of the fundamental threshold w.r.t. the proportion of informed investors is zero once the boundary is hit. Therefore, the thresholds in the general case of asymmetrically informed investors are always bounded, which proves Result (A).

Given boundedness, in turn, the derivatives of the fundamental threshold can be clearly signed, yielding Result (B). That is, given the result from step 1, the second line of each derivative drops and equations (11) and (12) follow.

We prove that $\Theta_t^*(1, \Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1, -s\Delta)$ for all $n_t$ if $\alpha$ sufficiently small. First, $\Theta_t^*(1, \Delta) < \Theta_t^*(\Delta) = \Theta_t^*(0) = \Theta_t^*(-s\Delta) < \Theta_t^*(1, -s\Delta)$ if $n_t = 0$, while $\Theta_t^*(1, \Delta) = \Theta_t^*(\Delta)$ and $\Theta_t^*(1, -s\Delta) = \Theta_t^*(-s\Delta)$ if $n_t = 1$. Second, $\frac{d\Theta_t^*(\Delta)}{dn_t} |_{n_t=0} < 0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t} |_{n_t=1} > 0$ and $\frac{d\Theta_t^*(\Delta)}{dn_t} |_{n_t=1} = \frac{d\Theta_t^*(-s\Delta)}{dn_t} |_{n_t=1} = 0$. Third, by continuity $\Theta_t^*(1, \Delta) \leq \Theta_t^*(\Delta), \Theta_t^*(-s\Delta) \leq \Theta_t^*(1, -s\Delta)$ and $\frac{d\Theta_t^*(\Delta)}{dn_t} |_{n_t=0} < 0$, $\frac{d\Theta_t^*(-s\Delta)}{dn_t} |_{n_t=0} > 0$ for small values of $n_t$. Fourth, if for any $\tilde{n}_t \in (0, 1]$ $\Theta_t^*(-s\Delta) \not\leq \Theta_t^*(1, -s\Delta)$ when
$n_t \to \hat{n}_t$, then for sufficiently small but positive values of $\alpha$ it has to be true that $\Theta^*_t(\Delta) \downarrow \Theta^*_t(1, \Delta)$ when $n_t \to \hat{n}_t$. This is because of the result in step 1. Fifth, given that the derivatives of the fundamental thresholds flip when both are outside of the bounds we have $\Theta^*_t(1, \Delta) = \Theta^*_t(1, -s\Delta) = \Theta^*_t(-s\Delta)$ for all $n_t \geq \hat{n}_t$. In conclusion, $\Theta^*_t(1, \Delta) \leq \Theta^*_t(\Delta), \Theta^*_t(-s\Delta) \leq \Theta^*_t(1, -s\Delta)$ for all $n_t \in [0, 1]$ if $\alpha$ sufficiently small.

**Result (C).** From equation (27),

$$\frac{dx^+_t(m)}{dn_t} = \frac{d\Theta^+_t(m)}{dn_t} \left( \frac{\beta}{\alpha + \beta} \right)^{-1}. \quad (51)$$

Therefore, by continuity, there exists a sufficiently small but positive value of $\alpha$ that implies the required inequality, taking into account the monotonicity of the fundamental thresholds. The distance between the fundamental thresholds is monotone for any $n_t > 0$, which implies $\frac{d(x^+_t(n_t, -s\Delta) - x^+_t(n_t, \Delta))}{dn_t} \geq 0 \ \forall \ n_t \in [0, 1)$. Furthermore, $x^+_t(n_t, -s\Delta) - x^+_t(n_t, \Delta)) > 0 \ \forall \ n_t \in [0, 1]$. This completes the proof.