A Model of Liquidity Provision with Adverse Selection

Christoph Bertsch*

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Abstract

This paper analyzes a model of liquidity provision where liquidity risk is shared in two distinct spot markets. One of them is a market for asset sales prone to an adverse selection problem and the other is a collateralized credit market, which is not subject to an adverse selection problem. I find that the increased availability of collateralized credit (ex-post) may make the adverse selection problem in the asset market more severe. As a result, the relationship between the completeness of markets and equilibrium welfare, as well as efficiency, is non-monotone. Furthermore, I generate a financial crisis by introducing an aggregate liquidity or solvency shock, which amplifies the adverse selection problem, leading to a market failure. A central bank can address this market failure by using existing market institutions to reallocate liquidity in the economy. Interestingly, the central bank has to be willing and able to incur a loss.

Keywords: Liquidity, asymmetric information, open market operations

JEL classification: D82, E58, G01, G12, G18

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1 Introduction

At the beginning of the financial crisis in mid 2007 a predominant problem was the shortage of liquidity and markets with asymmetric information problems were most affected. I develop a model of liquidity provision that incorporates this features and demonstrates how a detrimental interaction between an asymmetric information friction in asset markets and a shortage of aggregate liquidity or a solvency shock can generate a market dry-up stemming from an adverse selection problem. Through this mechanism my model offers an explanation for the sharp drop in prices for securitized US sub-prime mortgage loans and high-yield corporate bonds at the beginning of the crisis. The model is used to address several questions. Can the increased availability of collateralized credit (ex-post) make the adverse selection problem in the asset market more severe? What are the policy implications for liquidity requirements and leverage? How can central banks effectively respond to a financial crisis by using existing market institutions to reallocate liquidity?

To answer these questions, I propose a model with two different spot markets that can be used by agents to share their liquidity risk, which arises from idiosyncratic cost shocks. The two spot markets have distinct characteristics. One of them is an asset market that is prone to an adverse selection problem, as agents have private information on the valuation of assets. This asset market can dry up in a liquidity or solvency crisis. A phenomenon similar to what we saw in the market for securitized sub-prime mortgage loans where originators had private information on the loan quality. The other spot market is a collateralized credit market in the prime segment. As distinct from the asset market, the credit market is not subject to an adverse selection problem. However, access to collateralized credit is restricted due to limited pledgeability and so markets are incomplete.

This novel setup allows me to analyze not only the role of asymmetric information in the asset market, but also the role of market completeness, which is reflected in the restricted access to collateralized credit. Interestingly, the role of market completeness is ambiguous, since the adverse selection problem in asset markets is amplified when markets are more complete. Therefore, a better access to collateralized credit is beneficial if the adverse selection problem is mild. Instead, it turns out to be harmful when the functioning of the asset market is endangered. This is due to a financing asymmetry. Agents with a liquidity need and a high quality asset in hand have to sell less of their assets at a discount when having a better access to collateralized credit via the prime market, while agents with low quality assets in hand are still willing to sell their assets since they can gain from trading on their private information. Consequently, more complete markets can lead to a deterioration in the average quality of assets traded in the market and amplify the adverse selection problem.

Finally, the proposed model with two spot markets is able to capture essential features of the initial crisis response of central banks in 2007 and 2008. The initial response of central banks aimed at re-allocating the available liquidity in the economy between different markets to overcome a dry-up of sub-prime and interbank markets. In particular, I focus on the asset purchase program of the U.S. Federal Reserve (henceforth FED) that was financed by selling or borrowing against short-term government debt. I find that such an asset purchase program can in my model be effective in overcoming a market failure by channeling liquidity from the prime market segment, collateralized credit, to the asset market and thereby mitigating the adverse selection problem. Furthermore, I argue that an asset purchase program has an advantage over a guarantee scheme, as the policy maker requires less information. However, the central bank has to face losses from intervening.

1.1 The initial financial crisis period and policy interventions

At the beginning of the Financial crisis in 2007 the prices of securitized loans dropped sharply (see left graph in figure 1). Simultaneously the 3-month Libor-OIS spread shot up, indicating that the interbank market was distressed (see right graph in figure 1). While many risky market segments showed a considerable drop in market activity, often referred to as the dry-up of sub-prime, the trading and issuing activity in prime markets was not majorly affected in 2007.[1] My model is consistent with this fact, as in the model the collateralized credit market, itself not subject to an adverse selection problem, exhibits a high turnover during the financial crisis.

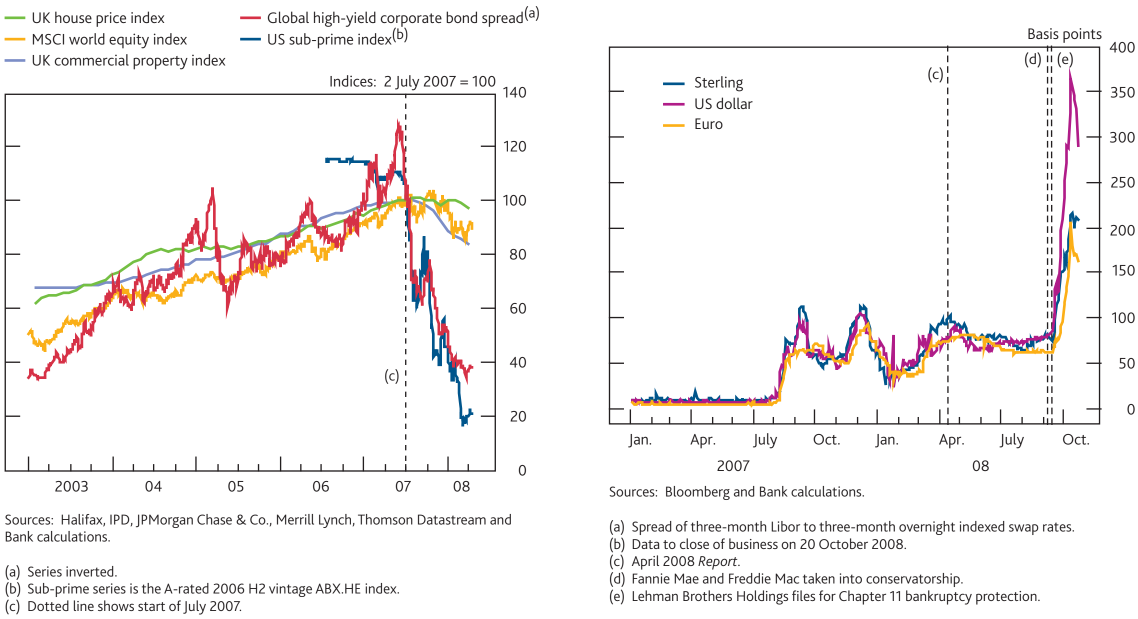

Figure 1: The large drop in asset prices (left-hand side: graph taken from page 6 of BoE Financial Stability Report, April 2008) and the elevated LIBOR-OIS spread (right-hand side: graph taken from page 18 of BoE Financial Stability Report, October 2008).

Gorton (2008) argues that it is not clear whether a drop in liquidity supply relative to aggregate liquidity demand or a rise in perceived counter-party risk, possibly amplified by a fear of contagion, is responsible for the market distress. As Borio (2010) puts it, "[i]n all such crisis, counter-party risk either triggers or amplifies the original disturbance" (page 71). However, there is evidence that market liquidity played a crucial role at the beginning of the crisis and my model is motivated by the observations that *(a)* at the beginning of the crisis the predominant problem was a shortage of liquidity and *(b)* markets with asymmetric information problems were most affected.[2] This initial crisis period, which can be dated from July 2007 until April 2008, is the period of interest in my paper. At that

[1] Taking the German Pfandbrief market as an example in the prime segment, the total private placements were with 77.4 billion Euros in 2007 about 14.2 % short of the previous year but still well above the average of 73.2 billion Euros in the years 2003-2006 (see statistics of Verband Deutscher Pfandbriefbanken: http://www.pfandbrief.de/).

[2] A Bank of England Financial stability Report from April 2008 presents conservative estimates of US sub-prime losses. Their approximations lead to estimates for the accumulated sub-prime losses of up to 170 billion US Dollars. Further, would AAA-rated securities bear losses only in extremely pessimistic scenarios. However, mark-to-market losses are more than twice as high (pages 18 et. seqq.) and highly rates securities are also affected. This suggests that not only credit risk, but also a scarcity of market liquidity are the driving forces behind the drop in US sub-prime prices. In a recent empirical study Schwarz (2010) tries to disentangle market liquidity and credit risk. She finds that market liquidity can explain roughly two-thirds of the widening in interbank market spreads. Finally, Allen and Carletti (2008) argue that cash-in-the-market pricing could offer an explanation for the observed asset price movements during the crisis. The underlying reasons for the shortage of liquidity can be numerous. It may be attributed to margin calls, drawn credit lines from structured investment vehicles or the flight to quality.

time policy markers perceived the problem to be a failure of markets to allocate liquidity and so the interventions of leading central banks aimed at re-allocating the liquidity available in the economy, rather than providing additional liquidity.[3]

The way I model the policy intervention is motivated by the initial crisis response of leading central banks. I demonstrate how a central bank facing a market failure can improve upon the allocation of liquidity by intervening ex-post, using a policy that channels liquidity from the collateralized credit market to the market for asset sales. The type of intervention I consider shares similarities with the interventions of the FED and the European Central Bank (henceforth ECB). Cecchetti (2009) gives an insightful review of the FED policy response. He argues that the key characteristic of the FED's intervention in 2007 and early 2008 lies in a change of its balance sheet decomposition, rather than in an expansion of the balance sheet size. By providing liquidity to the markets for commercial paper and collateralized mortgage loans, using the Term Auction Facility (TAF) and the Term Asset-Backed Securities Loan Facility (TASF) and, simultaneously, raising liquidity by selling safe government securities using the Term Securities Lending Facility (TLF), the FED put upward pressure on the price of borrowing against government bonds and downward pressure on the price for borrowing against asset-backed securities. Borio and Nelson (2008) provide a review complementing Cecchetti (2009) in that they also discuss the central bank policies in the euro area, Japan, the United Kingdom, Canada, Australia and Switzerland. On page 45 of their paper they find that "on net, liquidity was only temporarily, if at all, injected in larger amounts than usual in line with the fundamental characteristics of the demand for reserve balances".

Interestingly in my model a similar central bank intervention, characterized by a change in the decomposition of the balance sheet (rather than an injection of additional liquidity or quantitative easing), can only be effective if the central bank is willing to incur future losses on its investments in the market for asset-backed securities.[4] Otherwise the intervention only reshuffles liquidity without improving its allocation.

1.2 Literature review

This paper is related to the literature on liquidity provision and draws from the contributions of Allen and Gale (henceforth A&G). Borrowing from earlier work of Shleifer and Vishny (1992), A&G develop the idea of liquidity supply being inelastic in the short-run, namely the idea of *cash-in-the-market* pricing.[5] In my model agents face liquidity risk that can be shared in an interim market where long-term assets are traded against cash. As in A&G the asset market exhibits cash-in-the-market pricing, but the liquidity risk does not arise from random consumer liquidity demand. Instead it arises from a "cost shock" in the intermediate period that could be considered as a random profitable investment opportunity similar to Plantin and Parlour (2008) or Kahn and Wagner (2010).

As in Plantin and Parlour, lucrative and safe investment opportunities can be financed by selling risky long-term projects in an asset market that is prone to an Akerlof (1970) type adverse selection problem. However, I allow for an additional way of financing. Namely, I give agents a constrained access to borrowing against the proceeds of the safe investment opportunity. This feature is similar to the

[3]The latter took place at a later point in the crisis in the form of quantitative easing with the aim of stimulating the economy.

[4]Up to now the FED's haircuts were sufficient and it did not incur losses from the TASF but a considerable fraction of the repayments are still due. See Federal Reserve Board publication on transactions data: http://www.federalreserve.gov/newsevents/reform_talf.htm.

[5]In their work, they analyze financial fragility in models with and without financial intermediation where liquidity risk is shared in an asset market. See Allen and Gale (2007) for an overview.

approach used by Kahn and Wagner, where agents can pledge a constant fraction of their investment opportunity's total return as collateral. The rational for the restricted access to collateralized credit arises from a limited pledgeability of the proceeds of the investment opportunity. Combining these features allows me to analyze how the collateral constraint interacts with the adverse selection problem in the asset market.

Furthermore, my model is related to recent papers addressing the possibility of an interbank market failure due to an adverse selection problem. I review this literature below. Notably all of these papers offer similar explanations for the market failure phenomenon during the financial crisis. However, the models and, hence, the policies that can be considered differ from mine. Heider et al. (2009) study the role of asymmetric information in a competitive inter banking credit market and develop a Diamond and Dybvig (1983) type model that can generate three different interbank market regimes (pooling, separating, market break-down with liquidity hoarding). In a variation of the model Heider and Hoerova (2009) show that interest rates of secured and unsecured debt can decouple after an adverse shock to credit risk. Their focus is on explaining market phenomena and how central bank policy can improve the functioning of the secured market segment which is not subject to a private information problem. My paper, on the other hand, focuses on the welfare consequences of the severity of an adverse selection problem in the asset market.[6]

There is a close relationship between the modelling of credit markets as in Heider et al. and the modelling of asset markets. Several recent papers elaborate on the relation between the "lemons problem" and the liquidity of asset markets. Kirabaeva (2010) develops a model similar to Heider et al. but without financial intermediation, where liquidity risk is shared in an asset market. Her model has some parallels to a special case of my model where borrowing against the proceeds of the investment opportunity is ruled out. In a related model Malherbe (2009) examines how hoarding behavior can arise in a model where cash holdings pose a negative externality. Instead, in my model the aggregate level of cash is endogenous leading to a detrimental interaction between cash-in-the-market pricing and the severity of the adverse selection problem. Different to Malherbe cash holdings are not necessarily a bad thing in my model, which gives rise to different implications. My paper is also related to the work of Freixas and Holthausen (2004), who find that a better access of agents to a secured credit market has an ambiguous effect on welfare. While they consider a change in the income structure, I examine the role played by better abilities to borrow against future income. Still, their result shares some similarity with mine, but their mechanism differs and I consider the role of market completeness directly. Moreover, my focus is on policy implication that hinge on trading on private information and cash-in-the-market pricing, which Freixas and Holthausen don't have in their model. Eventually, Taddei (2010) and Kurlat (2009) construct more macro-type models inspired by Eisfeldt (2004) and investigate the relationship between the liquidity in the economy and macroeconomic fluctuations.

Another main contibution of my paper lies in the analysis of the FED's initial response to the crisis, as discussed by Cecchetti (2009). The policy response was unconventional at its time but we can find parallels in the earlier theoretical literature. There are a number of papers where the government/central bank provides its cash reserves or creates new cash to counteract an aggregate liquidity shortage.[7] In contrast, my paper focuses on a central bank intervention that, like the initial

[6] In my paper a market failure cannot arise as consequence of liquidity hording as in Heider et.al.. Instead the phenomenon occurs because the "good" types who have a liquidity need do not want to liquidate their long-term assets of high quality at low equilibrium liquidation prices.

[7] See for instance Allen et al. (2009) where the central bank raises the cash needed for the state with high aggregate liquidity demand by taxing agents ex-ante.

FED intervention, reallocates the liquidity available in the economy, using existing market institutions, rather than providing additional liquidity to the economy. I show that the intervention can only be effective when the central bank incurs a loss, as in Gorton and Huang (2004) where costly bail-outs of banks trough purchases of distressed assets are financed by issuing government bonds.

