State aid and tacit collusion*

Christoph Bertsch Claudio Calcagno
Mark Le Quement

European University Institute, Economics Department
via della Piazzuola 43, 50133 Firenze, Italy
Ph. +39 055 4685 927; Fax: +39 055 4685 902

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Abstract

Both literature and policy debate on State aid (or government subsidies) have focused on the trade-off between the potential inefficiencies caused by state intervention (inefficient allocation of resources, moral hazard) and the potential gains from intervention (whether related to the resolution of market failures or to the achievement of some dimension of social equity).

By contrast, the debate has ignored another important negative effect of State aid: governments, by setting up aid schemes to ailing firms, may increase the likelihood of (tacit) collusion in an industry characterised by idiosyncratic shocks.

Specifically, we show that, in a repeated-game setting, a systematic bailout regime increases the expected profits from cooperation and simultaneously raises the probability that competitors will still be in business to carry out punishment against cheaters.

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Despite the generality of the model and of its key insight, we study this problem through an application to the banking sector, as it has recently been subject of much attention within the context of the economic and financial crises.
1 Introduction and motivation

The financial crisis and its aftermath have been affecting the global economy since 2007. This has involved the bankruptcies of a large number of global firms, including major financial institutions. The rapid deterioration of the financial and economic situation has prompted massive interventions from Treasuries (and Central Banks) the world over. State intervention, and in particular State aid, has been playing a major role, with governments rushing to rescue not only a large fraction of their financial industries, but also other key industries like the automotive one. Apart from the fiscal burden imposed, State aid entails some thorny legal considerations, especially in the European Union (EU), where Art. 107 of the EU Treaty explicitly forbids measures that confer (through public resources) economic advantages to selected entities, affecting trade. The European Commission (EC) was therefore under pressure to soften its stance. In October 2008, owing to the gravity of the situation, it published a communication on the application of State aid rules to financial institutions.¹

To understand the magnitude of the aid involved, it is worth noting that in the EU the number of newly introduced state aid measures is steadily increasing. The number of cases where aid was legally granted grew from 202 in 2003 to 636 in 2007 and to 964 in 2009.² Moreover, during the crisis, EU Member States strongly supported the financial sector, with €300 billion in capital injections and almost €3 trillion of guarantees.³

From a theoretical perspective, most of the debate on State aid (or government subsidies) has focused on the trade-off between inefficiencies caused by intervention (inefficient allocation of resources, moral hazard) and potential gains following from aid measures. Arguments in favour of State aid range from equity considerations to the potential resolution of existing market failures. In the financial sector, for example, intervention (including that of a Central Bank) is often justified on the grounds of gains in the stability of the financial system. Bankruptcies of individual banks may trigger contagion effects across the sector (through the interbank and asset markets), and may also harm consumers directly through the loss of private deposits (subject to national deposit insurance schemes).

¹Official Journal C 270, 25.10.2008, pages 8–14. The accompanying press release explained that aid would be approved if: "Non-discriminatory access in order to protect the functioning of the Single Market by making sure that eligibility for a support scheme is not based on nationality; State commitments to be limited in time in such a way that it is ensured that support can be provided as long as it is necessary to cope with the current turmoil in financial markets but will be reviewed and adjusted as soon as improved market conditions so permit; State support to be clearly defined and limited in scope to what is necessary to address the acute crisis in financial markets while excluding unjustified benefits for shareholders of financial institutions at the taxpayer's expense; [a]n appropriate contribution of the private sector by way of an adequate remuneration for the introduction of general support schemes (such as a guarantee scheme) and the coverage by the private sector of at least a significant part of the cost of assistance granted; [s]ufficient behavioural rules for beneficiaries that prevent an abuse of state support, like for example expansion and aggressive market strategies on the back of a state guarantee; [a]n appropriate follow-up by structural adjustment measures for the financial sector as a whole and/or by restructuring individual financial institutions that had to rely on state intervention."


³The numbers include only measures approved by the EC by September 2009 and hence do not include the granted and planned capital injections after the European stress tests in 2010 and Basel III. Source: EC, DG Competition, http://ec.europa.eu/competition/recovery/financial_sector.html.
On the other hand, State aid can distort the competitive process (prompting a misallocation of resources), as well as create moral hazard: if firms expect that the Government will intervene to help them in case of failure (or in adverse circumstances more generally), these may have the incentive to take excessive risks.

The economic effects studied have always impinged on firms’ unilateral behaviour. In this article, in contrast, we develop a simple (infinite-horizon) model that sheds light on a result that to our knowledge has not been stressed before: a government policy aimed at systematically rescuing firms in the presence of negative idiosyncratic shocks facilitates (tacit) collusion. Collusion is easier to sustain because the punishment threat faced by a firm is more efficient, which is in turn due to the increased survival probability of firms. Indeed, when firms are guaranteed to be in business in the next periods, expected future cooperative profits increase, so that foregoing such profits implies an increased opportunity cost. Furthermore, the guaranteed presence of competitors in the future makes the expected punishment harsher than in an environment where competitors may exit the market due to an exogenous shock, which would leave the deviant firm unpunished. Our results impinge on the provision of systematic bailouts; the EC’s official policy of "one-time-last-time" aid would thus undermine our mechanism. However, the EC has departed from this principle a number of times (especially so in the latest financial crisis) where ex post this would have been detrimental for the economy; we discuss this in Section 3 in greater detail.

Although the model and its implications are general and can match many industries, we study this problem through an application to the banking sector, as it has recently received much attention due to large bailouts.

Some words of caution are in order. The key result of the paper - namely that State aid may facilitate tacit coordination - cannot be taken at face value in the real world. This is ultimately an empirical question, which regulators, policy-makers and courts would need to assess on a case-by-case basis. There are several industry features that may facilitate coordination, such as stable demand, homogeneous products, little innovation, market transparency. It is clearly arguable whether the banking industry actually meets these criteria. Our point, however, is that for given industry characteristics, an explicit (or implicit) commitment to bail firms out upon the realisation of shocks facilitates coordination.

This article proceeds as follows: in Section 2 we briefly consider the relevant literature; in Section 3, we present the model. Section 4 shows and discusses the results, including a welfare analysis. Section 5 concludes and suggests some extensions.