1.3 Organization of the paper

The paper proceeds as follows. Section 2 introduces the description of the economy and section 3 discusses a special case of the model where agents can share their liquidity risk only in one spot market, namely the market for trade in risky long-term assets. In section 4 the collateralized credit market is added. First, I analyze how the properties of equilibria depend on the agents' ability to leverage by using the collateralized credit market. Second, I solve the problem of a constrained planner and discuss when and how social welfare can be increased by appropriate ex-ante and ex-post policy intervention. In section 5 I extend the model by introducing aggregate liquidity and solvency risk. This allows to examine effective ex-post central bank responses to a market failure. Finally section 6 concludes.

2 Description of the economy

2.1 Setup of the model

Consider a three period economy ($t = 0, 1, 2$) with a continuum of *ex-ante identical* and *risk-neutral agents* of mass one that could be understood as firms or banks. They have an identical endowment of one unit of cash in the initial period and no endowment in subsequent periods. Agents derive utility from consumption at date $t = 2$ only.

Moreover, every agent is equipped with one unit of a project at the beginning of time. This project is agent or firm specific and yields a return in the terminal period. One part of the total return is a monetary payoff which is pledgeable and the other part comes as a private benefit to the agent. The firm specific project is called "internal project" for the remainder of the paper. The internal project does not require inputs in period 0 or period 2, however it is subject to a random refinancing need in the intermediate period. Specifically, each agent faces a financing need with probability $0 < \lambda < 1$. The total return per unit on the project is $\bar{R}$ at date $t = 2$ if the project was not subject to a financing requirement at date $t = 1$. Otherwise, the total return is proportional to the level of financing provided to the project at date $t = 1$. Let the total financing requirement be one unit of cash and let the actual amount of cash invested be denoted by l, where $l \in [0, 1]$. The total return at the terminal date is then given by $\left(\bar{R} * l\right)$. The financing requirement is effectively a *"cost shock"* but can also be interpreted as a *"liquidity shock"* as in Holmstrom and Tirole (1996) since it implies a demand for cash. Liquidity shocks are *independently distributed* across agents. Hence, at $t = 1$ a population fraction $(1 - \lambda)$ of agents do not have a liquidity shock, while the remaining fraction λ faces a liquidity need.

At date $t = 0$ agents anticipate their possible liquidity need in the future and decide how to invest their endowment of one unit of cash. Each agent has access to a storage technology and to a risky long-term asset with payoffs depicted in table 1. Let y denote the units of endowment invested in the risky long-term technology, with $(1 - y)$ denote the units invested in storage. The payoffs are given in table 1.

When held until maturity, the risky asset yields a return R^k, where $k = L, H$. Agent experience a low return of $0 \leq R^L < 1$ with probability $0 < \alpha < 1$ and a high return of $R^H > 1$ with probability

$(1-\alpha)$. For simplicity I set $R^L = 0$. The main result are unaffected by this simplification.[8]

The returns on the risky assets are *distributed independently* across agents (and also independently from the liquidity shock on the internal project). Hence α and $(1-\alpha)$ are the population fractions of agents facing a low return and a high return, respectively. Moreover, agents can trade risky long-term assets in the intermediate period at an endogenous price P.

investments	$t=0$	1	2
storage (cash)	-1	$+1$	
		-1	$+1$
risky long-term asset	-1		R^H w.p. $(1-\alpha)$
	-1		R^L w.p. α
	-1	$+P$	

Table 1: Risky long-term asset and storage

Assume that $\bar{R} > R^H$ and that the expected return on the risky long-term asset is larger than one, i.e. $ER = (1-\alpha)R^H > 1$. The former assumption assures that the realization of a liquidity need by the internal project offers agents a superior return on cash when compared to the risky asset (tantamount to a lucrative investment opportunity). The latter assumption, instead, implies that storage is dominated in terms of expected return. Effectively, the two assumptions on the relative returns give me a model of liquidity risk sharing where agents face a trade-off and typically do not find it optimal to hold all the cash at date $t=0$ that may be needed at date $t=1$. They rather choose to hold a mixed portfolio and then partially finance a liquidity need through sales in the asset market.

Information structure

At the beginning of $t=1$

- agents receive *private information* (for simplicity a perfect signal) on the quality of the risky long-term asset in their portfolio: R^H or $R^L = 0$.
- liquidity shocks are realized and agents have *private information* on their liquidity need.

The agents in this economy could be thought of as banks and the risky long-term asset as a portfolio of securitized loans that banks invest in at date $t=0$ and then hold on their trading book. Then they learn privately about the asset quality before a market for trade in securitized loans takes place in the intermediate period.

Alternatively, the internal project could be thought of as the sum of all other proprietary activities of a bank that are accounted for in the banking book. Due to drawn credit lines or margin calls, a liquidity need may materialize in the intermediate period. Arguably, banks have private information on their liquidity need. If they do not service all their commitments, then a proportion of their profits are lost. For simplicity, the per unit return on the internal project is safe and given by $\bar{R}$, conditional on the provision of financing at date $t=1$. This creates a stylized model where all the "pure" return risk is captured by the risky long-term asset, while all the liquidity risk is attached to the internal investment project.[9]

[8] For a more general version of the model please refer to an earlier version of this paper.

[9] Essentially, what is important is not so much that the return on the internal project is safe, but that there is no significant private information problem attached to it. This could be justified by the ability of banks to fully hedge all significant return risk (besides the liquidity shock) of their highest quality loan portfolio, financial investments and other

Market institutions

Due to private information, agents do not have the possibility to trade state contingent claims in the initial period to insure against the idiosyncratic liquidity risk, and so markets are endogenously incomplete. Instead, agents can share their idiosyncratic liquidity risk in two spot markets at $t = 1$: A lemons asset market where risky long-term assets can be traded and a collateralized credit market where agents can borrow against a fraction of the proceeds of their internal project.

Lemons asset market: Agents can trade risky long-term assets in an *anonymous* and *competitive* spot market.[10] However, the asset market is prone to an informational friction, as at the beginning of date $t = 1$ agents receive private information on the payoff at date $t = 2$. In addition, agents have private information on the realization of the liquidity shock. Consequently, agents with no liquidity need and a lemon in their portfolio can "trade on their private information" by selling their lemons.

Collateralized credit market: Alternatively agents can borrow against the return on their internal project in a bilateral credit market. However, as mentioned before, only a fraction $0 \leq \theta \leq 1$ of the future payoff can be pledged. If a borrower defaults, then the lender can seize the collateral. In case the value of the collateral at date $t = 2$ falls short of the promised repayment, the lender can come after the borrower and ask for the short-fall if the latter has any remaining cash.[11] This latter assumption may not be compatible with all observed collateralized borrowing agreements but it is consistent with many important ones, such as the Pfandbrief market, which is one of the world's largest fixed-income markets, and most private mortgage loan arrangements in Continental Europe.[12]

Returning to the setup, agents can borrow up to $\frac{R*l}{r}$, where $(R * l)$ is the pledgeable future income and r is the endogenous gross interest rate on secured borrowing. Hence, the borrowing constraint can be written as:

$$\frac{\theta\left(\bar{R} * l\right)}{r},$$

where θ is the fraction of the discounted total return on the internal project than can be pledged as

operational activities. However, this does not apply for banks' low quality (sub prime) loan portfolios where private information problems loom large.

[10] The risky long-term assets could be thought of as sub-prime asset-backed securities that are traded on a secondary market. Hence the assumption of an anonymous and competitive spot market is justified, since the majority of asset-backed securities are traded over-the-counter (OTC) where trades are not mutually observed. The important role played by trades in the OTC market and the unobservability of trades is pointed out by Sabarwal (2006) and in a mimeo of Acharya and Engle (2009). Concerning the market for mortgage-backed securities, the assumption of unobservability could also be justified, although a growing number of these fairly standardized securities are traded on electronic platforms. This is because a large number of competing platforms co-exist alongside OTC markets, making the market less transparent.

[11] Notably, I assume that agents are liable for the promise they give in the credit market. An implication of this is that agents with a liquidity need do find it optimal to invest the cash that they raise in the market. Furthermore, agents without a liquidity need do not have an incentive to raise costly financing by borrowing against the proceeds of their internal project. As will become clear later, this assures that there is no default in equilibrium. An equivalent outcome could be achieved if the assumption of private information on the realization of the liquidity shock is changed in a way such that lenders in a bilateral borrowing relation can ensure that the borrowers are investing the funds in the project. E.g. consider a two-stage game where at date $t = 1$ in the first stage the lemons asset market opens and in a second stage, after all trades have taken place and the liquidity shocks have become observable, the collateralized credit market opens. This alternative market setup is not unrealistic given the bilateral nature of borrowing compared to the anonymous nature of asset markets. Moreover it captures more closely the structure of the majority of collateralized borrowing contracts that are observed in reality, where the lender has only the right to seize the collateral.

[12] Regarding the predominantly German Pfandbrief market, the lawmaker assures a very standardized and comparably transparent and over-collateralized debt product in the prime segment. This demands that individual loans in the cover pool that turn out to be non-performing have to be replaced in good time by the issuer with new loans of high quality. In other words, the issuer is liable by more than just the cover pool and has to finance replacements of non-performing loans. Concerning the typical European mortgage loans, households cannot just default on their loan and walk away from their house, as it is the case in the United States. Instead, the lender can come after the households financial and non-financial wealth.

collateral.[13] Although the collateral is the borrower's pledge of ownership of the internal project, it is important to notice that its value depends crucially on the level of liquidity provision and can, in principle, differ from the promised repayment. Lenders cannot observe the actual liquidity provision to the internal project l. However, as long as borrowers have an incentive to invest all the cash they raise, it is easy to see that lenders are able to form correct perceptions on the optimal liquidity provision of borrowers. For this it is sufficient that the borrowers' financing possibilities are common knowledge.

Liquidity: The notion of "liquidity" used in this paper refers to the cost of converting the expected future income into cash. This cost of transferring future income is higher, the stronger the adverse selection problem in the asset market and the less cash is available in the economy.

2.2 Decision problems and time line

The description of the economy is summarized in figure 2.

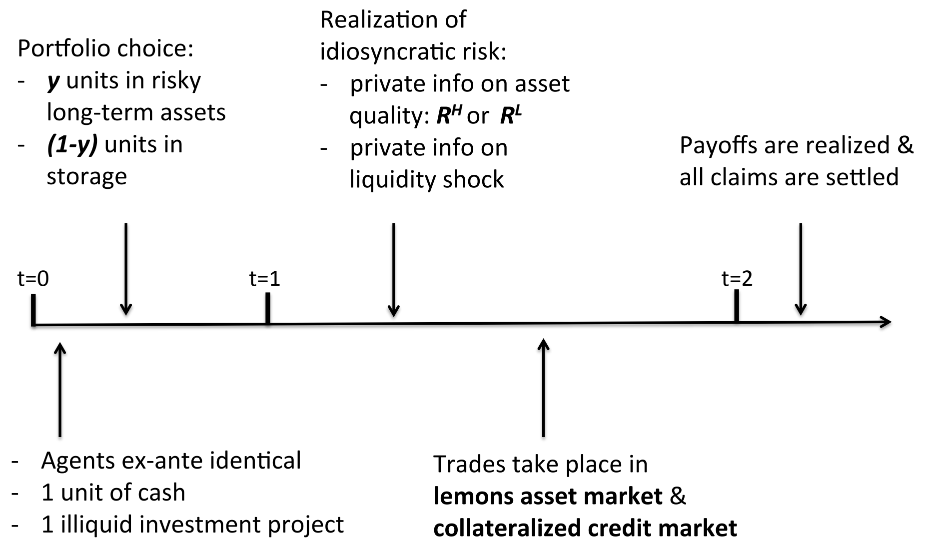

Figure 2: Time line

At the initial date, agents have one unit of cash endowment in hand and choose their optimal portfolio composition between storage $(1-y)$ and the risky long-term asset (y), knowing that they face liquidity risk at $t=1$. With this portfolio agents enter period 1, where they face the realization of "pure" *return risk* on the y units of long-term assets ("good type" R^H & "bad type" $R^L = 0$) and *liquidity risk* from the internal project which materializes as a cost shock. Agents *with a liquidity* need are henceforth called *"deficit agents"* and agents *without a liquidity* need are henceforth called *"surplus agents"*. The former agents are typically lacking in cash (and are the natural candidates to demand cash in the market) whereas the latter agents typically have a surplus of cash (and are the natural suppliers of cash in the market).

Decision problem of a surplus agent at date $t=1$: Surplus agents optimally supply their $(1-y)$ units of cash-in-hand to the market offering the highest return or alternatively store cash if the

[13]To be more specific, suppose that the market works as follows: a potential lender has a certificate of ownership for the internal project. This certificate is divisible and when a borrower meets a potential lender, she can pledge any fraction of the ownership certificate as collateral. In this way, every unit of the internal project can only be used once as collateral.

market return is not higher than 1. Furthermore, they sell their long-term asset if there are gains from "trading on private information" (TPI).

Decision problem of a deficit agent at date $t = 1$: Deficit agents optimally invest their $(1 - y)$ units of cash-in-hand in the internal project as long as the return of doing so is not dominated by the market return. In addition, they raise additional financing by selling their long-term asset and / or by borrowing against the internal project, as long as the cost of funding is lower than the return on being able to provide additional liquidity to the internal project. Deficit agents sell any remaining long-term assets if there are gains from TPI.

3 The special case with $\theta = 0$

In this section I analyze a special case of the model where agents have only access to the lemons asset market and not to the collateralized credit market. Starting with this simplified version of the model is not only useful to gain an understanding of the main mechanisms, but it also allows for a straightforward analysis of efficiency. I find that there is a tendency for over-investment in risky long-term assets which causes market equilibria to be constrained inefficient whenever this over-investment tendency prevails. Furthermore, I demonstrate how a policy maker can intervene ex-ante and implement the constrained efficient allocation. Section 3.1 derives results on the existence and characterization of equilibria. Then section 3.2 analyzes efficiency. The results derived in these two sections also apply to the more general case discussed in section 4 where I give agents access to the credit market, i.e. $\theta > 0$. While section 3 focuses on the role of asymmetric information in the asset market in isolation, section 4 is concerned with the interaction between the constrained access to collateralized credit and the asymmetric information problem. Finally, introducing the collateralized credit market allows me to analyze the initial crisis response of central banks in section 5, which is based on a re-allocation of liquidity from the prime market segments (collateralized credit market) to the market under distress (lemons asset market).

3.1 Liquidity risk sharing in the lemons asset market only

The model has to be solved backwards. First, I examine the decision problem of agents with and without a liquidity need at date $t = 1$. Second, I consider market clearing at date $t = 1$. Finally, the portfolio choice problem at date $t = 0$ is solved. Throughout the paper I am interested in symmetric equilibria, meaning equilibria where all agents choose an identical portfolio ex-ante. Although the main results hold more generally.

Definition. *(a)* There is *pooling* in the lemons asset market, if the equilibrium asset price is such that the deficit agents of "good type" are selling all their long-term assets. *(b)* There is *partial pooling* if the deficit agents of "good type" are willing to sell only a fraction of their long-term assets or are indifferent on whether to sell their long-term assets, while the "bad type" deficit agents are selling all their assets. *(c)* There is a *market failure* if the equilibrium asset price is such that the deficit agents of "good type" are not willing to sell their long-term assets.

It shows that the behavior of good type deficit agents plays a crucial role. Their decision on whether to finance their liquidity requirement by liquidating long-term assets depends on the return of providing liquidity to the investment project relative to the cost of raising cash by selling long-term assets in the lemons market.

In section 3.1.1 I argue that there exists a critical threshold for the asset price above which the good type deficit agents are willing to sell and below which they are not willing to sell their high quality assets. Moreover, the date $t = 1$ decision rules for all 4 types of agents (deficit and surplus agents of good and bad type) are derived formally. These decision rules are then used in section 3.1.2 to solve for the market clearing asset price as a function of the aggregate level of liquidity available in the economy $(1 - y)$. I provide a graphical illustration of market clearing at date $t = 1$ and summarize the main mechanics derived from the decision rules. Finally, I define a competitive equilibrium in section 3.1.3 and solve the date $t = 0$ problem for y in section 3.1.4.