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5 Clear and sound economic reasoning (presented in a suitable manner for policy-making) on the economics of tacit collusion can be found in Ivaldi et al (2003).
2 Related literature

There is not a vast literature on the economics of State aid as such, possibly reflecting the richness of the literature on subsidies and trade.\(^6\) State aid is typically criticised by economists, as it leads to a variety of inefficiencies. Besley et al (1999) discuss two broad classes of effects: externalities arising from aid (strategic trade policy, tax competition and economic geography considerations) and inefficient competition between governments. Dewatripont and Seabright (2006) go beyond intergovernmental issues and build a model where local politicians invest in wasteful projects purely to show their diligence and win votes. Collie (2000) instead proposes an economic explanation of why individual states may have an incentive to subsidise firms with the aim of reducing oligopolistic distortions. He shows that a multilateral institution responsible for prohibiting subsidies can increase welfare.

Friederiszick et al (2008) review the efficiency rationales for aid (tackling market failures such as externalities, public goods, asymmetric information and lack of coordination) as well as equity considerations. They also point towards cross-border (positive) externalities in the case of EU State aid. Their paper then highlights the potential costs of State aid (beyond the direct cost of intervention) such as anti-competitive effects, “picking wrong winners” and international spillover effects. Among the potential distortions of competition, they list the support of inefficient production; the distortion of dynamic (inter-temporal) incentives; the potential increase in market power; and the distortion of production and location decisions across EU countries. Finally, they propose an actual effects-based framework to assess whether particular State aid measures should be approved. Martin and Valbonesi (2008) develop a model of the impact of State aid on market structure and performance in an integrating market (i.e. a common market with increasing trade flows) and find that in equilibrium governments grant State aid, reducing common market welfare. However, they only focus on non-cooperative equilibria.

Finally, Hainz and Hakenes (2009) compare the efficiency properties of five options to grant State aid to firms (some of which would also feed through to the banking system). The most efficient option is shown to depend on the tax distortion and the informational cost needed to select the "good" firms.

In the case of banking, it may be beneficial to sacrifice some level of competition in the interest of financial stability.\(^7\) However, this relationship is rather complex and both empirical and theoretical results are far from being clear-cut.\(^8\) This debate is nevertheless beyond the scope of our paper, since we only take banking as an example and

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\(^6\) An extensive review of the role and the effects of State aid can be found in Nitsche and Heidhues (2006). For an equally policy-oriented approach based on economic theory, the reader is also directed to OFT (2004) and Buelens et al (2007).

\(^7\) See Carletti (2008) for an excellent survey on this trade-off.

\(^8\) Keeley (1990) was the first one to find a positive empirical relationship between more competition and more risk-taking but later studies came to mixed or even opposite results. The earlier theoretical literature mostly gave rationales for Keeley’s findings but also discussed why the opposite may be the case (most prominently Boyd and De Nicoló (2005)). For a theoretical discussion see Allen and Gale (2004). A more recent extensive review is given by Vives (2010).
our model is not sector-specific. Rather, we simply point towards some work on imperfect competition in banking in dynamic setups. Hellmann et al (2000) show that in a dynamic model deposit rate controls can be superior to costly regulation. They explore the interaction between financial liberalisation and prudential regulation and find that capital requirements (though costly) can prevent banks from excessive risk-taking in a static model. By contrast, this positive effect comes at a very high cost in a dynamic model because of the business-stealing effect that induces banks to opt for riskier strategies than their competitors and offer better interest rates to depositors in order to collect more deposits. Perotti and Suarez (2002) investigate the impact of competition on banks’ portfolio risk choices. In particular, they examine the relationship between the optimal portfolio risk and banking regulation (merger policy and market entry regulation) in an oligopoly context. The main mechanism in their model is the strategic substitutability between portfolio decisions of duopolistic banks. In particular, a given duopolist has an incentive to invest in the prudent asset if the competitor chooses a risky strategy (since she can expect large monopoly rents if the competitor fails).

In industrial economics, models of tacit collusion have been heavily applied, but not in the context of State aid. We take a standard framework of tacit collusion for repeated oligopoly interaction (originally due to Stigler (1964)) and analyse the effect of State aid in an application to the banking industry.

3 Model

We develop a model on the relation between State aid and tacit collusion through an application to the banking industry. The mechanism we describe is not specific to the financial industry and can be readily transposed to other industries.

The starting block of our model is Freixas and Rochet’s (2008) extension of the Klein-Monti model to Cournot oligopoly. The original monopolistic model features a single bank facing an upward-sloping demand for deposit and a downward-sloping demand for loans (as developed by Klein (1971) and Monti (1972)).

To shed light on the impact of State aid on tacit collusion and consumer welfare, our model will make simplifications with respect to Freixas and Rochet (2008). In our economy, the banking industry is characterised by a duopoly competing in the deposit market over an infinite horizon. We consider time in a discrete fashion. Production (management) costs are normalised to \( \text{nil} \). Banks simultaneously set interest rates \( r_1 \) and \( r_2 \), where

\[ \text{nil} \]

\[ 9 \text{Hellmann et al (2000) do not model imperfect competition explicitly but make assumptions on derivatives of the deposit demand function. See Repullo (2004) for an explicit modelling of imperfect competition that delivers some additional insights.} \]
\( r_1, r_2 > 0 \). The (linear) demand function is given by:

\[
Q(r_1, r_2) = \min \left\{ \bar{Q}, \max \{r_1, r_2\} \right\}.
\]

The environment is stochastic in the sense that there is a single asset (project) in which a bank invests its funds. This asset (project) is subject to the following idiosyncratic shock: it yields net return \( R_H \) (with probability \( p \)) and \( R_L \) (with probability \( 1 - p \)).\(^{10}\) We assume that \( R_H < \bar{Q} \) (this simply normalises static competitive profits to nil).

For simplicity, and without loss of generality, we normalise \( R_L \) to \(-1\). This means that all funds are lost in the presence of a bad shock and nothing is returned to depositors.

Banks have discount factor \( 0 < \delta < 1 \) and the timeline of our game, for each period \( t \in [1, \infty) \), is given in Figure 1.\(^{11}\)

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\[^{10}\] In another industry one could argue that firms can be hit with a certain probability in each period by a cost shock or a shock on their production technology that forces them to leave the market.

\[^{11}\] Discount factors over infinite-horizon games are often interpreted as the probability that the game will actually be played in a given period, so as to implicitly relax the infinite-horizon interpretation. Note that this is different from the adverse shocks that we introduce (with probability \( p \)) as these are idiosyncratic.

\[^{12}\] In general the game has multiple equilibria but we restrict ourselves to the best equilibrium in terms of profit maximization, i.e. banks collude for sufficiently high discount factors and both charge the monopoly interest rate.
**Condition 1** \( R_H > 4\frac{1-p}{p} \).

This condition is necessary to ensure positive levels of consumer welfare.\(^{13}\) Banks will set interest rates only taking the good state of the world \((R_H)\) into account. Let us adopt the superscript \(M\) for monopoly (or collusion) and \(C\) for competition, and denote profits with \(\pi\). If a bank is alone in the market, profit maximisation leads to \(r^M = \frac{R_H}{2}\), \(Q(r^M) = \frac{R_H}{2}\) and \(\pi^M = \left(\frac{R_H}{2}\right)^2\). Competition, by contrast, leads to \(r^C = R_H\), and \(Q(r^M) = R_H\) and \(\pi^C = 0\). Let us define \(W^M = \frac{1}{2}\left(\frac{R_H}{2}\right)^2\) as the consumer surplus (or welfare) at the monopoly (or collusive) price level and \(W^C = \frac{R_H^2}{2}\) as the consumer surplus under perfect competition.

The next element in our economy is a national deposit insurance scheme (NDIS), covering 100\% of deposits. Hence, when a bank goes bankrupt, depositors are returned their initial investments (without interest). The NDIS is funded by flat-rate taxes. We use the subscript "C" for collusion and "NC" for "no collusion". The size of the insurance scheme is: \(\Phi_{C1} = -\frac{R_H}{2}\) whenever one collusive duopolist bank fails; \(\Phi_{C2} = -\frac{R_H}{2}\) whenever a monopolist bank fails or both banks in a collusive duopoly fail; \(\Phi_{NC1} = -\frac{R_H}{2}\) whenever banks compete and one receives a bad shock; \(\Phi_{NC2} = -R_H\) whenever banks compete and both receive a bad shock.

Notice that the introduction of the deposit insurance scheme is not only crucial within the context of a banking model, but it also generates two important features. First, it enables us to isolate the effect of State aid on collective competitive behavior when we compare consumer welfare under a State aid and a No State aid regime. This follows from the fact that depositors get their deposits fully refunded under both regimes whenever a bank fails, while the financing of the scheme is done by a non-distortionary tax in both regimes. Second, the introduction of deposit insurance makes our model readily comparable to applications to other industries where the failure of a firm does not cause an immediate loss in wealth to its customers.

Figure 2 summarises, graphically, the above discussion. Notice that as the demand schedule is upward sloping, the various areas in the graph (profit \(\pi\), consumer surplus and deadweight loss, or DWL) are inverted (horizontally) with respect to a traditional diagrammatic analysis of linear demands. For simplicity, we have only depicted the potential loss from bankruptcy \((\Phi_{C2})\) in the case of monopoly. Notice the analogy to a model of duopolistic competition in a non-banking industry with a downward sloping demand curve.

Finally, we define State aid (also financed through lump-sum taxes) as the following, cumulative actions. First, the State renews the bank’s authorisation to operate even when the bank has gone bankrupt. Second, the State

\(^{13}\)Under reasonable parameter assumptions we have \(R_H > 4\frac{1-p}{p}\), e.g. for \(p = 0.9\) and \(R_H = 0.6\) we have \(0.6 > 0.4\) (recall \(R_L = -1\), so \(R_H = 0.6\) is not particularly high).
incurs a sunk cost $\gamma$ per bank rescue. We do not explicitly model how this cost arises but it could be justified, for instance, as the cost of resources devoted by the financial regulator to examine the books of a failing bank and facilitate its rescue. In a non-financial industry this cost could arise from expensive restructuring. Notice that we do not impose any assumption on $\gamma$. Instead, in the welfare analysis section, we determine upper bounds on its value such that the aid policy is welfare-enhancing to consumers.

As mentioned earlier, the EC is committed to a "one-time-last-time" approach to aid. But in practice there have been several exceptions to this rule, most notably during the financial crisis. Moreover, in the financial industry policy makers fear potentially strong negative externalities in the short-run (contagion effects, adverse impact on real economy) if they do not grant aid to an insolvent bank. For this reason policy makers consider bailouts as being necessary \textit{ex post}.$^{14}$ Consequently banks that are very interconnected or "too-big-to-fail" can expect to be rescued because policy makers can hardly make a credible commitment not to bail banks out. We therefore think that our approximation of systematic State aid is adequate for this type of implicit or explicit guarantees in the

$^{14}$See also Lyons (2010) for a discussion of this issue.
financial sector.\textsuperscript{15}

Finally, as we noted above, the NDIS is independent of whether there is a State aid policy (however, in total terms, the NDIS will be costlier in the presence of a State aid regime since banks are always rescued and can fail several times, whereas in absence of State aid there can be two bankruptcies at most in the economy). The NDIS is a banking-specific concept, but as the NDIS is exactly matched in size by a lump-sum tax, our framework is general and goes beyond the banking sector application.

4 Results

In this section we first demonstrate that the introduction of State aid decreases the minimal discount factor for which tacit collusion is possible. This implies that there is a range of values of the discount factor for which the implementation of State aid causes a change from a competitive outcome to a collusive one. Next, we examine the implications of this result on the welfare effects of a State aid policy. We demonstrate that for this range of values of the discount factor, State aid always affects consumer welfare negatively, while it has a positive effect for all other values of the discount factor if the intrinsic cost of rescuing ($\gamma$) is not too high.

4.1 State aid and collusion

Let us posit a collusive regime in which firms adopt a simple trigger strategy by charging the collusive price as long as no one in the cartel defects and reverting to the competitive price (forever) as soon as one firm has defected from the tacit collusive agreement.

The first step is to derive the critical discount factors above which tacit collusion is sustainable under each policy regime. In the case of no State aid, the collusive profits need to embed the probability that a firm becomes a monopolist (i.e. whenever the competitor receives a negative shock) as well as the probability that the firm itself goes bankrupt. The profits in the period of deviation also need to account for the probability that the deviating firm goes bankrupt, since the shock occurs after market conduct is chosen. Likewise, punishment profits (from the following period onwards) need to encompass the possibility that the deviating firm will actually be a monopolist for some time, as well as the possibility that the deviating firm goes out of business.