3.1.1 The problem at date $t = 1$

In period 1 agents take y, the market clearing asset price P and the average quality of assets traded in the market, say ER^M, as given. Their optimal decision rules are functions of P, y and ER^M.

Decision rules of surplus agents at $t = 1$: Surplus agents in possession of lemons can trade on their private information. Let f_S^L be the fraction of lemons sold (where the subscript S stands for surplus agents and the superscript L for the bad type with a low quality asset in the portfolio). It has to be true that $f_S^L = 1$ whenever $P > 0$ and $f_S^L \in [0, 1]$ otherwise.

On the other hand, surplus agents are the natural buyers in the lemons asset market. Both, good and bad type, surplus agents are in possession of $(1 - y)$ units of cash at the beginning of period 1. Furthermore, the bad type surplus agents may have up to yP units of additional cash from trading on private information which they can re-invest. When considering supplying cash to the lemons asset market, buyers have to form expectations on the average quality of assets traded in the market $\left(ER^M\right)$ based on the observables P and y. Surplus agents are willing to buy assets if the return on purchasing an asset exceeds 1, i.e. if $ER^M \geq P$.

More formally, let c_S^k denote the supply of cash by surplus agents of type k, then:

$$c_S^H\left(y, P, ER^M\right) = \begin{cases} 1 - y & \text{if } \frac{ER^M}{P} > 1 \\ \in [0, 1 - y] & \text{if } \frac{ER^M}{P} = 1 \\ 0 & \text{otherwise} \end{cases} \qquad c_S^L\left(y, P, ER^M\right) = \begin{cases} \begin{pmatrix} c_S^H\left(y, P, ER^M\right) + \\ f_S^L(P)\, yP \end{pmatrix} & \text{if } \frac{ER^M}{P} \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Decision rules of deficit agents at $t = 1$: As argued earlier, deficit agents provide their $(1 - y)$ units of cash-in-hand to the project whenever the return of doing so exceeds the market return of cash, i.e. if $\bar{R} > ER^M P^{-1}$. Furthermore, deficit agents compare the return of providing cash to the investment project with the cost of raising additional market-based funding. Notice that good type deficit agents can meet their liquidity need by liquidating their assets only partially if $P > 1$, due to the "capacity constraint" given by the total financing requirement of one unit of cash. Here they want to raise only as much market-based funding as needed because it is costly to sell high quality assets at a discount. In contrast, a bad type deficit agent sells all her assets to capture the gains from trading on private information.

Formally, the amount of liquidity provided to the project by a type k deficit agent, $l^k \in [0, 1]$, is given by:

$$l^k\left(y,P,ER^M\right)=\begin{cases} min\left\{1-y+f_D^k\left(P\right)yP,1\right\} & \text{if } \bar{R}>ER^M P^{-1}\\ \in\left[0,min\left\{1-y+f_D^k\left(P\right)yP,1\right\}\right] & \text{if } \bar{R}=ER^M P^{-1}\\ 0 & \text{otherwise,}\end{cases}$$

where f_D^L (f_D^H) denotes the fraction of assets sold by deficit agents (where the subscript D stands for deficit agent and the superscript indicates the quality of the agent's asset portfolio, i.e. $k=L,H$). Due to the asymmetry mentioned before, the bad and the good type deficit agents differ in their selling decisions:

$$f_D^L\left(P\right)=\begin{cases} 1 & \text{if } P>0\\ \in[0,1] & \text{if } P=0\end{cases} \qquad f_D^H\left(P\right)=\begin{cases} \frac{1}{P} & \text{if } R^H>P>1\\ 1 & \text{if } 1\geq P>\widehat{P^H}\\ \in[0,1] & \text{if } P=\widehat{P^H}\\ 0 & \text{otherwise,}\end{cases}$$

where the critical asset price threshold is defined as $\mathbf{\widehat{P^H}\equiv R^H\bar{R}^{-1}}$.

The two types of deficit agents also have different schedules for their optimal cash supply to the market. Whereas the good type deficit agents never supply cash, i.e. $c_D^H\left(y,P,ER^M\right)=0$, the bad types deficit agents may do so. This is the case if they raise more cash than needed, i.e. if $P\geq ER^M>1$. Here we have that $c_D^L\left(y,P,ER^M\right)=\left(1-y+f_D^L\left(P\right)yP\right)-l^L\left(y,P,ER^M\right)\geq 0$, which holds with strict inequality if $ER^M>P$.

3.1.2 Market clearing at date $t=1$

Having derived the agents' decision rules in period 1, I can now analyze market clearing. In particular I examine, for a given value of y, if there exits an equilibrium with (partial) pooling in the asset market, with a market failure or with both.

A graphical illustration: Figure 3 depicts the market clearing price P (thick solid line) and the average return on assets traded in the market ER^M (thick dot-dashed line) as a function of y. The left graph is constructed using a parametrization with a low α, meaning a low number of bad types. Consequently the adverse selection problem is "mild" and ER^M is larger than 1 as long as the good type deficit agents are selling all their assets, i.e. for all $P>\widehat{P^H}$.

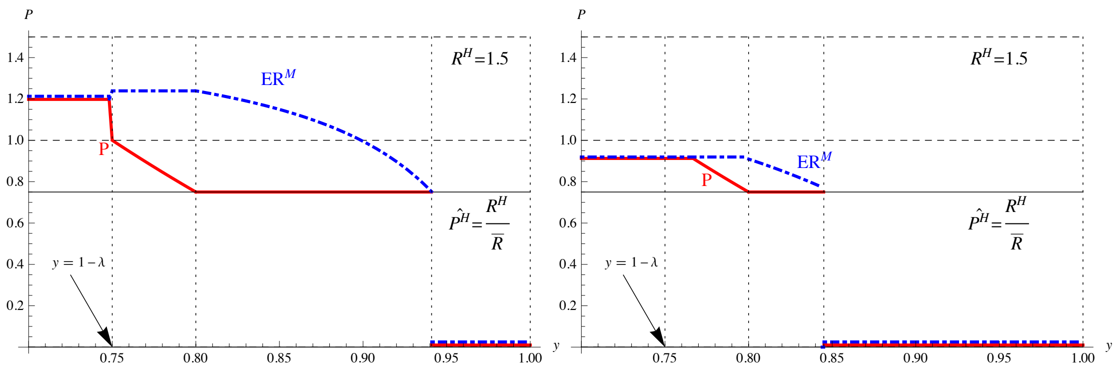

Figure 3: Date $t=1$ price (P) and average return on traded assets $\left(ER^M\right)$ as a function of y. The left graph uses a parameterization with a low level of α and the right graph uses a high level of α.

It can be seen that: *(1)* The market clearing price P has to be weakly smaller than ER^M. Otherwise nobody would be willing to buy assets, as we saw from the decision rules for the supply of cash (c_S^H and

c_S^L). If the adverse selection problem is severe, i.e. high α (right-hand side graph), then a market price above one cannot be supported. *(2)* P is weakly decreasing in y because of cash-in-the-market pricing. *(3)* If $P = \widehat{P^H}$ the good type deficit agents are indifferent between selling their assets or not. Here a further reduction of liquidity causes a deterioration of ER^M up to the point where $ER^M(y) = \widehat{P^H}$. *(4)* For even lower levels of aggregate liquidity (higher y) we have a drop in the asset price to zero due to a *market failure.* *(5)* We have *pooling* in the asset market for all $P \in \left(\widehat{P^H}, 1\right]$, and *partial pooling* for $P = \widehat{P^H}$ and for all $P > 1$.

A formal analysis in Appendix section A.1 reveals that the existence of equilibria at date $t = 1$ not only crucially depends on the cash in the market, but also on the average quality of assets traded.

The role of ER^M: Notice that a high level of α (right-hand side graph) implies that the average return on traded assets is small even when aggregate liquidity is plentiful, i.e. if $y < (1 - \lambda)$. This is because we are facing a "severe" adverse selection problem. It is evident that a higher α amplifies the trading on private information and thereby deteriorates the average quality of assets in the market. As a result, deficit agents are never able to fully cover their liquidity need for all $y > 0$. In the extreme there cannot exist an equilibrium with pooling if α is too high relative to λ. In that case ER^M falls short of the critical threshold $\widehat{P^H}$ independent of the available amount of liquidity in the economy. A formal condition is given by equation (22) in Appendix section A.1. However, the interest of this paper is to analyze the model in a range of parameter values, where the severity of the adverse selection problem does not prevent the existence of a pooling equilibrium per se. Accordingly, I impose a relatively mild condition which restricts α from being "too high" relative to λ.

Condition 1. $\overline{ER^M} > \widehat{P^H}$, where $\overline{ER^M} \equiv \frac{\lambda(1-\alpha)R^H}{\lambda+(1-\lambda)\alpha}$.

3.1.3 Definition of a competitive equilibrium

Before solving the period 0 problem I define a competitive equilibrium for the economy with $\theta = 0$.

A competitive equilibrium consists of

- An optimal portfolio allocation (y) that is feasible and solves the problem of agents at date $t = 0$, taking the period 1 liquidation price P and the average quality of assets traded ER^M as given.
- A date $t = 1$ liquidation price P and the average quality of assets traded in the market ER^M.
- Type dependent decision rules at date $t = 0$ as function of y, P and ER^M: $c_S^H\left(y, P, ER^M\right)$, $c_S^L\left(y, P, ER^M\right)$, $c_D^H\left(y, P, ER^M\right)$, $c_D^L\left(y, P, ER^M\right)$, $l^H\left(y, P, ER^M\right)$, $l^L\left(y, P, ER^M\right)$, $f_S^L(P)$, $f_D^H(P)$ and $f_D^L(P)$.

Given the asset price and the initial portfolio allocation, the optimal decision rules solve the problem of surplus agents and deficit agent at date $t = 1$. The date $t = 1$ market for long-term assets clears. Finally the date $t = 0$ perceptions on P and ER^M have to be correct.

3.1.4 The problem at date $t = 0$

The agents' objective is to maximize their expected payoff in the terminal period, taking the asset price, P, and the expected return on assets traded in the period 1 market, ER^M, as given. The functional form of period 1 decision rules differs for different price intervals. Given the discussion in section 3.1.2, it is useful to divide the relevant price range into the following four intervals:

$$\left[0, \widehat{P^H}\right), \widehat{P^H}, \left(\widehat{P^H}, 1\right], (1, \bar{P}]$$

Each of this intervals differ in the functional form of at least one decision rule.

To find all possible symmetric equilibria I proceed as follows. First, I solve the period 0 portfolio choice problem by conjecturing that the asset price is in a certain interval of the price range. The solution is summarized in the pair (y, P). Second, I check if the solution is consistent with the conjectured asset price interval and the implied period 1 decision rules. If the answer is yes, I have found an equilibrium. I can find all possible symmetric market equilibria by following this procedure for all different asset price intervals in the relevant range $P \in [0, \bar{P}]$, where $\bar{P}$ is the highest possible asset price. The date $t = 0$ problem can be stated as:

$$(P1) \qquad \underset{0 \leq y \leq 1}{max} \left\{ \begin{array}{c} \lambda \left[\alpha D^L\left(y, P, ER^M\right) + (1-\alpha) D^H\left(y, P, ER^M\right)\right] \\ + (1-\lambda) \left[\alpha S^L\left(y, P, ER^M\right) + (1-\alpha) S^H\left(y, P, ER^M\right)\right] \end{array} \right\}$$

$$\begin{array}{ll} s.t. & D^L\left(y, P, ER^M\right) \equiv \bar{R} * l^L\left(y, P, ER^M\right) + \frac{ER^M}{P} * c_D^L\left(y, P, ER^M\right) \\ & D^H\left(y, P, ER^M\right) \equiv \bar{R} * l^H\left(y, P, ER^M\right) + yR^H * \left(1 - f_D^H(P)\right) \\ & S^L\left(y, P, ER^M\right) \equiv \bar{R} + \frac{ER^M}{P} * c_S^L\left(y, P, ER^M\right) \\ & S^H\left(y, P, ER^M\right) \equiv \bar{R} + yR^H + \frac{ER^M}{P} * c_S^H\left(y, P, ER^M\right). \end{array}$$

The first term in square brackets represents the expected return of a deficit agent who turns out to be in possession of a lemon $\left(D^L\left(y, P, ER^M\right)\right)$ and of a high quality asset $\left(D^H\left(y, P, ER^M\right)\right)$, weighted by the respective probabilities α and $(1-\alpha)$. The second term in square brackets represents the expected return of a surplus agents who possesses a lemon $\left(S^L\left(y, P, ER^M\right)\right)$ and a high quality asset $\left(S^H\left(y, P, ER^M\right)\right)$, weighted with the respective probabilities α and $(1-\alpha)$. Both terms in square brackets are again weighted by the probabilities of being a deficit agent (λ) and a surplus agent $(1-\lambda)$. From section 3.1.1 we know the functional forms of the decision rules on how much liquidity to provide to the internal project, on how much assets to sell and on how much liquidity to provide to the market. Consequently the payoff in each of the 4 possible events is given by the sum of the returns on the internal project, the share of long-term assets held after period 1 and the cash invested at date $t = 1$ in storage or to purchase additional long-term assets.

The results are summarized in Proposition 1 below. They emerge from an analysis of *(P1)*, using the findings of section 3.1.2. Before stating the results, it is useful to define the first derivative of the objective function of *(P1)* under the conjecture of pooling:

$$I(P) \equiv \left\{\lambda \bar{R}(P-1) + (1-\lambda)\left[\alpha \overline{ER^M} - \overline{ER^M} P^{-1} + (1-\alpha) R^H\right]\right\}. \tag{1}$$

The first summand reflects the cost of portfolio illiquidity of deficit agents and the second summand reflects the benefits to surplus agents of having more long-term assets. Notice that equation (1) is increasing in P.

Proposition 1 (Results on the existence and characterization of date $t = 0$ equilibria). *Given Condition 1 holds, there can exist three different categories of market equilibria characterized by (i) pooling, (ii) partial pooling and (iii) market failure in the period 1 asset market. In addition, there can exist an equilibrium with (iv) no trade.*

(i) *If* $\overline{ER^M} \geq 1$*, then there exists an equilibrium with pooling characterized by* $P^* \in \left(\widehat{P^H}, 1\right]$ *and* $y^* > 1 - \lambda$ *if the following necessary and sufficient condition holds:*

$$I\left(\widehat{P^H}\right) \leq 0. \tag{2}$$

If $1 > \overline{ER^M} > \widehat{P^H}$*, then there exits an equilibrium with pooling characterized by* $P^* \in \left(\widehat{P^H}, \overline{ER^M}\right]$ *and* $y^* \geq (1-\lambda)\left(\lambda\overline{ER^M} + (1-\lambda)\right)^{-1}$ *if the following necessary and sufficient conditions hold:*

$$I\left(\widehat{P^H}\right) \leq 0 \quad \text{and} \quad I(P)\Big|_{P=\overline{ER^M}} \geq 0. \tag{3}$$

(ii) *There may exist an equilibrium with partial pooling characterized by* $y^* \in \left[\underline{y}, \overline{y}\right]$ *and* $P^* = \widehat{P^H}$*. A necessary condition is given by equation* (2)*.*

(iii) *There always exists an equilibrium where the market fails characterized by* $y^* \in [0,1]$ *and* $P^* = 0$*.*

(iv) *There exists an equilibrium with no trade characterized by* $y^* = 0$ *and* $P^* \in \left(\widehat{P^H}, \overline{ER^M}\right]$ *if the following necessary and sufficient conditions hold:*

$$I\left(\widehat{P^H}\right) \leq 0 \quad \text{and} \quad I(P)\Big|_{P=\overline{ER^M}} < 0. \tag{4}$$

Proof. See Appendix section A.2.