\textsuperscript{15}The existence of implicit guarantees is of course hard to prove empirically. Nevertheless there is some indirect evidence. Rating agencies publish "external support" ratings which reflect their expectations on the likelihood of a bailout. See Gropp et al (2010) for a paper that uses this data to estimate how bank risk-taking behaviour is affected by the presence of a guarantee.
Without State aid the incentive compatibility constraint (ICC) for tacit collusion - which we fully derive in Appendix A.1 - looks as follows:

\[
\sum_{t=1}^{\infty} \delta^{t-1} \left( p^{2t-1} \frac{\pi^M}{2} + p^t (1 - p^{t-1}) \pi^M \right) \geq \frac{p\pi^M}{2} \quad + \sum_{t=2}^{\infty} \delta^{t-1} \left( p^{2t-1} \pi^C + p^t (1 - p^{t-1}) \pi^M \right).
\]

From now onwards, we use the superscripts "NA" for "no aid" and "A" for "aid". Noting that \( C = 0 \), tacit collusion can be sustained for all \( \delta \geq \delta^{\text{NA}} = \frac{1}{2p} \).

With State aid, by contrast, a bank that has received a bad shock is rescued at no cost to it. There is no profit (nor loss) in the period of failure. Noting once again that the competitive profit is zero every period, we set up the following ICC:

\[
\sum_{t=1}^{\infty} \delta^{t-1} \left( p^{2t-1} \frac{\pi^M}{2} + (1 - p) \ast 0 \right) \geq p\pi^M \ast + \sum_{t=2}^{\infty} \delta^{t-1} (p\delta^{t-1} \ast \pi^C + (1 - p)\delta^{t-1} \ast 0).
\]

So tacit collusion can be sustained for all \( \delta \geq \delta^{\text{A}} = \frac{1}{2} \). This is the traditional result obtained in a supergame where symmetric duopolists compete on price. The only difference is that both the collusive profit and the deviation profit have to be scaled by the probability of receiving a good shock; however the actual probability \( p \) cancels out.

Figure 3 provides a graphical representation of this result. When banks place little value on the future (left half of the chart) tacit collusion cannot be sustained, regardless of whether there is State aid. When banks care much about the future for a given likelihood of a good shock (\( \delta \geq \frac{1}{2p} \), i.e. the area at the top-right corner) tacit collusion can be sustained in either regime. Finally, for intermediate values of the discount factor (\( \frac{1}{2} \leq \delta < \frac{1}{2p} \), i.e. the area at the bottom-right) only the competitive outcome is sustainable in the absence of an aid policy, while tacit collusion is made possible by a State aid policy.
The results are summarised in Proposition 1:

**Proposition 1 State aid and collusion**

In the presence of idiosyncratic shocks, a State aid policy that keeps banks (firms) in business after a shock facilitates tacit collusion; that is, tacit collusion can be sustained for lower values of discount factors.

**Proof.** The proof of is trivial, as \( \frac{1}{2} = \delta^A < \delta^{NA} = \frac{1}{2p^2} \), \( \forall p < 1 \).

### 4.2 Welfare impact of State aid

In the last section we examined the relation between competitive behaviour and the exogenous discount factor for the two regimes, State aid and no State aid. We derived two critical discount factors that defined three regions in the \([p \times \delta]\) parameter space. We next perform a comparison of consumer welfare under the two regimes and summarise our results in Proposition 2 at the end of the section.
4.2.1 No State aid

Under collusion ($\delta \geq \frac{1}{2p^2}$): The exercise here is to capture all possible scenarios, namely collusive duopoly and monopoly, considering all the possible combinations of events (good and bad shocks). Each scenario will occur with a certain probability and will entail a certain level of consumer welfare (including the tax levied). We provide a full derivation in Appendix A.2. Expected consumer welfare under a high enough discount factor to sustain tacit collusion and in the absence of State aid is computed to be:

$$E(W_{NC}^N) = \frac{R_H \left( \frac{p}{4} - \frac{\sqrt{8p} R_H}{2} \right)}{2 (1 - \delta p^2)} + \frac{R_H \left( \frac{p^2 R_H}{2} - \frac{2}{2} (1 - p) \right)}{2 (1 - \delta p)}. \quad (4)$$

Note that Condition 1 ensures positive consumer welfare levels.\(^{16}\)

Under no collusion ($\delta < \frac{1}{2p^2}$): Here, we carry out the analogous exercise; however, due to the lower discount factor, there can only be either a monopoly (if the competitor has received a bad shock) or a competitive duopoly. In either case there can be good or bad shocks (in the latter case it could be that only one bank received the bad shock in the last period). The expected consumer welfare in the absence of aid, under a low enough discount factor to guarantee competition and in the presence of both banks on the market, is given as follows:\(^ {17}\)

$$E(W_{NC}^N) = \frac{p \left( R_H \right)^2}{4 (1 - \delta p^2)} + \frac{R_H \left( \frac{p^2 R_H}{2} - \frac{2}{2} (1 - p) \right)}{2 (1 - \delta p)}. \quad (5)$$

4.2.2 State aid

We proceed in the same fashion as in subsection 4.2.1. However, here, banks never exit. When consumers’ deposits are lost due to the bad shocks, the Government refunds them the original capital, as well as paying the direct rescuing costs $\gamma$. To close the model, we need to compute the total expected stream of aid and set up a corresponding lump-sum tax (which includes the financing of both the NDIS and the direct rescuing costs $\gamma$) on consumers, thus reducing their welfare.

Under collusion ($\delta \geq \frac{1}{2}$): In the presence of State aid, the expected consumer welfare under a high enough discount factor to sustain tacit collusion is:

$$E(W_{NC}^A) = \frac{R_H \left( \frac{p}{4} R_H - (1 - p) \right)}{2 (1 - \delta)} - \frac{2 (1 - p) \gamma}{1 - \delta}. \quad (6)$$

\(^{16}\)To see this notice that the terms in brackets are negative since $R_H > \frac{4 (1 - p)}{p}$. Further $\frac{R_H / 2}{1 - \delta p^2} < \frac{R_H / 2}{1 - \delta p}$ whenever $p < 1$. Eventually for expected welfare to be positive we need to have $\frac{R_H / 2}{1 - \delta p^2} \left( (1 - p) - \frac{p}{4} R_H \right) > \frac{R_H / 2}{1 - \delta p^2} \left( \frac{p}{2} R_H - 2 (1 - p) \right)$ which is ensured since $\frac{1 - \delta p^2}{1 - \delta p} < 2$.

\(^{17}\)See in Appendix A.2 for the derivation. Again, Condition 1 ensures positive consumer welfare levels.
**Under no collusion** \((\delta < \frac{1}{2})\): In the presence of State aid, the expected consumer welfare under a discount factor low enough to guarantee competition (when both banks are in the market) is:

\[
E(W_{NC}^A) = \frac{R_H (pR_H - (1 - p))}{2 (1 - \delta)} - \frac{2(1 - p)\gamma}{1 - \delta}.
\]

\(^{(7)}\)

### 4.3 Summary and discussion of the results

Having derived these four benchmark levels of consumer welfare corresponding to the two possible policy regimes and the two possible competitive regimes, it is instructive to plot them jointly in a diagram. We do so in Figure 4. The horizontal axis corresponds to the discount factor \(\delta\). Three relevant regions can be identified. In the left region \((\delta < \frac{1}{2})\) banks compete with each other regardless of whether there is State aid. In the right region \((\delta \geq \frac{1}{2p})\) tacit collusion can be sustained in either regime. However, in the central region, tacit collusion can *only* be sustained in the presence of State aid. In the left and in the right regions, State aid can be consumer welfare-enhancing and this depends on whether the vertical distance between the broken and the solid line (for any given discount factor) exceeds the (expected tax bill due to the) direct costs of rescuing banks \(\gamma\). In the central region, by contrast, State aid *decreases* consumer welfare.

Figure 4 abstracts from the cost \(\gamma\) of rescuing a bank. In what follows, we compute the upper limits on \(\gamma\) such that the expected consumer welfare under State aid is larger than under no State aid, in the cases where public intervention does not affect the competitive state of the economy. Call \(\hat{\gamma}_{NC}\) this limit value of \(\gamma\) for the case of no collusion in both policy regimes, i.e. for \(\delta < \frac{1}{2}\):

\[
E(W_{NC}^A) \geq E(W_{NC}^{NA}) \iff 
\gamma \leq \hat{\gamma}_{NC} = \frac{1 - \delta}{2(1 - p)} \left( \frac{R_H (pR_H - 2(1 - p))}{2 (1 - \delta)} - E(W_{NC}^{NA}) \right)
\]

\(^{(8)}\)

In Appendix A.3 we show that \(\hat{\gamma}_{NC} \geq 0\). We now compute the upper limit \(\hat{\gamma}_C\) corresponding to the case of collusion in both regimes, i.e. for \(\delta \geq \frac{1}{2p}\):

\[
E(W_C^A) \geq E(W_C^{NA}) \iff 
\gamma \leq \hat{\gamma}_C = \frac{1 - \delta}{2(1 - p)} \left( \frac{R_H \left( \frac{p}{4}R_H - (1 - p) \right)}{2 (1 - \delta)} - E(W_C^{NA}) \right)
\]

\(^{(9)}\)

Again we show in Appendix A.3 that \(\hat{\gamma}_C \geq 0\).

Proposition 2 summarises the results on the welfare effects of State aid.
Proposition 2 \textit{State aid and consumer welfare}

(i) In the range of discount factors such that there is competition in absence of State aid but tacit collusion with State aid, such a policy reduces consumer welfare. That is, \( E(W_C^A) < E(W_{NC}^A) \), \( \forall \frac{1}{2} \leq \delta < \frac{1}{2p^2} \). This is true even in the absence of direct rescuing costs.

(ii) In an environment where duopolistic banks (firms) compete regardless of whether there is State aid \( (\delta < \frac{1}{2}) \), such a policy reduces consumer welfare if and only if the direct costs of rescuing a bank (firm) exceed \( \hat{\gamma}_C \).

(iii) In an environment where duopolistic banks (firms) can sustain tacit collusion regardless of whether there is State aid \( (\delta \geq \frac{1}{2p^2}) \), such a policy reduces consumer welfare if and only if the direct costs of rescuing a bank (firm) exceed \( \hat{\gamma}_{NC} \).

\textbf{Proof.} The proof of (i) is in Appendix A.4. For the proof of (ii) and (iii) see above inequalities.

Proposition 2 introduces a simple dichotomy between two scenarios that summarises the effect of State Aid on consumer welfare. In one scenario (cases ii) and iii) above), State aid does not influence the competitive state of the economy, and is welfare-improving as long as its intrinsic cost is not too large. Indeed, in such a case, the
only effect of State aid is to beneficially preserve the existence of the market, which is advantageous to both firms and consumers. In the other scenario (case i)), where State aid affects the competitive state of the economy by triggering collusion, we find that its overall effect is always negative. This means that in such a scenario, the adverse collusion-creating effect of State aid dominates its beneficial market-preserving effect. The dominance of the first force over the second force does not appear like an a priori necessity. We attribute this feature to the fact that this scenario corresponds to relatively low values of the discount factor. For such values of the discount factor, the adverse price effect of collusion thus dominates the long run positive benefits related to the preservation of the market.

5 Conclusion

The literature on State aid and the related policy debate have typically focused on the adverse efficiency effects of such policies (misallocation of resources, moral hazard) and on countervailing arguments typically (though not exclusively) based on social policy.

Here we developed a very simple infinite-horizon model that sheds light on a separate result that to our knowledge had not been stressed before: a government policy aimed at systematically rescuing firms in the presence of negative idiosyncratic shocks facilitates (tacit) collusion. This is because expected future co-operative profits increase (since firms are ensured to be in business in the subsequent periods); and because the guaranteed presence of competitors in the next periods makes the (expected) punishment phase harsher than under an environment where competitors may exit the market due to an exogenous shock, which would leave the deviant firm unpunished.

Examining the implications of this result for the welfare effects of State aid policy, our main result is the identification of a range of discount factors for which an aid policy is always detrimental for consumer welfare because of its collusion-facilitating effect. Yet, in the real world, this link would actually have to be shown empirically and regulators, policy-makers and courts would need to assess this on a case-by-case basis.