Expression (2) is key for understanding the results of Proposition 1 on the existence of pooling equilibria summarized under *(i)*. Recall that a higher level of aggregate liquidity (a smaller y) implies a higher asset price. Hence, equation (2) intuitively demands that the benefit for deficit agents to hold more cash in period 1, weighted by the probability of being deficit agent, has to be sufficiently high, as to induce agents in period 0 to hold a level of aggregate liquidity that can sustain an equilibrium with pooling in the asset market. The condition tends to be satisfied if λ and $\bar{R}$ are sufficiently big such that holding cash is attractive in period 0. As indicated earlier, a crucial role is played by $\overline{ER^M}$. Recall that $\overline{ER^M}$ constitutes an upper bound on the average quality of assets traded in the market. Consequently for $1 > \overline{ER^M} > \widehat{P^H}$ an equilibrium with pooling in the asset market can only exits if the upper bound is not binding, i.e. if $I\left(\overline{ER^M}\right) \geq 0$ (see equation (3)). Otherwise the risk neutral agents optimally choose a fully liquid portfolio (corner solution) and there is no trade in equilibrium, (see *(iv)*). Intuitively, a high value of cash holdings (sufficiently high λ and $\bar{R}$) together with a limited ability of the asset market to be used for liquidity risk sharing because of a severe adverse selection problem (low $\overline{ER^M}$) makes it attractive for agents to hold a fully liquid portfolio in order to be able to face the cost shock. Proposition 1 finally summarizes the results on the existence of an equilibrium where the market fails under *(iii)*. This latter equilibrium always exists. However, the focus of this paper is on the conditions for the existence of pooling equilibria and their characteristics rather than on coordination failure.

3.2 Efficiency & Policy

It is easy to see that an unconstrained planner can maximize the total period 2 payoff and, hence, social welfare if he chooses $y^*_{FB} = (1-\lambda)$ at date $t = 0$ and then allocates all liquidity to deficit agents. In this way deficit agents are enabled to fully finance their internal projects and the total payoff in period

2 is maximized. However, the aim of this section is to study welfare from a second-best perspective. For this reason I consider a constrained planner problem to derive a welfare benchmark. In particular, I analyze the problem of a planner who can only choose y in period 0 and can neither infer in period 1, nor in period 2. The idea is that the planner has to "work through" the same market institutions and faces the same private information problem as the individual agents in the decentralized economy.

Constrained inefficiency: The results are summarized formally in Proposition 2 below. In short, the market equilibrium is characterized by inefficient over-investment in long-term assets whenever the adverse selection problem is comparably mild, i.e. $\overline{ER^M} \geq 1$. Otherwise, if $1 > \overline{ER^M} > \widehat{P^H}$, the market equilibrium is characterized by inefficient over-investment only if the population fraction of agents with a liquidity shock (λ) is sufficiently small, such that it is unattractive to hold a very liquid portfolio. Moreover the constrained planner can implement the first-best whenever $\overline{ER^M} \geq 1$ by choosing $y^*_{planner} = (1-\lambda)$. I show that the latter is not possible when $1 > \overline{ER^M} > \widehat{P^H}$ since the constrained planner faces the same constraints from the private information problem. Hence there is a "true" second-best as a welfare benchmark. I argue that market equilibria are constrained inefficient for a large range of parameter values. The reason is that the "trading on private information" results in a tendency for over-investment since it increases the expected return of the illiquid investment in the risky long-term technology compared to the liquid storage technology. Due to a price externality together with the adverse selection problem, agents do not take into account how their collective liquidity choice affects the aggregate liquidity and, hence, the asset price and average quality of assets traded in the market. The constrained planner internalizes this externality. So far the result for the special case with $\theta = 0$ is not surprising and similar to Lorenzoni (2008) or Kurlat (2009). However, it complements the welfare analysis in section 4 when I allow for collateralized borrowing which then generates novel implications concerning efficiency and policy.

Proposition 2 (Results on efficiency). *The constrained planner can implement the first-best level of welfare by choosing* $y^*_{planner} = (1-\lambda)$ *if* $\overline{ER^M} \geq 1$. *If, instead,* $1 > \overline{ER^M} > \widehat{P^H}$, *then the constrained planner cannot implement the first-best level of welfare and optimally chooses:*

$$y^*_{planner} = \begin{cases} (1-\lambda)\left(\lambda\overline{ER^M} + (1-\lambda)\right)^{-1} & \text{if } I\left(\overline{ER^M}\right) \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Pooling equilibria can be characterized by (i) inefficient over-investment or (ii) an efficient level of investment in the risky long-term technology. In equilibrium there is an:

(i) inefficient over-investment if either $\overline{ER^M} \geq 1$, *or* $1 > \overline{ER^M} > \widehat{P^H}$ *and* $I\left(\overline{ER^M}\right) > 0$.

(ii) efficient level of investment if $1 > \overline{ER^M} > \widehat{P^H}$ *and* $I\left(\overline{ER^M}\right) \leq 0$.

Proof. See Appendix section A.3.

Policy intervention: In the previous section I demonstrated that the decentralized market equilibrium with pooling is characterized by an *inefficient over-investment* in the risky long-term technology if $\overline{ER^M} \geq 1$. It is easy to show that the constrained efficient allocation can be implemented by a central bank or government authority that can tax the purchase of long-term assets in period 0 and refund the tax proceeds instantaneously lump sum to all agents. After introducing a proportional tax

τ in the period 0 problem of agents *(P1)*, one can solve for a positive proportional tax on investments given by:

$$\tau = (1-\lambda)\,\frac{\left(\alpha ER^M + (1-\alpha)\,R^H\right) - ER^M}{\lambda \bar{R} + (1-\lambda)\left(\alpha ER^M + (1-\alpha)\,R^H\right)}, \quad \text{where} \quad ER^M = \frac{\lambda\,(1-\alpha)\,R^H}{\lambda + (1-\lambda)\,\alpha}.$$

Notice that the constrained inefficiency disappears if the adverse selection problem vanishes. That is $\tau \to 0$ if $\alpha \to 0$. Moreover, the constrained efficient allocation can be implemented by a proportional tax constructed in an analogous way if $1 > \overline{ER^M} > \widehat{P^H}$.

4 Introducing the bilateral collateralized credit market, i.e. $\theta > 0$

In the last section the focus was on the role of asymmetric information in the asset market in isolation. Now an additional spot market is introduced, namely the collateralized credit market. The two markets can interact in a harmful way, in that a better access to collateralized credit can make the adverse selection problem in the asset market more severe. Thereby offering a new argument in favor of a regulation that aims at reducing agents' possibilities to leverage. Furthermore, this new modelling feature allows me to analyze the initial crisis response of central banks in section 5, which is based on a re-allocation of liquidity from the prime market segments (the collateralized credit market) to the market under distress (the lemons asset market).

Financing asymmetry - the pecking order

What changes when introducing the second spot market to share liquidity risk? Agents gain the possibility to borrow against the future income from their investment project. The direct effect on equilibrium welfare is positive because markets become more complete. In addition, there can be an indirect effect that negatively impacts upon equilibrium welfare. The key mechanism lies in a financing asymmetry between deficit agents with high quality long-term assets and deficit agents with lemons. Good and bad types deficit agents have a different "pecking order". On the one hand, a good type always faces a discount when financing through the asset market due to the adverse selection problem. On the other hand, there is no such discount when borrowing against a fraction of the future income from the investment project since the latter market is not subject to an adverse selection problem. Consequently, good type deficit agents prefer to borrow up to their constraint before seeking additional financing through asset sales. In contrast, deficit agents in possession of a lemon gain from selling it. That is why their financing choice is contrary and they prefer to first finance by selling assets before starting to borrow.

By allowing agents to borrow more against their investment project, the good type deficit agents become less dependent on financing themselves in the lemons asset market. As a consequence, the adverse selection problem in the lemons asset market may become more severe which in turn has negative implications for the ability of agents to share liquidity risk.

Preview of main results

The main results can be summarized graphically. Figure 4 depicts the social welfare associated with the Pareto dominant market equilibrium over the full range of θs for a numerical example.[14]

[14] The graph is based on the following parameter values: $\left\{\alpha, \lambda, R^L, R^H, \bar{R}\right\} = \{0.15, 0.25, 0, 1.5, 2.3\}$. The corresponding expected return on the long-term asset is given by 1.275 and $\overline{ER^M} = 0.88$.

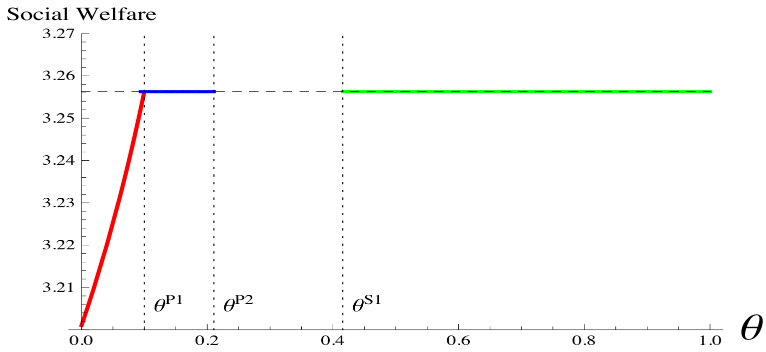

Figure 4: Equilibrium welfare over the range of θs

It is useful to distinguish 4 segments which are separated by dashed vertical lines. In the leftmost segment θ is close to zero. Here there exists an equilibrium with inefficient over-investment in liquidity (this is true also for the special case with $\theta = 0$). Equilibrium welfare increases as markets become more complete, i.e. as θ increases. In the second segment, for all $\theta^{P1} \leq \theta \leq \theta^{P2}$, the market outcome coincides with the constrained efficient allocation (for this numerical example, the first-best level of social welfare is achieved). When θ reaches the third segment an equilibrium with pooling in the lemons asset market does not exist. Here only the collateralized credit market is active and the market equilibrium is associated with a much lower level of welfare (for readability not depicted in the graph). The reason for this discontinuity is that the better access to borrowing (higher θ) makes the good type deficit agents less dependent on the asset market and, hence, exacerbates the adverse selection problem up to the point where the average quality of assets traded in the market falls short of $\widehat{P^H}$. Finally, the forth segment represents sufficiently high values of $\theta > \theta^{S1}$ such that both types of deficit agents can fully refinance by using the collateralized credit market only. Again the first-best level of welfare is achieved.

The remainder is organized as follows. First, I describe in section 4.1 how to solve the model with $\theta > 0$. The key difference with the special case of $\theta = 0$ lies in the more complex period 1 decision problems. Second, I present results on the properties and characterization of equilibria in section 4.2. Third, I explore welfare and efficiency in section 4.3. Finally, I discuss the main empirical predictions of the model and relate them to the model's novel policy implications in section 4.4.

4.1 Solving the model

As before the model has to be solved backwards.

4.1.1 No-arbitrage condition at date $t = 1$

Since I now have to deal with an additional market there is an additional market price r, which was introduced earlier as the gross interest rate in collateralized credit market. In equilibrium, the relationship between the two market prices P and r is pinned down by a no-arbitrage condition whenever a pooling equilibrium exists:

$$\frac{ER^M}{P} = r. \tag{5}$$

If equation (5) holds, then the risk-neutral suppliers of cash are indifferent between providing liquidity to the lemons asset market or to the collateralized credit market.

4.1.2 The problem at date $t = 1$

In period 1 agents take y, P and r as given. Their optimal decisions are functions of y, P, r and ER^M. The decision rules of surplus agents stay the same as in the special case with $\theta = 0$. Good type surplus agents don't have an incentive to sell their high quality assets at depressed prices, while bad type surplus agents sell their lemons. Notice that both do not have an incentive to borrow against their internal project when $r \geq 1$, because borrowing is collateralized and their investment project can be seized in case of default. In summary, the fraction of lemons sold by the bad type surplus agents $f_S^L(P)$, as well as the decision rules for the total supply of cash $c_S^L\left(y, P, ER^M\right)$ and $c_S^H\left(y, P, ER^M\right)$ stay the same. Due to no-arbitrage, surplus agents are indifferent between providing their cash to one market or the other.

Impact of the financing asymmetry on the decision rules of deficit agents: The crucial difference lies in the decision rules for deficit agents reflecting the financing asymmetry discussed earlier. Define b_D^k as the amount of cash borrowed by a deficit agent of type k and define l^k as her optimal amount of liquidity provision to the project which can be derived as:

$$l^k\left(y, P, ER^M\right) = \begin{cases} min\left\{1 - y + f_D^k(y, P)\, yP + b_D^k\left(y, P, ER^M\right), 1\right\} & \text{if } \bar{R} > \frac{ER^M}{P} = r \\ \in \left[0, min\left\{1 - y + f_D^k(y, P)\, yP + b_D^k\left(y, P, ER^M\right), 1\right\}\right] & \text{if } \bar{R} = r \\ 0 & \text{otherwise,} \end{cases}$$

where f_D^k denotes the fraction of assets sold by a deficit agent of type k and b_D^k the amount borrowed.

When considering the optimal fraction of assets sold by a deficit agent in possession of high quality assets f_D^H and her schedule for collateralized borrowing b_D^H (which is a function of θ), one can see the crucial role played by the borrowing constraint. The borrowing constraint is given by $l^H * \theta \bar{R} r^{-1}$. Hence, there is a possible feedback effect since a higher liquidity provision to the project allows for more collateralized borrowing, which in turn allows deficit agents to raise more cash in the market and thereby improves the liquidity provision to the project. To see this formally, consider for example the decision of a good type deficit agent on how much to borrow if the asset price is given by P. In period 1 the deficit agent faces a lack of y units of cash. What is the maximum possible amount of collateralized borrowing? It is indicated by an upper bar and given by:

$$\overline{b_D^H\left(y, P, ER^M\right)} = \begin{cases} \theta \bar{R} r^{-1} & \text{if } \left\{(1 - y) + yP + \theta \bar{R} r^{-1}\right\} \geq 1 \\ \frac{1-y+yP}{r-\theta\bar{R}}\theta \bar{R} & \text{otherwise.} \end{cases}$$

Notice that she is able to fully finance all her liquidity need if $yP + \theta \bar{R} r^{-1} \geq y$.

The feedback effect and its implications for the critical price threshold: While the feedback effect does not have important implications for the main mechanics of the model, it does affect the critical price threshold above which good type deficit agents are willing to sell their high quality assets. The feedback effect is present, whenever the good type deficit agents are not able to fully finance their liquidity need. In the model with borrowing good type deficit agents are willing to accept lower asset prices if they cannot fully meet their liquidity need, i.e. if $yP + \theta \bar{R} r^{-1} < y$. For the reason that in this situation the return from raising additional liquidity by selling assets exceeds $\bar{R}$ due to the feedback effect. Hence, the new critical price threshold denoted by $\widetilde{P^H}$ is lower, so that $\widetilde{P^H} < \widehat{P^H}$:[15]

$$\widetilde{P^H}\left(\theta, \overline{ER^M}\right) \equiv \widehat{P^H} \frac{\overline{ER^M}}{(1-\theta)\,\overline{ER^M} + \theta R^H}. \tag{6}$$

Changes in the decision rules of deficit agents: A full schedule for the decision rules over the relevant price range is relegated to Appendix section A.5. Demand and supply of cash have the same functional form as before with the difference being that $l^k\left(y, P, ER^M\right)$, $f_D^k\left(y, P, ER^M\right)$ and $c_D^L\left(y, P, ER^M\right)$ are now functions of the optimal amount of collateralized borrowing $b_D^H\left(y, P, ER^M\right)$. The main changes lie in the financing asymmetry discussed above and the different critical price threshold below which deficit agents of good type refuse to sell. Therefore, it is important to distinguish the case where deficit agents are not able to fully refinance (i.e. $yP + \theta\bar{R}r^{-1} < y$) from the case where they can (i.e. $yP + \theta\bar{R}r^{-1} \geq y$). In the former case, both types of deficit agents are borrowing up to their constraint whenever the cost of borrowing does not exceed the benefit of liquidity provision to the investment project. The feedback effect is present here. In the later case, however, deficit agents of bad type prefer to sell all their lemons and do not need to borrow up to their constraint, while good type deficit agents prefer borrowing to selling high quality assets.