The analysis in this paper has focused on the basics of the mechanism to highlight the key insight. However, this paper can set the stage for interesting extensions. One possible direction is to devise more complex aid policies (stochastic, with repayments, limited to the last failing firm) or to consider a more complex competitive setup (introducing asymmetries, entry, a richer menu of contracts). Another possibility would be to embed a true banking model within our framework, for instance by endogenising portfolio choice and modelling an interbank (wholesale) market.
References


A Appendix

A.1 Derivation of the critical discount factors

A.1.1 No State aid

We start by deriving the expected collusive profit (LHS of the ICC). At each period $t$, a bank gets collusive profits $\delta^t \pi^M$ with probability $p_1(t)$; monopoly profits $\delta^t \pi^M$ with probability $p_2(t)$; and 0 with probability $p_3(t)$. The respective probabilities can be written as follows:

$$p_1(t) = p^t (p^{t-1}) = p^{2t-1}$$
$$p_2(t) = p^t [(1-p) + p(1-p) + p^2(1-p) + \ldots + p^{t-2}(1-p)] = p^t (1-p) \frac{1-p^{t-1}}{1-p} = p^t (1-p^{t-1})$$
$$p_3(t) = 1 - p^{2t-1} - p^t (1-p^{t-1}) = 1 - p^t.$$

This yields:

$$LHS = \sum_{t=1}^{\infty} \delta^{t-1} \left( p^{2t-1} \frac{\pi^M}{2} + p^t (1-p^{t-1}) \pi^M \right)$$
$$= \left( \frac{p}{2(1-\delta p^2)} + \frac{p}{1-\delta p} - \frac{p}{(1-\delta p^2)} \right) \pi^M$$
$$= \left( \frac{p}{1-\delta p} - \frac{p}{2(1-\delta p^2)} \right) \pi^M.$$

Next, we turn to the right-hand side of ICC, i.e. the immediate deviation profit (obtained with probability $p$ since the shock occurs after the interest rate decision) plus the expected punishment stream from the following period onwards. The former profit is simply $px^M$. As for the latter, there are four possible events at each time $t \geq 2$:

1. Both banks are in the market at the beginning of the period and the deviating bank has a good shock in that period: $p_4(t) = p^{2(t-1)} \ast p$.

2. Both banks are in the market at the beginning of the period and the deviating bank has a bad shock in that period: $p_5(t) = p^{2(t-1)} \ast (1-p)$.

3. The deviating bank will be in the market at time $t \geq 2$ and earn monopoly profit alone:

$$p_6(t) = p^t [(1-p) + p(1-p) + p^2(1-p) + \ldots + p^{t-2}(1-p)] = p^t (1-p) \frac{1-p^{t-1}}{1-p} = p^t (1-p^{t-1}).$$

\footnote{Notably the probability of the competitor being in the market in period $t$ can be computed as $p^{t-1}$. The fact that the competitor might have to leave the market in period $t$ does not affect the profits of the other bank in period $t$.}
4. The deviating bank will not be in the market: $p_7(t) = 1 - p^{2(t-1)} - p^t(1 - p^{t-1})$.

However, it is only $p_6(t)$ that is associated to a non-zero payoff ($p_4(t)$ and $p_5(t)$ are associated to $\pi^C = 0$ and $p_7(t)$ to previous exit). Thus, adding over $t$:

$$RHS = p\pi^M + \sum_{t=2}^{\infty} \delta^{t-1} p^t (1 - p^{t-1}) \pi^M$$

$$= p\pi^M + \frac{\delta p^2}{1 - \delta p} \pi^M - \frac{\delta p^3}{1 - \delta p^2} \pi^M$$

Constructing the overall ICC by comparing LHS against RHS (i.e. (2)), solving for $\delta$ and noticing that $\pi^M$ falls through, one gets that tacit collusion is sustainable if:

$$\left( \frac{1}{1 - \delta p} - \frac{1}{2(1 - \delta p^2)} \right) p\pi^M \geq \left( 1 + \frac{\delta p}{1 - \delta p} - \frac{\delta p^2}{1 - \delta p^2} \right) p\pi^M$$

i.e. $\delta \geq \frac{1}{2p^2}$.

A.1.2 State aid

With State aid a bank that has received a bad shock is rescued and allowed to operate in the following period. There is no profit (nor actual loss) in the period of failure. Setting up the ICC and solving for the critical discount factor, we obtain the traditional supergame result in a symmetric price-setting duopoly:

$$\left( \frac{p \sum_{t=1}^{\infty} \delta^{t-1} \frac{\pi^M}{2}}{1 - \delta p} + (1 - p) \sum_{t=1}^{\infty} \delta^{t-1} * 0 \right) \geq \left( p\pi^M + (1 - p) * 0 + p \sum_{t=2}^{\infty} \delta^{t-1} * \pi^C + (1 - p) \sum_{t=2}^{\infty} \delta^{t-1} * 0 \right)$$

i.e. $\frac{p\pi^M}{2(1 - \delta)} \geq p\pi^M$.

That is, collusion is sustainable for $\delta \geq \frac{1}{2}$.

A.2 Derivation of the consumer welfare equations

A.2.1 No State aid

Under collusion ($\delta \geq \frac{1}{2p^2}$):

From a consumer welfare perspective, there are six possible states at time $t$: (I) there is a collusive duopoly
and the deposits are returned with interest by both banks; (II) there is a collusive duopoly and all deposits are lost because of the bad shocks to both duopolists; (III) there is a collusive duopoly and only one bank receives a bad shock; (IV) there is a monopoly and the deposits are returned with interest; (V) there is a monopoly and the deposits are lost because of a bad shock; (VI) there is no market at all (banks have exited and consumer welfare is nil).

\[ p_I(t) = p^{2t} \]

\[ p_{II}(t) = p^{2(t-1)}(1-p)^2 \]

\[ p_{III}(t) = 2 \left( p^{2(t-1)}p(1-p) \right) = 2 \left( p^{2t-1}(1-p) \right) \]

\[ p_{IV}(t) = 2 \left[ p^{t-1} \left( (1-p) + p(1-p) + p^2(1-p) + ... + p^{t-2}(1-p) \right) \right] * p \]
\[ = 2p^t(1-p^{t-1}) \]

\[ p_{V}(t) = 2 \left[ p^{t-1} \left( (1-p) + p(1-p) + p^2(1-p) + ... + p^{t-2}(1-p) \right) \right] * (1-p) \]
\[ = 2p^{t-1}(1-p)(1-p^{t-1}) \]

\[ p_{VI}(t) = 1 - (p_I(t) + p_{II}(t) + p_{III}(t) + p_{IV}(t) + p_{V}(t)). \]

The next step is to associate consumer welfare values to each state:

\[ p_I(t) : \quad W_C = \frac{1}{2} \left( \frac{R_H}{2} \right)^2 \]

\[ p_{II}(t) : \quad \Phi_{C2} = -\frac{R_H}{2} \]

\[ p_{III}(t) : \quad \frac{W^M}{2} + \Phi_{C1} = \frac{1}{4} \left( \frac{R_H}{2} \right)^2 - \frac{R_H}{4} \]

\[ p_{IV}(t) : \quad W^M = \frac{1}{2} \left( \frac{R_H}{2} \right)^2 \]

\[ p_{V}(t) : \quad \Phi_{C2} = -\frac{R_H}{2} \]

\[ p_{VI}(t) : \quad 0. \]