4.1.3 Market clearing at date $t = 1$

As before, the period 1 equilibrium asset price is derived by solving the market clearing condition for $P(y)$. The expected return of assets traded in the market becomes a function of θ. In particular, the good type deficit agents become less dependent on the asset market for higher levels of θ as this allows them to finance a larger fraction of their liquidity need by borrowing against their future income of the internal project. Hence, they optimally decide not to sell all their high quality long-term assets whenever they are able to fully finance their short-fall in cash (y units). This can be seen formally from expressions (33) and (34) in Appendix section A.5. As a result, the average quality of assets traded in the market under pooling is (conditional on y) smaller than in the special case with $\theta = 0$ whenever the good type deficit agents can fully refinance their liquidity need.

4.1.4 The problem at date $t = 0$

The period 0 problem is constructed in a similar fashion as in section 3.1.4. The only difference between $(P3)$ and $(P1)$ lies in the terms representing the amount of collateralized borrowing by deficit agents:

$$(P3) \qquad \underset{0\leq y\leq 1}{max} \left\{ \begin{array}{c} \lambda\left[\alpha D^L\left(y, P, ER^M\right) + (1-\alpha)\, D^H\left(y, P, ER^M\right)\right] \\ +(1-\lambda)\left[\alpha S^L\left(y, P, ER^M\right) + (1-\alpha)\, S^H\left(y, P, ER^M\right)\right] \end{array} \right\}$$

$$\begin{aligned} s.t. \quad D^L\left(y, P, ER^M\right) &\equiv \bar{R} * l^L\left(y, P, ER^M\right) + \tfrac{ER^M}{P} * c_D^L\left(y, P, ER^M\right) - r * b_D^L\left(y, P, ER^M\right) \\ D^H\left(y, P, ER^M\right) &\equiv \bar{R} * l^H\left(y, P, ER^M\right) + yR^H * \left(1 - f_D^H(y, P)\right) - r * b_D^H\left(y, P, ER^M\right) \\ S^L\left(y, P, ER^M\right) &\equiv \bar{R} + \tfrac{ER^M}{P} * c_S^L\left(y, P, ER^M\right) \\ S^H\left(y, P, ER^M\right) &\equiv \bar{R} + yR^H + \tfrac{ER^M}{P} * c_S^H\left(y, P, ER^M\right). \end{aligned}$$

As before, the first term in square brackets represents the sum of the expected return of a deficit agents who turns out to be in possession of a lemon, weighted by probability α, and of a a high quality

[15] See Appendix section A.4 for more details on the feedback effect and the computation of the new critical price threshold.

asset, weighted by probability $(1-\alpha)$. The second term in square brackets represents the sum of the expected return of a surplus agents possessing a lemon, weighted by probability α, and a high quality asset, weighted by probability $(1-\alpha)$. By applying the same procedure as before, the model can be solved for different intervals in the relevant price range to find all possible symmetric market equilibria.

4.2 Properties and characterization of equilibria

Recall that the decision rules of deficit agents on how much liquidity to provide to the project and on how this should be financed crucially differ depending on whether deficit agents are able to meet all their liquidity need or not, i.e. if $yP + \frac{\theta\bar{R}}{r} > y$ or $yP + \frac{\theta\bar{R}}{r} \leq y$. For this reason it is useful to consider these two different cases in turn. Section 4.2.1 discusses a pooling equilibrium characterized by *scarce aggregate liquidity* when deficit agents are not able to fully refinance their projects. Section 4.2.2 then discusses a pooling equilibrium characterized by *plentiful aggregate liquidity* when deficit agents can fully refinance all their projects. The former case requires $y^* > (1-\lambda)$ and the latter case requires $y^* \leq (1-\lambda)$. Finally, section 4.2.3 discusses equilibria with a failure of the lemons asset market. The main results of each section are summarized at the end in a proposition.

4.2.1 The pooling equilibrium characterized by a scarcity of aggregate liquidity, i.e. $y^* > (1-\lambda)$

For the special case of $\theta = 0$ I showed in Proposition 1 that, given $\overline{ER^M} \geq 1$, there exists a market equilibrium with pooling characterized by $y^* > (1-\lambda)$ and $\widehat{P^H} \leq P^* < 1$ when the necessary and sufficient condition stated in equation (2) is satisfied. If, instead, $1 > \overline{ER^M} > \widehat{P^H}$, then there exists a market equilibrium with pooling characterized by $y^* > (1-\lambda)$ and $\widehat{P^H} \leq P^* < \overline{ER^M}$ when the necessary and sufficient conditions summarized in equation (3) hold.

In this section I examine to what extend the results from the special case apply when $\theta > 0$. I show analytically that, under "mild conditions" on the relative returns of the long-term technologies, there is a unique threshold $\theta^{P1} \in (0,1)$, such that there exists for all $\theta < \theta^{P1}$ a market equilibrium characterized by pooling in the asset market and scarce aggregate liquidity, that is $y^* > (1-\lambda)$. When $\theta \geq \theta^{P1}$, however, this type of equilibrium does not exist.

Firstly, I assume that $yP + \frac{\theta\bar{R}}{r} < y$ holds in equilibrium. That is the aggregate level of liquidity is scarce enough so that deficit agents cannot fully refinance even if selling all their assets and borrowing up to their constraint. The relevant price range can be derived as $P \in \left[\widetilde{P^H}\left(\theta, \overline{ER^M}\right), 1\right)$. Supposing there exists an interior solution to *(P3)*. For a given θ the corresponding first-order condition can derived as:

$$III\left(P;\theta\right) \equiv \left\{ \begin{array}{c} \lambda\left[\frac{(1-\theta)\bar{R}(P-1)}{1-\theta\bar{R}P(ER^M)^{-1}}\right] + \\ (1-\lambda)\left[\alpha ER^M + (1-\alpha)R^H - \frac{ER^M}{P}\right] \end{array} \right\} = 0. \tag{7}$$

A necessary condition for the existence of an interior solution is given by (notice the similarity to equation (2)):

$$III\left(\widetilde{P^H}\left(\theta, \overline{ER^M}\right); \theta\right) \leq 0. \tag{8}$$

Since both types of deficit agents are selling *all* their assets at the pooling price, the expected return on assets traded in the market is the same as in the special case (i.e. $ER^M = \overline{ER^M}$). From the market clearing condition I can derive a relation between the price and the aggregate investment level for a given θ:

$$P(y;\theta) = \frac{(1-\lambda)(1-y)\overline{ER^M}}{\lambda y\overline{ER^M} + (1-\theta)\bar{R}(1-y)}. \tag{9}$$

Finally, I can solve equations (7) and (9) for the unknowns, namely the price P and the aggregate investment level y, to find a candidate equilibrium.[16]

Characterization and properties of the equilibrium

Suppose there exists a unique threshold $\theta^{P1} \in (0,1)$ such that the equilibrium level of aggregate liquidity is scarce for all $\theta \in \left[0, \theta^{P1}\right)$. Denote by $P^{C1}(\theta)$ and $y^{C1}(\theta)$ the expressions for the candidate equilibrium price and allocation as a functions of θ, respectively. Then the solution can be derived in four steps.

First, the candidate equilibrium price can be derived from equation (7) as:

$$P^{C1}(\theta) = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2}, \tag{10}$$

$$\begin{aligned} \text{where}\quad a_2 &\equiv \left\{\theta\left[\lambda + (1-\lambda)\alpha + (1-\lambda)(1-\alpha)R^H\overline{ER^M}^{-1}\right] - \left[(1-\lambda)\alpha + (1-\lambda)(1-\alpha)R^H\overline{ER^M}^{-1}\right]\right\}\bar{R} \\ b_2 &\equiv -\theta\bar{R} + (1-\lambda)\left[\left(\alpha\overline{ER^M} + (1-\alpha)R^H\right)\bar{R}\right] \\ c_2 &\equiv -(1-\lambda)\overline{ER^M}. \end{aligned}$$

Second, the candidate for the aggregate equilibrium investment level can be derived by solving equation (9) for y and plugging in for $P^{C1}(\theta)$:

$$y^{C1}(\theta) = \frac{(1-\lambda)\overline{ER^M} - \theta\bar{R}P^{C1}(\theta)}{(1-\lambda)\overline{ER^M} + \left(\lambda\overline{ER^M} - \theta\bar{R}\right)P^{C1}(\theta)}. \tag{11}$$

Third, the following condition must hold (i.e. it has to be verified that deficit agents cannot fully refinance):

$$y^{C1}(\theta)P^{C1}(\theta) + \theta\bar{R}\overline{ER^M}^{-1}P^{C1}(\theta) < y^{C1}(\theta). \tag{12}$$

Finally, the upper bound on the asset price must not bind, so as to assure an interior solution:

$$P^{C1}(\theta) \le \overline{ER^M}. \tag{13}$$

If the solutions to equations (10) and (11) do not violate equations (12) and (13), then there exists an equilibrium characterized by $y^*(\theta) = y^{C1}(\theta)$ and $P^*(\theta) = P^{C1}(\theta)$. As in the special case with $\theta = 0$ the level of $\overline{ER^M}$ plays a crucial role. When $\overline{ER^M} > 1$, equation (13) is always satisfied, otherwise it may be violated. In the latter case, there does not exist an equilibrium characterized by scarce aggregate liquidity with $y^* > (1-\lambda)$. Furthermore, notice that the corner solution with $y = 0$ is not consistent with the conjecture I used to solve the problem because a fully liquid portfolio allows deficit agents to fully finance their liquidity need in period 1.

[16] Notice that agents are indifferent on how much to invest in the risky long-term technology (indeterminacy on the individual level) and recall that we are interested in symmetric equilibria. Nevertheless it is easy to verify that the result holds more generally as, given that liquidity is scarce in the aggregate, an individual agent does not have an incentive to choose a portfolio that is sufficiently liquid such that $yP + \frac{\theta\bar{R}}{r} \ge y$.

For the remainder of this section I am interested in the characterization and properties of the described market equilibrium when equation (13) holds and, hence, trivial no trade equilibria as in Proposition 1 *(iv)* are ruled out. Lemma 3 demonstrates that the equilibrium investment level is decreasing in θ. Lemma 4 then establishes that there exists a unique threshold θ^{P1} which pins down when equation (12) holds. Finally, Proposition 5 draws from the results of Lemma 3 and 4. It shows that the equilibrium under consideration can only exist for $\theta < \theta^{P1}$ and gives a full characterization.

Lemma 3 (Change in the equilibrium investment level). *In the limit, the optimal aggregate level of investment (y) is decreasing when moving from an economy where liquidity risk is shared only in an asset market* ($\theta = 0$) *to an economy where markets are more complete, that is when* θ *is increasing. Formally,*

$$\left.\frac{\partial y^*}{\partial \theta}\right|_{lim_{\theta \to 0}} < 0.$$

Given that there exists a unique threshold $\theta^{P1} \epsilon \, (0, 1)$*, the result extends to:*

$$\frac{\partial y^*}{\partial \theta} < 0 \quad for\ all \quad 0 < \theta \leq \overline{\theta^{P1}}$$

if the following (sufficient) condition holds:

$$\bar{R} \leq \frac{2-\lambda}{1-\lambda} \overline{ER^M}, \tag{14}$$

where $\overline{\theta^{P1}}$ *is the highest possible* θ *solving equation* (12) *.*

Proof: See Appendix section A.6.

Notice that the condition in equation (14) is stronger than actually needed for the results of Lemma 3 to hold and was chosen for its analytical simplicity. In a numerical analysis over a broader range of parameter values I found no cases where the results of Lemma 3 and 4 did not hold.

Lemma 4 (Unique threshold). *Given that the condition in equation* (14) *holds, there exists a unique threshold* $\theta^{P1} \in (0, 1)$*. It is defined implicitly by:*

$$\theta = \frac{(1-\lambda)\,\overline{ER^M}\left(1 - P^{C1}(\theta)\right)}{\bar{R} P^{C1}(\theta)}.$$

Proof: See Appendix section A.7.

Using these results, I discuss existence and characterization of the equilibrium in Proposition 5.

Proposition 5 (Existence and characterization of a pooling equilibrium with scarce aggregate liquidity).

(a) *Given the condition in equation* (14)*, the two conditions expressed in equations* (8) *and* (13)*, when evaluated at* θ^{P1}*, are necessary and sufficient for the existence of a market equilibrium with pooling in the asset market for all* $\theta \in \left[0, \theta^{P1}\right)$*. The equilibrium is characterized by an aggregate level of liquidity that falls short of the total liquidity need, i.e.* $y^* > (1-\lambda)$*. Moreover, the equilibrium level of investment (y) is increasing in the tightness of the exogenous borrowing constraint (decreasing in* θ*). In the limit,* $\theta \to \theta^{P1}_{-}$*, the aggregate level of liquidity is just about sufficient to meet the total liquidity need, i.e.* $\lim\limits_{\theta \to \theta^{P1}_{-}} y^*(\theta) = (1-\lambda)$*.*

(b) *The type of pooling equilibrium described in (a) does not exist for any* $\theta \geq \theta^{P1}$.

Proof: See Appendix section A.8.

4.2.2 The pooling equilibrium characterized by sufficient aggregate liquidity, i.e. $y^* \leq (1-\lambda)$

Suppose that there exists a market equilibrium with pooling where aggregate liquidity is plentiful, i.e. $y^* < (1-\lambda)$, or just about sufficient, i.e. $y^* = (1-\lambda)$, to meet the total liquidity need. In section 3.1.4 I showed that such a market equilibrium where all deficit agents are able to fully finance their internal projects cannot exist in the special case with $\theta = 0$. Here, I demonstrate that this result is not longer true if $\theta > 0$. Intuitively, a market equilibrium where the aggregate level of cash holdings in the economy exceeds the total liquidity need of deficit agents implies that deficit agents do not have to exhaust all available financing opportunities to raise the cash needed. Consequently, the good type deficit agents do not have to sell all their high quality assets, as was previously the case. A direct implication of this is that the average quality of assets traded in the market deteriorates to a level strictly below $\overline{ER^M}$.

I consider the period 0 problem ($P3$) when $yP + \frac{\theta \bar{R}}{r} \geq y$ so that in equilibrium the aggregate level of liquidity is sufficient and, hence, deficit agents can fully refinance. Furthermore, I conjecture that $\frac{\theta \bar{R}}{r} < y$ must hold in the described equilibrium. Otherwise, good type deficit agents would not want to sell their assets, something which is inconsistent with the conjecture of pooling in the period 1 asset market.[17] Given these two conjectures, the first-order condition can be derived as (supposing an interior solution):

$$IV(P) \equiv \left\{ \begin{array}{c} \lambda \left[\alpha ER^M(y,\theta) + (1-\alpha) R^H\right] \left(\frac{P(y,\theta)-1}{P(y,\theta)}\right) \\ + (1-\lambda) \left[\alpha ER^M(y,\theta) + (1-\alpha) R^H - \frac{ER^M(y,\theta)}{P(y,\theta)}\right] \end{array} \right\} = 0. \tag{15}$$

Let P^{C2} and y^{C2} be the candidate equilibrium price and allocation. $P^{C2}\left(ER^M(y,\theta)\right)$ can be derived by solving equation (15) for the asset price:

$$P^{C2}\left(ER^M(y,\theta)\right) = \frac{ER^M(y,\theta) + \lambda (1-\alpha) \left(R^H - ER^M(y,\theta)\right)}{\alpha ER^M(y,\theta) + (1-\alpha) R^H}. \tag{16}$$

It is necessary to consider two cases in turn. First, I solve the problem with $y^* < (1-\lambda)$ to see for which values of θ an equilibrium of this type exists. Second, I solve the problem letting $y^* = (1-\lambda)$. The crucial difference between the two cases lies in the market clearing condition. Given the excessive availability of cash in the aggregate, the market price for cash has to be such that surplus agents are indifferent between supplying their liquidity to the market and storing it. Hence, the return on cash has to be $r = \frac{ER^M(y,\theta)}{P(y,\theta)} = 1$ in the first case with $y^* < (1-\lambda)$. In contrast, the second case with just about sufficient liquidity, i.e. $y^* = (1-\lambda)$, does not imply that $r = 1$ but allows for $r > 1$. The derivation of the results for both cases is relegated to Appendix section A.9 and a summary of the results is given in Proposition 6 below. While the pooling equilibria characterized by $y^* = (1-\lambda)$ always exists at least for some θ, the pooling equilibria with $y^* < (1-\lambda)$ requires that the adverse selection problem is weak, but not too weak (see equation (42)).

[17] The case when the latter condition is violated is discussed in section 4.2.3.

Proposition 6 (Existence and characterization of pooling equilibria with sufficient aggregate liquidity).

(a) *Given the necessary and sufficient condition expressed in equation* (42)*, there exists a market equilibrium with pooling characterized by* $y^* = (1-\lambda)$ *for all* $\theta \in \left[\theta^{P1}, \theta^{P2}\right]$*, where* $\theta^{P2} > \theta^{P1}$*.*

(b) *Given the necessary and sufficient condition expressed in equation* (42)*, there exists a market equilibrium with pooling characterized by* $y^* < (1-\lambda)$ *for all* $\theta \in \left(0, \theta^{P2}\right)$*. The aggregate investment level* (y) *is weakly increasing in* θ*.*

(c) *If equation* (42) *is violated but equation* (8)*, evaluated at* θ^{P1}*, is satisfied, there exists a market equilibrium with pooling characterized by* $y^* = (1-\lambda)$ *for a smaller range of* $\theta \in \left[\theta^{P1}, \theta'\right]$*, where* $\theta' < \theta^{P2}$*.*

Proof: See Appendix section A.9.

4.2.3 Failure of the lemons asset market

As in the special case with $\theta = 0$ we have that there exist equilibria where the lemon asset market fails. As long as, the access to the collateralized credit market is limited these equilibria are associated with a lower level of social welfare. But when the lemons asset market is redundant, then there exists an equilibrium with an inactive lemons asset market which reaches the first-best level of welfare. To see this consider the corresponding period 0 problem *(P3)* under the condition that $P = 0$:

$$\max_{y} \left\{ \begin{array}{c} \lambda\left[\alpha\left(\bar{R} - r * y\right) + (1-\alpha)\left(\bar{R} + yR^H - r * y\right)\right] \\ + (1-\lambda)\left[\alpha\left(\bar{R} + r * (1-y)\right) + (1-\alpha)\left(\bar{R} + r * (1-y) + yR^H\right)\right] \end{array} \right\}.$$

Supposing an interior solution, I can solve the first-order condition to find that $r^* = (1-\alpha) R^H > 1$. For consistency reasons an equilibrium with these prices has to be characterized by $y^* = (1-\lambda)$ and it can only exist for high values of θ, because otherwise deficit agents are not fully able to finance their liquidity need. In particular we need that $\theta \geq \theta^{S1}$, where:[18]

$$\theta^{S1} \equiv \frac{(1-\lambda)(1-\alpha) R^H}{\bar{R}}.$$

4.2.4 Summary and discussion

In the analysis of the model with the collateralized credit market it becomes clear that there are several discontinuities in θ, meaning that certain categories of equilibria do not exists over the full range of θs, but only for some θs.

This main result is illustrated in figure 4 which displays the pooling equilibria over the range of θs that yield the highest level of welfare, as well as the equilibrium with an inactive lemons asset market which achieves the first-best for all $\theta \geq \theta^{S1}$. Firstly, Proposition 5 (a) shows that for $\theta < \theta^{P1}$, there exists a pooling equilibrium characterized by a level of aggregate liquidity which insufficient to finance all liquidity demand at date $t = 1$ (the associated welfare level is depicted in red in figure 4).

[18] This can be proven by contradiction. Suppose that $\theta < \theta^{S1}$. Here deficit agents would only be able to fully refinance if the interest rate were lower because $\frac{\partial}{\partial r}\left(\frac{1-y}{r-\theta \bar{R}}\theta\right) > 0$. However, a lower interest rate would move us to a corner case and imply a higher y, leading to a contradiction.

Proposition 5 (b) shows that this type of equilibrium cannot exist for all $\theta \geq \theta^{P1}$. However, there always exists a pooling equilibrium characterized by aggregate liquidity being just about sufficient, i.e. $y^* = 1 - \lambda$, for some $\theta \geq \theta^{P1}$ (the associated welfare level is depicted in blue in figure 4). Furthermore, Proposition 6 provides conditions under which the pooling equilibrium characterized by $y^* = 1 - \lambda$ and another pooling equilibrium characterized by excessive aggregate liquidity, i.e. $y^* < 1 - \lambda$, can co-exist for any $\theta^{P1} \leq \theta \leq \theta^{P2}$.[19] Finally, the lemons asset market is redundant if $\theta \geq \theta^{S1}$. Here the first-best welfare level is reached, as shown in green in figure 4.

The question arises where this crucial discontinuity on the existence of pooling equilibria comes from. Intuitively, the adverse selection problem becomes worse if θ increases. This is because the average quality of traded assets, ER^M, is successively deteriorating up to the point where a pooling equilibrium cannot exist and so we end up with an equilibrium where the lemons asset market fails. In other words, for a pooling equilibrium to exist, good type deficit agents must be sufficiently dependent on financing through the sale of their high quality assets.

4.3 Social welfare and policy implications

4.3.1 Equilibrium welfare

Taken together, the results of Propositions 5 and 6 imply that within each class of equilibria, welfare is weakly increasing in θ. This is because, within each class of equilibria, there is a one-to-one mapping between the equilibrium level of investment and social welfare. Moreover, I showed in section 4.2 that for some θs there exists a market equilibrium that reaches the first-best level of welfare. However, this does not hold for all θs, laying the ground for potential policy intervention.

4.3.2 Constrained efficiency

In the discussion of the special case with $\theta = 0$, I demonstrated the important role played by $\overline{ER^M}$. If $\theta = 0$, then a constrained planner can obtain the first-best whenever $\overline{ER^M} \geq 1$. Otherwise we have a "true" second-best when $\overline{ER^M} < 1$. How does this change in the full fledged model where I allow for borrowing against the proceeds of the internal project?

1. If θ takes on a sufficiently low value, then the results derived in the special case with $\theta = 0$ go through. Namely there is a tendency for inefficient over-investment in risky long-term assets.

2. The constraint planner can now implement the first-best allocation for high enough values of θ even if $\overline{ER^M} < 1$. This is because she can use the additional market to share liquidity risk. However, it must be the case that θ is not too high, i.e. $\theta \leq \theta^{P3} < \theta^{S1}$, where θ^{P3} is derived below.

3. Better access to borrowing can make the adverse selection problem more severe. This is the indirect effect of a higher θ. As a consequence, the constrained planner cannot obtain the first-best for all values of θ even if $\overline{ER^M} \geq 1$. In particular, the constrained planner cannot implement the first-best if $\theta \in \left(\theta^{P3}, \theta^{S1}\right)$.

The second result is easy to see when looking at the financing possibilities of deficit agents. Deficit agents can fully refinance if $yP + \theta \bar{R} r^{-1} \geq y$. When the constrained planner chooses a level of aggregate liquidity that is sufficient to meet the total liquidity need of deficit agents (i.e. $y \leq (1 - \lambda)$), then

[19] The pooling equilibrium characterized by $y^* < 1 - \lambda$ is not depicted in figure 4. Extensive numerical analysis suggests that the associated welfare level is inferior to the welfare levels of other pooling equilibria.

the $t = 1$ asset price (which is increasing in the level of aggregate liquidity) reaches its upper bound $P = \overline{ER^M}$. Now suppose that $\overline{ER^M} = 1 - \varepsilon < 1$, where $\epsilon > 0$ is small. A deficit agents can raise at most $y(1-\varepsilon) < y$ units of cash by selling all her long-term assets. Consequently she can cover the shortfall of $y\varepsilon$ units as long as $\theta \geq y\varepsilon$. Hence, the negative indirect effect of a higher θ on the severity of the adverse selection problem does not play a role for small θ since good type deficit agents still rely on the lemons asset market for almost all of their financing. In sum, the constrained planner can implement the first-best for low values of θ even if $\overline{ER^M} < 1$. Nevertheless this conclusion does not necessarily hold for higher values of θ since the constraint faced by the social planner has more bite in the model with $\theta > 0$. This gives rise to the third result.

The third result is derived below by showing that the constrained planner cannot obtain the first-best for all values of θ even if $\overline{ER^M} \geq 1$. Again, the intuition for this result is that better access to the market for collateralized borrowing can make the adverse selection problem in the asset market more severe. Since the constrained planner relies on the functioning of the asset market, I find that she is not always able to mitigate the adverse selection problem. If the latter is true, then the constrained planner cannot implement a pooling equilibrium and is restricted to implement an equilibrium where only the collateralized credit market is used by agents.

The formal argument goes as follows. Suppose that the planner implements $y = 1 - \lambda$. From the market clearing condition we have that the interest rate $r \in \left[1, \bar{R}\right]$ is indeterminate. Furthermore the following financing constraint must bind for the good type deficit agents:

$$f_D^H (1-\lambda) P + \frac{\theta \bar{R}}{ER^M\left(f_D^H\right)} P = (1-\lambda). \tag{17}$$

From the proof of result (b) in Proposition 6 it can be seen that the highest possible value of θ that allows equation (17) to hold can be derived by setting $P = ER^M = \widehat{P^H}$. Isolating θ yields:

$$\theta = \frac{1-\lambda}{\bar{R}} \left(\frac{ER^M\left(f_D^H\right)}{P} - f_D^H ER^M\left(f_D^H\right) \right),$$

where:

$$ER^M\left(f_D^H\right) = \frac{\lambda(1-\alpha) R^H f_D^H}{\lambda\left[\alpha + (1-\alpha) f_D^H\right] + (1-\lambda)\alpha}.$$

Since $ER^M\left(f_D^H\right)$ is increasing in f_D^H, a conservative upper bound for the highest possible value of θ such that equation (17) holds can be found by setting $\widehat{P^H} = ER^M\left(f_D^H\right)$ and solving for $\underline{f_D^H}$ to get:

$$\underline{f_D^H} = \frac{\alpha \widehat{P^H}}{\lambda(1-\alpha)\left(R^H - \widehat{P^H}\right)}. \tag{18}$$

Finally, we have that the constrained planner cannot implement the first-best for all θ in the following interval:

$$\theta \in \left[\theta^{P3}, \theta^{S1}\right], \tag{19}$$

where

$$\theta^{P3} \equiv (1-\lambda)\left(1 - \underline{f_D^H}\widehat{P^H}\right)\left(\bar{R}\right)^{-1}.$$

Notice that $\theta^{S1} > \theta^{P3} \geq \theta^{P2}$.

4.4 Empirical predictions & policy implications

4.4.1 Prediction 1

When alternative sources of finance become more easily available, the functioning of an asset market prone to adverse selection problems may or may not improve:

If the adverse selection problem is relatively mild (strong), then asset prices increase (asset prices decrease or the asset market dries up completely) after alternative sources of finance become more easily available.

Leverage & re-financing operations: Changing the agents' abilities to leverage can increase equilibrium welfare.[20] When $\theta \in \left[0, \theta^{P1}\right)$ an increase in θ leads to an increase in asset prices. As a result, a central bank re-financing operation that makes finance more easily available by widening the collateral requirements (i.e. by increasing θ) can improve the market functioning and increase welfare. However, when the adverse selection problem is relatively strong (e.g. high α), then the functioning of the lemons asset market is more easily threatened by a high level of θ due to the financing asymmetry between good and bad type deficint agents. Here a central bank or financial regulator can increase equilibrium welfare by decreasing the agents' possibilities to leverage at date $t = 1$ (i.e. by decreasing θ below θ^{P2}) if the latter helps to restore market functioning. By reducing leverage, agents rely more on the asset market for financing. Thereby the adverse selection problem can be mitigated. This result is similar to Eisfeldt (2004), who shows that the liquidity of assets is crucially determined "by the amount of trade for reasons other than private information" (page 1).[21]

Take as an example a large-scale re-financing operation such as the ECB's Long-term Refinancing Operation (LTRO). It can have positive or negative implications for market functioning depending on the market characteristics. In market segements where adverse selection problems are mild the intervention may be beneficial and lift asset prices. Instead in market segements with severe adverse selection problems LTRO may be harmful for market functioning. If the latter problem persists and affects important market segements, then my model would suggest that the central bank should provide liquidity only at a sufficiently high penalty rate. In this way it can ensure that agents still find it attractive enough to use the lemons asset market for financing.

4.4.2 Prediction 2

Imposing minimum liquidity requirements supports prices and improves the market functioning as long as the adverse selection problem is relatively mild. This is because agents tend to under-invest in cash, which leads to depressed asset prices due to cash-in-the-market pricing.

[20]The result that tighter borrowing constraints can in some circumstances increase welfare is a not such novelty in the literature, nevertheless the underlying mechanics differ. See for instance Diamond and Rajan (2011) and Farhi and Tirole (2009) on regulation of bank's leverage or Gottardi and Kuebler (2011) who show under what conditions tightening of collateral constraints can improve welfare in a model with complete markets.

[21]In my model the ability to leverage by borrowing against the investment opportunity reduces the amount of trade by good type deficit agents. Consequently, market liquidity, when defined as "the cost of transferring the value of expected future payoffs from long-term assets into current income" (ibid.), deteriorates. To see this, suppose that parameters are such that there exists a unique equilibrium with a market failure and that the constrained planner cannot implement a market equilibrium with pooling, i.e. if $\theta \in \left[\theta^{P3}, \theta^{S1}\right]$. In this case a tightening of the exogenous borrowing constraint θ, which could be understood as a reduction of leveraging possibilities, can enable the constrained planner to implement a market equilibrium with pooling and $y^* = (1 - \lambda)$.

Minimum liquidity requirements: As in the special case of $\theta = 0$, the constrained social planner can, in general, improve upon the equilibrium level of welfare. For instance, for θ close to zero, the best market equilibrium is characterized by inefficient over-investment in risky long-term assets (in line with what we have seen in section 3.2). Here minimum liquity requirements can support market prices and increase equilibrium welfare up to its efficient level. Following the same steps as in section 3.2, it is possible to design a proportional tax on investments that achieves this goal.

5 Extension: a financial crisis

In this section we generate a financial crisis in two different ways. First, I consider an aggregate liquidity shock and, second, an aggregate shock to solvency. Both types of aggregate shocks can, if sufficiently large, cause a failure of markets to allocate the liquidity available in the economy. While the solvency shock has a direct effect on the severity of the adverse selection problem in the lemons asset market, the aggregate liquidity shock has a more indirect effect through cash-in-the-market pricing due to a scarcity of aggregate liquidity.

As far as policy makers are concerned about a scenario where the economy faces a "rare" shock to aggregate liquidity demand or to solvency, ex-ante liquidity regulation shows not to be suitable and the policies considered up to now are largely ineffective. Instead ex-post policy implications are of key interest. In particular, I give policy makers the options to intervene in markets at date $t = 1$ by buying or guaranteeing assets. The intervention can be financed by borrowing against the policy maker's future income (using existing market institutions). I demonstrate how a re-allocation of liquidity at date $t = 1$ from the prime market to the asset market under distress can overcome a market failure due to an adverse selection problem.