Next, we simply sum up these welfare levels (adjusted by the probabilities) over time, accounting for the discount factors. Notice that the sum of the losses is the same as the total size of the NDIS and thus the expected present
discounted value of total taxes in the economy, which thus enter as negative terms ($\Phi < 0$):

$$E(W_{C}^{NA}) = \sum_{t=1}^{\infty} \delta^{t-1} \left\{ p^{2t} W^C + p^{2(t-1)}(1-p)^2 \Phi_{NC2} + 2 \left( p^{2t-1}(1-p) \right) \left( \frac{W^M}{2} + \Phi_{C1} \right) \right\} +$$

$$+ \sum_{t=1}^{\infty} \delta^{t-1} \left\{ 2p^t(1-p^{t-1}) W^M + 2p^{t-1}(1-p)(1-p^{t-1}) \Phi_{C2} \right\}$$

$$= \frac{R_H/2}{1-\delta p^2} \left[ (1-p) - \frac{p}{4}R_H \right] + \frac{R_H/2}{1-\delta p} \left[ \frac{p}{2} R_H - 2(1-p) \right].$$

**Under no collusion ($\delta < \frac{1}{2p^2}$):**

From a consumer welfare perspective, there are again six possible states at time $t$: (I) there is a competitive duopoly and the deposits are returned with interest by both banks; (II) there is a competitive duopoly and all deposits are lost because of the bad shocks to both duopolists; (III) there is a competitive duopoly and only one bank receives a bad shock; (IV) there is a monopoly and the deposits are returned with interest; (V) there is a monopoly and the deposits are lost because of a bad shock; (VI) there is no market at all (banks have exited and consumer welfare is nil). These events occur, respectively, with the same probabilities $p_I(t)$ through $p_{VI}(t)$ that we discussed above; it is just that "collusive duopoly" has to be replaced with "competitive duopoly". However, the welfare levels associated to each probability are different:

$$p_I(t) : \ W^C = \frac{R_H^2}{2}$$
$$p_{II}(t) : \ \Phi_{NC2} = -R_H$$
$$p_{III}(t) : \ \frac{W^C}{2} + \Phi_{NC1} = \frac{R_H^2}{4} - \frac{R_H}{2}$$
$$p_{IV}(t) : \ W^M = \frac{1}{2} \left( \frac{R_H}{2} \right)^2$$
$$p_{V}(t) : \ \Phi_{C2} = -\frac{R_H}{2}$$
$$p_{VI}(t) : \ 0.$$

We sum again these probability-adjusted welfare levels over time, to obtain:

$$E(W_{NC}^{NA}) = \sum_{t=1}^{\infty} \delta^{t-1} \left\{ p^{2t} W^C + p^{2(t-1)}(1-p)^2 \Phi_{NC2} + 2 \left( p^{2t-1}(1-p) \right) \left( \frac{W^M}{2} + \Phi_{NC1} \right) \right\} +$$

$$+ \sum_{t=1}^{\infty} \delta^{t-1} \left\{ 2p^t(1-p^{t-1}) W^M + 2p^{t-1}(1-p)(1-p^{t-1}) \Phi_{C2} \right\}$$

$$= \frac{R_H/2}{1-\delta p^2} \left( \frac{p}{2} R_H \right) + \frac{R_H/2}{1-\delta p} \left[ \frac{p}{2} R_H - 2(1-p) \right].$$
A.2.2 State aid

We proceed in the same fashion as before. However, here, banks never exit. When consumers’ deposits are lost due to the bad shocks, the Government refunds them the original capital, as well as paying the direct rescuing costs $\gamma$.

Under collusion ($\delta \geq \frac{1}{2}$):

There are three scenarios that can characterise the economy at any period $t$: (I) both banks have a good shock and return deposits with interest (which occurs with probability $\tilde{p}_I(t) = p^2$); (II) only one bank receives a bad shock (probability $\tilde{p}_{II}(t) = 2p(1 - p)$); (III) both banks receive a bad shock ($\tilde{p}_{III}(t) = (1 - p)^2$). This is true every period and each probability is associated to the following welfare levels:

$$\tilde{p}_I(t) : \quad W^M = \frac{1}{2} \left( \frac{R_H}{2} \right)^2$$
$$\tilde{p}_{II}(t) : \quad \frac{W^M}{2} + \Phi_{C1} = 1 \left( \frac{R_H}{2} \right)^2 - \frac{R_H}{4} - \gamma$$
$$\tilde{p}_{III}(t) : \quad \Phi_{C2} - 2\gamma = -\frac{R_H}{2} - 2\gamma.$$

We can therefore sum this stream of expected payoffs and then subtract the present discounted value of the total tax bill (NDIS and direct aid):

$$E(W^A_C) = \sum_{t=1}^{\infty} \delta^{t-1} \{ p^2 W^M + 2p(1 - p) \left( \frac{W^M}{2} + \Phi_{C1} - \gamma \right) + (1 - p)^2 \left( \Phi_{C2} - 2\gamma \right) \}$$
$$= \frac{R_H / 2}{1 - \delta} \left[ \frac{p}{4} R_H + p - 1 \right] - \frac{2(1 - p)\gamma}{1 - \delta}.$$

Under no collusion ($\delta < \frac{1}{2}$):

We proceed exactly as in the case of collusion. The probabilities are the same as those derived above but the associated welfare levels are different:

$$\tilde{p}_I(t) : \quad W^C = \frac{R_H^2}{2}$$
$$\tilde{p}_{II}(t) : \quad \frac{W^C}{2} + \Phi_{NC1} = 1 \left( \frac{R_H}{4} \right)^2 - \frac{R_H}{2} - \gamma$$
$$\tilde{p}_{III}(t) : \quad \Phi_{NC2} - 2\gamma = -R_H - 2\gamma.$$
Summing up over time:

\[
E(W_{NC}^A) = \sum_{t=1}^{\infty} \delta^{t-1} \left\{ p^2WC + 2p(1-p) \left( \frac{WC}{2} + \Phi_{NC1} - \gamma \right) + (1-p)^2 (\Phi_{NC2} - 2\gamma) \right\}
\]

\[
= \frac{R_H/2}{1-\delta} [pR_H - 2(1-p)] - \frac{2(1-p)\gamma}{1-\delta}.
\]