5.1 Introduction of aggregate shocks

In the first part of the paper agents faced idiosyncratic liquidity and solvency risk. Now aggregate liquidity and solvency risk is introduced by assuming that the population fraction of agents facing a liquidity shock and the population fraction of low quality assets is random and can take on one of two values:[22]

$$\widetilde{\lambda} = \begin{cases} \lambda_L & w.p.\ \gamma \\ \lambda_H & w.p.\ (1-\gamma) \end{cases} \qquad \text{and} \qquad \widetilde{\alpha} = \begin{cases} \alpha_L & w.p.\ \gamma \\ \alpha_H & w.p.\ (1-\gamma) \end{cases},$$

where $\lambda_H \geq \lambda_L > 0$ and $\alpha_H \geq \alpha_L > 0$.

To simplify the analysis, I assume that the probability of the crisis event is relatively low. For analytical simplicity suppose that $\gamma \to 1$, which implies that neither agents nor the central bank do have an incentive to take this "low probability event of a bad state of the world" into account in their ex-ante optimization problem. Accordingly, the $t = 0$ decision problem of agents is *approximately* the same as in section 4. This stylized scenario is chosen to allow for analytical simplicity, however the qualitative results hold more generally.

[22] I do not attempt to model deeper causes that can trigger an aggregate liquidity shock, but instead I take a reduced-form approach similar to models with aggregate shocks to consumer liquidity demand. One of the possible stories behind the aggregate liquidity shock could be that banks encountered a strong liquidity demand from the investment side at the outbreak of the crisis as they had to provide cash to their structured off-balance sheet vehicles and take them on the bank's balance sheet (compare Brunnermeier (2009), p. 95).

5.2 Market failure

In this section I show that a sufficiently strong shock to aggregate liquidity or to solvency can cause a market failure. As shown in section 4.3, the central bank can implement the constrained efficient allocation for a large range of parameter values by taxing (subsidizing) long-term asset purchases (reserve holdings) in period 0. For simplicity consider a scenario where the central bank can implement the efficient allocation for the aggregate state with low liquidity demand (state L) by inducing agents to optimally choose $y^* = (1 - \lambda_L)$.

The interesting question is under what conditions on the size of the aggregate shocks there is a market failure in state H. Intuitively, we need that the two types of aggregate shocks need to be sufficiently large. This is illustrated in a two numerical examples. Take as an example the parametrization used in figure 4 of section 4 and consider a value of θ that ensures an efficient market equilibrium in state L: $\left\{\theta, \alpha_L, \lambda_L, R^L, R^H, \bar{R}\right\} = \{0.15, 0.15, 0.25, 0, 1.5, 2.3\}$.

Numerical example 1: aggregate liquidity shock in isolation Suppose that $\lambda_H = 0.41$ and $\alpha_H = \alpha_L$. Since $\gamma \to 1$, the agent (as well as the policy maker) only care about state L. As shown in section 4, agents choose the first-best level of investment, $y^* = 0.75$, which ensures that all investment opportunities are taken in state L. However, if state H is realized, there is a shortage of aggregate liquidity of $\lambda_H - \lambda_L = 0.16$ units and we face an equilibrium with adverse selection where the lemons asset market fails. In particular we have that in equilibrium 0.0865 out of 0.25 units of aggregate liquidity remain unused, resulting in a welfare loss. Notice that this welfare loss does not occur if the aggregate shock is only slightly smaller, i.e. for $\lambda_H = 0.40$.

Numerical example 2: aggregate solvency shock in isolation Now suppose that $a_H = 0.25 > \alpha_L = 0.15$ and $\lambda_H = \lambda_L$. Again it can be shown that agents choose the first-best level of investment $y^* = 0.75$. However, if state H is realized the solvency shock causes a failure of the lemons asset market and we have that in equilibrium 0.1546 out of 0.25 units of aggregate liquidity remain unused.

5.3 Policy intervention

If the market outcome in state H is a pooling equilibrium (e.g. if $\lambda_H = 0.40$ in the first numerical example), then there is no way for a central bank to improve on the market equilibrium if it cannot create cash instantly at no cost. This is because in a pooling equilibrium, market institutions do not fail to allocate all available liquidity to deficit agents. Adverse selection instead can cause markets to fail in allocating the available liquidity, as argued above (e.g. if $\lambda_H = 0.41$). Next, I discuss ex-post policy options for a central bank facing a market failure due to an aggregate shock to liquidity demand. The main focus is on an asset purchase program.

5.3.1 Design of the asset purchase program

During the financial crisis of 2007 and 2008 the main intervention of the U.S. Federal Reserve consisted of a change of its balance sheet composition (see Cecchetti (2009)). By providing liquidity to the markets for commercial paper and collateralized mortgage loans (using the Term Auction Facility) and, at the same time, raising liquidity by selling safe government securities (using the Term Securities Lending Facility), the FED increased the price for borrowing against government bonds and decreased the price for borrowing against asset-backed securities. I consider a similar type of intervention in my modeling framework. Here, the central bank can raise cash in the collateralized credit market and provide this money to the lemons asset market by buying risky long-term assets.

5.3.2 Asset purchase program after an aggregate liquidity shock

Notice that an asset purchase program can only be effective if the intervention of the central bank is not "netted out" by counteracting trades of agents (arbitrage trades). This requires a "large intervention" that **completely crowds-out the private liquidity supply to the lemons asset market**, as it is necessary to induce deficit agents with high quality assets to supply them to the market. To see how this could work, consider the financing choice of deficit agents with high quality long-term assets:

$$f_D^H = 0 \quad \text{and} \quad b_D^H = \begin{cases} \in \left[0, \frac{1-y}{r-\theta\bar{R}}\theta\bar{R}\right] & if\ r = \bar{R} \\ \frac{1-y}{r-\theta\bar{R}}\theta\bar{R} & if\ r \in \left(1, \bar{R}\right) \end{cases}$$

and compare it to the financing choice of "bad type" deficit agents in the financial crisis:

$$f_D^L = 1 \quad \text{and} \quad b_I^L = \begin{cases} \in \left[0, \frac{1-y+yP}{r-\theta\bar{R}}\theta\bar{R}\right] & if\ r = \bar{R} \\ \frac{1-y+yP}{r-\theta\bar{R}}\theta\bar{R} & if\ r \in \left(1, \bar{R}\right). \end{cases}$$

In an environment where the scarcity of aggregate liquidity (high λ_H) is the key driver of depressed asset prices, a successful intervention has to *(a)* provide cash to the distressed lemons asset market and *(b)* make the good type deficit agents more dependent on the lemons asset market. Following this intuition I claim that a central bank may be able to implement a pooling equilibrium (which is superior in terms of welfare) by soaking up all available liquidity and providing it to the lemons asset market. In particular, it is necessary to "break" the no-arbitrage condition that ties together the expected returns in the two markets. Otherwise, any intervention is completely counteracted by agents. This demands that the "private" liquidity supply to the lemons asset market has to be driven-out completely and replaced by "public" liquidity supply of the central bank. Moreover, the central bank's liquidity demand has to drive up the interest rate in collateralized credit market to $r = \bar{R}$ such that the ability of good type deficit agents to borrow is reduced as b_D^H is decreasing in r. The lemons asset market in turn becomes more attractive if the central bank supports the price, allowing for $f_D^H > 0$.

Result (Effective central bank intervention after an aggregate liquidity shock): *Given the parameters of the model are such that there only exists an equilibrium with adverse selection where the market fails in the state of high aggregate liquidity demand, a central bank can successfully intervene by borrowing all liquidity in the economy at the interest rate of $r^* = \bar{R}$ and using it to purchase long-term assets if:*

$$\lambda_H \leq \frac{\lambda_L}{\lambda_L + (1-\lambda_L)\,\widetilde{P^H}\left(\theta, \overline{ER^M}\right)}. \tag{20}$$

The central bank incurs a loss of:

$$\overline{ER^M}\left(\frac{(1-\lambda_H)\,\lambda_L}{\lambda_H\,(1-\lambda_L)}\right)^{-1} - \bar{R} < 0 \tag{21}$$

per unit of cash provided to the lemons asset market, which needs to be financed by a tax in period 2.

To see this, notice that the return on purchasing assets is by definition smaller than $\bar{R}$ in any pooling equilibrium. Hence, surplus agents prefer to provide their cash-in-hand to the central bank at the interest rate $\bar{R}$. Furthermore, we have from market clearing that the equilibrium interest rate is indeed $r^* = \bar{R}$, as long as the central bank is demanding more cash than available in the economy. Given this equilibrium interest rate both, the good and the bad type deficit agents, strictly prefer to raise financing by selling their assets as long as $P^* = \frac{(1-\lambda_H)\lambda_L}{\lambda_H(1-\lambda_L)} > \widetilde{P^H}\left(\theta, \overline{ER^M}\right)$, which leads to the condition given in equation (20).

5.3.3 Alternative policy intervention

An alternative policy intervention could be to provide a guarantee scheme. In particular, a policy maker could guarantee a minimum payoff in period $t = 2$ on all assets that are purchased by agents at date $t = 1$. It can be shown that this type of intervention can restore a pooling equilibrium in a way analogous to the asset purchase program. It is crucial that the guaranteed payoff on long-term assets purchased in the market has to be set sufficiently high. Let π be the payoff guaranteed by the governmental scheme, then the policy maker has to ensure that $(\pi/P) \geq r$.

Compared to the asset purchase program, the guarantee scheme requires more information on the side of the policy maker. While under the asset purchase program the policy maker just needs to act as a buyer of all available assets and a borrower in the credit market, the policy maker needs a lot of information to compute the critical price threshold $\widetilde{P^H}\left(\theta, \overline{ER^M}\right)$ in order to correctly set the guarantee scheme π.

5.3.4 Numerical example

Returning to the numerical examples from section 5.2, the asset purchase program described above shows to be an effective tool when facing both, aggregate liquidity shocks and solvency shocks.[23]

Aggregate liquidity shock: The policy maker can improve welfare in state H from 2.79 to 2.90 units. The equilibrium asset price after intervention is given by $P^* = \widetilde{P^H}\left(\theta, \overline{ER^M}\right) = 0.48$.

Aggregate solvency shock: The policy maker can improve welfare in state H from 2.94 to 3.14 units. The equilibrium asset price after intervention is given by $P^* = 0.8$.

5.3.5 Discussion

As argued earlier, the "large" intervention that completely crowds-out the private liquidity supply resembles the initial crisis response of the *FED*, which consisted of a change of the central bank's portfolio decomposition: purchasing assets under distress and financing the intervention by selling government bonds. The central bank provides an "attractive asset to the private sector" (Borio (2010), page 86) in raising cash and reducing financing constraints of market participants by "lending at more attractive terms than the market" (Borio and Disyatat (2010), page 25). Interestingly, my model delivers the result that the central bank has to be prepared to incur losses in order to ensure the effectiveness of the intervention.

I focus in this paper on the role of the adverse selection problem at the presence of aggregate liquidity and solvency shocks. By doing that I abstract from other mechanisms that may have been

[23]Notably solvency shocks cannot be too large, otherwise it is not possible to restore pooling. The same is true for aggregate liquidity shocks (see equation (20)).

at work during the crisis such as moral hazard problems, etc.. Obviously these other mechanisms may alter the policy implications derived in this section. Nevertheless, I believe that the idea of a financial crisis as a rare event and the fact that the ex-post policy interventions at the beginning of the financial crisis in 2007 were not aimed at providing additional liquidity to the market support my approach of putting the adverse selection problem in center stage and abstracting from a moral hazard problem.

6 Conclusion

This paper develops a model of liquidity provision where liquidity risk can be shared in two distinct spot markets: an asset market which is prone to an informational friction and a collateralized credit market. Interestingly, an increased ability to borrow can make the adverse selection problem more severe due to a financing asymmetry between good and bad type deficit agents. I demonstrate that there is a non-monotonic relationship between market completeness and equilibrium welfare. Moreover, I offer a detailed social welfare analysis and derive policy implications.

In my model, market equilibria are often constrained inefficient. Furthermore, the degree to which borrowing is available crucially influences the efficiency of equilibria. Hence, there is a clear role for the central bank to regulate ex-ante cash holdings. In addition, it is evident that the asset market cannot function well if deficit agents do not depend on it to a sufficient degree because they have alternative financing possibilities, namely collateralized credit. For this reason, a regulatory restriction on leverage, i.e. on the degree to which agents can borrow against the investment opportunity, can be beneficial.

Finally, I demonstrate how a liquidity crisis or a solvency crisis can lead to a market failure. I examine a central bank intervention motivated by the initial crisis response of the Federal Reserve in 2007 and 2008. The findings are that a central bank can, under certain conditions, successfully intervene by purchasing risky long-term assets and thereby re-allocate the liquidity available in the economy between the two spot markets. But the effectiveness of the intervention depends crucially on the willingness of the central bank to incur losses, and its ability to cover these losses by ex-post taxation.

A Appendix

A.1 Market clearing at date $t = 1$

This section formally states the results on existence and then offers a discussion.

Results on the existence and characterization of date $t = 1$ market equilibria: *For a given y, there exists a date $t = 1$ equilibrium with (partial) pooling if and only if:*

$$P^*(y) = min\left\{ER^M\left(y, P^{pool}(y)\right), P^{pool}(y)\right\} \geq \widehat{PH}. \tag{22}$$

It is characterized by the equilibrium asset price $P^*(y)$, where $P^{pool}(y)$ solves:

$$P = \frac{(1-\lambda)\left[\alpha * c_S^L\left(y, P, ER^M\right) + (1-\alpha) * c_S^H\left(y, P, ER^M\right)\right] + \lambda y P * c_D^L\left(y, P, ER^M\right)}{\lambda y\left[\alpha * f_D^L(P) + (1-\alpha) * f_D^H(P)\right] + (1-\lambda)\alpha y * f_S^L(P)} \tag{23}$$

and

$$ER^M\left(y, P^{pool}(y)\right) = \begin{cases} \frac{\lambda(1-\alpha)R^H * f_D^H}{\lambda\left[\alpha * f_D^L + (1-\alpha) * f_D^H\right] + (1-\lambda)\alpha * f_S^L} & \text{if } P^{pool}(y) > \widehat{P^H} \\ \\ \frac{(1-\lambda)\bar{R}(1-y) - \alpha y \lambda \left(R^H\right)^2}{(1-\lambda)\left(\bar{R}(1-y) + \alpha y R^H\right)} & \text{if } P^{pool}(y) = \widehat{P^H} \end{cases} \tag{24}$$

For a given y, there always exists a date $t = 1$ equilibrium with a market failure. It is characterized by an asset price $P^{**}(y) = 0$.

Supply & demand for cash: First, consider demand and supply for cash given y (which is determined in period 0). Demand for cash can be written as $\lambda y P \left[\alpha * f_D^L + (1-\alpha) * f_D^H\right] + (1-\lambda)\,\alpha y P * f_S^L$. The first summand is the value of liquidated assets by deficit agents, whereas the second summand is the value of assets sold by surplus agents who are in the possession of a lemon and raise additional cash by trading on their private information. Meanwhile, the supply of cash is given by $(1-\lambda)\left[\alpha * c_S^L + (1-\alpha) * c_S^H\right] + \lambda y P * c_D^L$. By equalizing supply and demand, one can derive the market clearing price under pooling, i.e. equations (23).

The average quality of assets traded in the market: The total supply of and demand for cash increases by the same amount when there is trading on private information, i.e. if the bad type surplus agents are selling their lemons. For this reason the trading on private information does not directly influence the liquidity available in the market and, hence, the market price. However, trading on private information does affect the average quality of assets traded in the market. As a result, there is an indirect effect on market liquidity as the average quality of traded assets constitutes and upper bound for the asset price. To see this more clearly, examine the expected return on assets traded in the market, i.e. equation (24). The average return of assets traded in the market is computed by dividing the aggregate return of all traded assets by the proportions of assets sold by the different types, weighted by the respective population fractions.