A.3 On the direct cost of rescuing

In this section we show the positivity of \( \gamma_{NC} \) and \( \gamma_C \). As for the threshold \( \gamma_{NC} \):

\[
\tilde{\gamma}_{NC} = \frac{1- \delta}{2(1-p)} \left\{ \frac{R_H/2}{1-\delta} [pR_H - 2(1-p)] - \frac{R_H/2}{1-\delta p^2} \left[ \frac{p}{2} R_H \right] - \frac{R_H/2}{1-\delta p} \left[ \frac{p}{2} R_H - 2(1-p) \right] \right\} \geq 0
\]

or

\[
pR_H \left( \frac{1}{1-\delta} - \frac{1/2}{1-\delta p^2} - \frac{1/2}{1-\delta p} \right) + 2(1-p) \left( -\frac{1}{1-\delta} + \frac{1}{1-\delta p} \right) \geq 0.
\]

It can be shown that the first term is larger in absolute terms. As a result the threshold is positive. To see this remember (Condition 1) that \( R_H > \frac{4}{1-p} \). Consequently we have that \( pR_H > 2(1-p) \). Further notice that

\[
\frac{1}{1-\delta} - \frac{1/2}{1-\delta p^2} - \frac{1/2}{1-\delta p} > - \left( -\frac{1}{1-\delta} + \frac{1}{1-\delta p} \right)
\]

which gives us the result that \( \tilde{\gamma}_{NC} \) is positive. As for the threshold \( \tilde{\gamma}_C \):

\[
\tilde{\gamma}_C = \frac{1- \delta}{2(1-p)} \left\{ \frac{R_H/2}{1-\delta} \left[ \frac{p}{4} R_H - (1-p) \right] - \frac{R_H/2}{1-\delta p^2} \left[ (1-p) - \frac{p}{4} R_H \right] - \frac{R_H/2}{1-\delta p} \left[ \frac{p}{2} R_H - 2(1-p) \right] \right\} \geq 0
\]

or

\[
\frac{p}{4} R_H \left[ \frac{1}{1-\delta} + \frac{1}{1-\delta p^2} - \frac{1}{1-\delta p} \right] + (1-p) \left[ -\frac{1}{1-\delta} - \frac{1}{1-\delta p^2} + \frac{2}{1-\delta p} \right] \geq 0.
\]

For the same argument as before we have \( \frac{p}{4} R_H > (1-p) \). Moreover

\[
\frac{1}{1-\delta} + \frac{1}{1-\delta p^2} - \frac{1}{1-\delta p} > - \left[ -\frac{1}{1-\delta} - \frac{1}{1-\delta p^2} + \frac{2}{1-\delta p} \right]
\]

\[
\frac{1}{1-\delta p} > 0
\]
which is true since $\delta p < 1$, hence $\tilde{\gamma}_C$ is positive.

### A.4 Proof of (i) in Proposition 2

**Proof.** By contradiction. Suppose that

$$E(W_C^A) = \frac{R_H}{1 - \delta} \left[ \frac{p}{4} R_H + p - 1 \right] - \frac{2(1 - p)\gamma}{1 - \delta} \geq 0$$

$$E(W_{NC}^A) = \frac{R_H}{1 - \delta p^2} \left[ \frac{p}{2} R_H \right] + \frac{R_H}{1 - \delta p^2} \left[ \frac{p}{2} R_H - 2(1 - p) \right]$$

or

$$\frac{p}{4} R_H - (1 - p) \geq \frac{1 - \delta}{1 - \delta p^2} \left[ \frac{p}{2} R_H \right] + \frac{1 - \delta}{1 - \delta p^2} \left[ \frac{p}{2} R_H - 2(1 - p) \right]. \tag{10}$$

Note that RHS in equation (10) is continuous and decreasing in $\delta$ if $\frac{p}{4} R_H + p - 1 > 0$, which holds by assumption.\(^ {19} \)

To see this take the derivatives:

$$\frac{\partial}{\partial \delta} \left( \frac{1 - \delta}{1 - \delta p^2} \right) = -\frac{(1 - \delta p^2) + (1 - \delta)p^2}{(1 - \delta p^2)^2} = \frac{p^2 - 1}{(1 - \delta p^2)^2} < 0$$

$$\frac{\partial}{\partial \delta} \left( \frac{1 - \delta}{1 - \delta p} \right) = -\frac{(1 - \delta p) + (1 - \delta)p}{(1 - \delta p)^2} = \frac{p - 1}{(1 - \delta p)^2} < 0.$$  

Yielding:

$$\frac{\partial \text{RHS}}{\partial \delta} = \begin{cases} 
\frac{p^2 - 1}{(1 - \delta p^2)^2} \left[ \frac{p}{2} R_H \right] & < 0 \\
\frac{p - 1}{(1 - \delta p)^2} \left[ \frac{p}{2} R_H + 2p - 2 \right] & < 0 
\end{cases} < 0.$$

Moreover:

$$\frac{1 - \delta}{1 - \delta p^2} < \frac{1 - \delta}{1 - \delta p}.$$

Consequently the RHS of equation (10) is biggest for the smallest value of $\delta$ in the given range, which is $\delta = \frac{1}{2}$.

After plugging in we arrive at:

$$\frac{1/2}{1 - p^2/2} \left[ \frac{p}{2} R_H \right] + \frac{1/2}{1 - p/2} \left[ \frac{p}{2} R_H - 2(1 - p) \right]$$

$$= \frac{1}{1 - p^2/2} \left[ \frac{p}{4} R_H \right] + \frac{1}{1 - p/2} \left[ \frac{p}{4} R_H - (1 - p) \right].$$

\(^ {19} \)Recall that $R_H > 4 \frac{1 - p}{p}$.
Thus we have:

\[
\frac{1}{1 - p^2/2} \left[ \frac{p}{4} R_H \right] + \frac{1}{1 - p^2/2} \left[ \frac{p}{4} R_H + p - 1 \right] \leq \frac{p}{4} R_H + p - 1
\]

\[
R_H \leq 4 \frac{(1 - p) (1 - p^2/4)}{p (p - p^2/4)}.
\]

Leading to a contradiction since \( \frac{1}{2} - \frac{p^2}{2} < 1 \) has to hold since \( p \in [0, 1] \) and because of the assumption that \( R_H > 4 \frac{(1 - p)}{p} \). We thus reach the result stated in Proposition 2(i): \( E(W^A_C) < E(W^A_{NC}) \), \( \forall p \in (0, 1) \) \( \wedge \frac{1}{2} \leq \delta < \frac{1}{2p^2} \).