For asset prices larger than one the good type deficit agents only partially liquidate their long-term assets to meet their liquidity need. This is because they face discounted asset prices. In particular, the good type deficit agents need to sell less long-term assets, the higher the liquidation price. Consequently, there exists a highest possible market price at which surplus agents are just willing to purchase assets. It can be computed by solving $ER^M(y, P)\,|_{P>1} = P$ for the asset price. Define the solution as $\bar{P}$ (the upper bound on the market price for assets), then the asset price range takes on the form $\left(1, \bar{P}\right]$.

In contrast, when considering a lower range of asset prices in the interval $\left(\widehat{P^H}, 1\right]$ we have that the good type deficit agents need to sell all their assets in order to raise cash for their investment project. At the critical threshold $\widehat{P^H}$ we have partial pooling since the good type deficit agents are indifferent between selling or not. In equilibrium, just as many good types are selling to make markets clear. By evaluating the market clearing condition at $\widehat{P^H}$, one can derive the average quality of assets traded in the market. It can be shown that the ER^M is increasing in the available liquidity (decreasing in y).

Existence of date $t = 1$ equilibria: From equation (23) on can see that the candidate equilibrium asset price under (partial) pooling is decreasing in y. Consequently, an equilibrium with (partial) pooling only exists if equation (22) holds. ER^M serves as an upper bound on the pooling asset price. Thus, there cannot exist an equilibrium with pooling if the "trading on private information" problem is too severe. Particularly if **Condition 1** is violated.

Finally, the equilibrium with a market failure *always* exists because the good type deficit agents do not sell their assets if they correctly perceive a market price which is lower than the critical threshold $\widehat{P^H}$. This implies that even if the available liquidity is very high, e.g. for $y \to 0$, there exists an equilibrium with $\lim_{y\to 0} P^{**}(y) = 0$. Here the market fails to allocate any of the economy-wide available liquidity to deficit agents.

A.2 Proof of Proposition 1

The results on the existence and characterization of equilibria in Proposition 1 can be derived by analyzing the date $t = 0$ problem over the relevant price range. As argued earlier, it is useful to divide the range in four intervals. Let's label the four price intervals with Roman letters from ***I*** to ***IV***, starting from the highest. The proof consists of a detailed analysis of the respective price intervals in turn.

(I) Suppose there exists a market equilibrium with partial pooling and $P \in \left(1, \bar{P}\right]$. According to the mapping between P and y derived from market clearing, this corresponds to an aggregate investment level in the interval $y \in [0, 1-\lambda)$. The first-order condition of *(P1)* under this conjecture is:

$$\left\{ \begin{array}{c} \lambda\left[\alpha\left(\frac{ER^M}{P}(P-1)\right) + (1-\alpha)\left(R^H\left(1-\frac{1}{P}\right)\right)\right] \\ +(1-\lambda)\left[\alpha\left(\frac{ER^M}{P}(P-1)\right) + (1-\alpha)\left(-\frac{ER^M}{P} + R^H\right)\right] \end{array} \right\} + \mu_1 - \mu_2 = 0,$$

where μ_1 and μ_2 are the multipliers on the first and second inequality constraint, i.e. $0 \leq y$ and $y \leq 1-\lambda$ respectively. Since the term in curly brackets is larger than zero, it must be true that $\mu_2 > 0$ and, hence, the solution is in the corner $y = 1-\lambda$. However, this cannot be a market equilibrium because it is inconsistent with the solution for the range $P \in \left(\widehat{P^H}, 1\right]$, as will become clear below.

(II) Suppose there exists a market equilibrium with pooling and $P \in \left(\widehat{P^H}, 1\right]$. This corresponds to the interval $(1-\lambda) \leq y < (1-\lambda)\left(\lambda\widehat{P^H} + (1-\lambda)\right)^{-1} \equiv \widehat{y^{H1}}$. The first-order condition is:

$$\left\{\lambda\bar{R}(P-1) + (1-\lambda)\left[\alpha ER^M - \frac{ER^M}{P} + (1-\alpha)R^H\right]\right\} + \mu_1 - \mu_2 = 0. \tag{25}$$

where μ_1 and μ_2 are the multipliers on the first and second inequality constraint, i.e. $1-\lambda \leq y$ and $y \leq \widehat{y^{H1}}$ respectively. The first summand is the benefit of deficit agents from having a more liquid portfolio in period 1, whereas the second summand is the cost for surplus agents from having a more liquid portfolio in period 1. For $P = 1$ the first summand in the curly bracket is zero and the second summand is strictly positive. Hence there does not exist a market equilibrium characterized by $y = (1-\lambda)$. Furthermore, notice that both terms are decreasing in P since the average quality of traded assets is constant in the given price interval, i.e. $ER^M = \overline{ER^M} > \widehat{P^H}$. I distinguish between two cases, $\overline{ER^M} \geq 1$ and $1 > \overline{ER^M} > \widehat{P^H}$, and look at each in turn.

(IIa) If $\overline{ER^M} \geq 1$, then the upper bound on the asset price does not play a role in the price interval under consideration. As equation (25) is increasing in P and $ER^M = \overline{ER^M}$ in the given price interval, there exists an interior solution if:

$$I(P)\Big|_{P\to\widehat{P^H}} = \left\{\lambda\bar{R}(P-1) + (1-\lambda)\left[\alpha\overline{ER^M} - \frac{\overline{ER^M}}{P} + (1-\alpha)R^H\right]\right\}\Big|_{P\to\widehat{P^H}} \leq 0. \tag{26}$$

The solution is characterized by an asset price:

$$P^* = \frac{-b_1+\sqrt{b_1^2-4a_1c_1}}{2a_1} < 1, \text{ where } \begin{array}{l} a_1 \equiv \lambda\bar{R} \\ b_1 \equiv \lambda\bar{R} - (1-\lambda)\left[\alpha\overline{ER^M} + (1-\alpha)R^H\right] \\ c_1 \equiv (1-\lambda)\overline{ER^M} \end{array}$$

and an aggregate investment level y^*, which can be derived by solving equation (23) for y. For the relevant price interval the expression can be simplified to:

$$y^* = \frac{(1-\lambda)}{\lambda P^* + (1-\lambda)}. \tag{27}$$

Due to risk-neutrality, the investment level is in equilibrium indeterminate on the individual level. Since I am interested in symmetric equilibria, I assume that all agents choose the same initial portfolio y^*. *In summary, given* $\overline{ER^M} > 1$, *the pooling equilibrium described in part (i) of Proposition 1 exists if equation* (2) *is satisfied. Furthermore,* $y^* > (1-\lambda)$ *and* $P^* \in \left(\widehat{P^H}, 1\right]$.

(IIb) If $1 > \overline{ER^M} > \widehat{P^H}$, then the upper bound on the asset price plays a crucial role. First, we have that the two cases $\left(\widehat{P^H}, 1\right]$ and $\left(1, \bar{P}\right]$ merge and hence the interval under consideration expands to $y \in \left[0, \widehat{y^{H1}}\right)$.

Second, I find that the upper bound is binding if:

$$I(P)\Big|_{P=\overline{ER^M}} = \left\{\lambda\bar{R}\left(\overline{ER^M}-1\right) + (1-\lambda)\left[\alpha\overline{ER^M} - 1 + (1-\alpha)R^H\right]\right\}\Big|_{P=\overline{ER^M}} \leq 0. \tag{28}$$

In case equation (28) holds with strict inequality, the resulting equilibrium is given by $y^* = 0$. All prices in the interval $\left(\widehat{P^H}, \overline{ER^M}\right]$ are consistent with this equilibrium investment level, although there is no trade in equilibrium because agents decide optimally to hold a fully liquid portfolio. Nevertheless, the asset prices that are consistent with the described equilibrium are such that both, good type deficit agents and bad type deficit agents would be willing to sell their assets. If expression (28) holds with equality, the resulting equilibrium is characterized by an investment level in the interval:

$$y^* \in \left[0, (1-\lambda)\left(\lambda\overline{ER^M} + (1-\lambda)\right)^{-1}\right].$$

Otherwise, if condition (28) is violated, there exists an equilibrium as described in ***(IIa)*** if in addition condition (26) is satisfied, which assures an interior solution. *In summary, given* $1 > \overline{ER^M} > \widehat{P^H}$, *the pooling equilibrium described in part (i) of Proposition 1 exists if, first, condition* (26) *holds and, second, equation* (3) *is satisfied (or equivalently equation* (28) *is violated). If, instead, condition* (26) *holds and condition* (3) *is violated, there exists an equilibrium with no trade characterized by* $y^* = 0$ *and* $P^* \in \left(\widehat{P^H}, \overline{ER^M}\right]$. *This equilibrium is described in part (iv) of Proposition 1.*

(III) Suppose that there exists a market equilibrium with $P = \widehat{P^H}$. Moreover, suppose that the solution is interior, then the first-order condition reads:

$$\left\{\lambda\bar{R}\left(\widehat{P^H}-1\right) + (1-\lambda)\left[\alpha ER^M - \frac{ER^M}{\widehat{P^H}} + (1-\alpha)R^H\right]\right\} = 0, \tag{29}$$

where

$$ER^M(y) = \frac{R^H(1-\lambda-(1+\alpha-\lambda)y)}{(1-\lambda)(1-y)} = \begin{cases} \overline{ER^M} & \text{if } y = \underline{y} \equiv \frac{(1-\lambda)\bar{R}}{(1-\lambda)\bar{R}+R^H\lambda} \\ \widehat{P^H} & \text{if } y = \overline{y} \equiv \frac{\bar{R}}{(1-\lambda)\bar{R}+R^H\lambda\alpha-\alpha R^H\left(\bar{R}-1\right)^{-1}} > \underline{y}. \end{cases}$$

Consequently, the term in curly brackets of equation (29) has to be evaluated over the interval $y \in \left[\underline{y}, \overline{y}\right]$. Since $ER^M(y)$ is decreasing in y (as I showed in section 3.1.2) there exists at most one value of $y \in \left[\underline{y}, \overline{y}\right]$ that solves equation (29). If there exists such a y, then there also exists a market equilibrium with partial pooling characterized by $P^* = \widehat{P^H}$. However, such an equilibrium can only exit if equation (26) is satisfied: the reason is that it gives a necessary condition for existence of both, the equilibria described in paragraphs ***(II)*** and ***(III)***.[24] *In summary, condition* (3) *is a necessary condition for the existence of the equilibrium with partial pooling as described in part (ii) of Proposition 1. If it exists, then it is characterized by a* $y^* \in \left[\underline{y}, \overline{y}\right]$.

(IV) Notice that there cannot exist a market equilibrium with $P \in \left(0, \widehat{P^H}\right)$ because good type deficit agents do not sell their assets if they face a price below the critical threshold. Hence, there can only exist the equilibrium where the market fails characterized by $y^* \in [0,1]$ and $P^* = 0$. As argued earlier, this type of equilibrium always exits. *This equilibrium is described in part (iii) of Proposition 1. (q.e.d.)*

A.3 Constrained efficiency

I first describe formally the constrained planner problem in section A.3.1 before presenting the proof of Proposition 2 in section A.3.2.

A.3.1 Constrained planner problem

The constrained planner faces the same problem as *(P1)* with the only difference being that the planner does not take the asset price P and the average quality of assets traded in the market ER^M as given. In other words, the planner internalizes the effect of the portfolio choice on the period 1 market price and how the severity of the adverse selection problem is affected. Since the constrained planner problem is complex, I first argue that it can be simplified by focusing on the relevant price range. Intuitively, the constrained planner has the objective of allowing deficit agents to meet as much as possible of their liquidity need. Hence, the planner attempts to implement a pooling equilibrium, which is feasible as long as $\overline{ER^M} > \widehat{P^H}$. Therefore, I can give a simplified representation of the constrained planner problem:

[24] Notice that the type of equilibrium described in this paragraph only exists in the special case (for $\theta = 0$).

$$
\begin{aligned}
(P2)\quad & \max_{y}\left\{\begin{array}{c} \lambda\left[\alpha D^{L}\left(y,P,ER^{M}\right)+(1-\alpha)\,D^{H}\left(y,P,ER^{M}\right)\right] \\ +(1-\lambda)\left[\alpha S^{L}\left(y,P,ER^{M}\right)+(1-\alpha)\,S^{H}\left(y,P,ER^{M}\right)\right] \end{array}\right\} \\
s.t.\quad & 0\leq y\leq(1-\lambda)\left(\lambda\widehat{P^{H}}+(1-\lambda)\right)^{-1} \\
& P=\min\left\{ER^{M}\left(y,P^{pool}(y)\right),P^{pool}(y)\right\} \\
& P^{pool}=\frac{(1-\lambda)\left[\alpha*c_{S}^{L}\left(y,P,ER^{M}\right)+(1-\alpha)*c_{S}^{H}\left(y,P,ER^{M}\right)\right]+\lambda yP*c_{D}^{L}\left(y,P,ER^{M}\right)}{\lambda y\left[\alpha*f_{D}^{L}(P)+(1-\alpha)*f_{D}^{H}(P)\right]+(1-\lambda)\alpha y*f_{S}^{L}(P)} \\
& ER^{M}=\frac{\lambda(1-\alpha)R^{H}*f_{D}^{H}(P)}{\lambda\left[\alpha*f_{D}^{L}(P)+(1-\alpha)*f_{D}^{H}(P)\right]+(1-\lambda)\alpha*f_{S}^{L}(P)},
\end{aligned}
$$

where the expected return of both types of deficit agents $D^{k}\left(y,P,ER^{M}\right)$ and surplus agents $S^{k}\left(y,P,ER^{M}\right)$ is defined as before in $(P1)$. The market clearing pooling price P^{pool} is given by the ratio of supply and demand, while ER^{M} is determined by the relative fraction of high quality assets sold in the lemons market. Both are developed in Appendix section A.1.

A.3.2 Proof of Proposition 2

The proof proceeds as follows. In step 1 the constrained planner problem *(P2)* is solved under the conjecture of pooling in the date $t=1$ asset market. Step 1(a) deals with the case when $\overline{ER^{M}}\geq 1$ and step 1(b) to the case when $1>\overline{ER^{M}}>\widehat{P^{H}}$. Then, in step 2, I compare the welfare of the equilibria described in Proposition 1 to the welfare attained by the constrained planner.

Step 1(a): If $\overline{ER^{M}}\geq 1$, then the constrained planner can obtain the first-best level of welfare by choosing $y^{*}_{planner}=(1-\lambda)$. To see this, notice that $P=\min\left\{\overline{ER^{M}},P^{pooling}\left(y=(1-\lambda)\right)\right\}=1$, which allows all deficit agents to fully refinance their internal projects.

Step 1(b): If $1>\overline{ER^{M}}>\widehat{P^{H}}$, then the constrained planner cannot obtain the first-best level of welfare by choosing $y=(1-\lambda)$ because market institutions fail to allocate the available liquidity to deficit agents. Consequently, the constrained planner finds a higher or lower investment level optimal. Formally, the constrained planner problem needs to be solved under pooling by conjecturing a period 1 asset price in the range $\left(\widehat{P^{H}},\overline{ER^{M}}\right]$. Let μ_1 and μ_2 be the Lagrange multipliers on the inequality constraints $\widehat{y^{ER}}\leq y$ and $y\leq\widehat{y^{H1}}$, respectively. Where $\widehat{y^{H1}}\equiv(1-\lambda)\left(\lambda\widehat{P^{H}}+(1-\lambda)\right)^{-1}$ and $\widehat{y^{ER}}=(1-\lambda)\left(\lambda\overline{ER^{M}}+(1-\lambda)\right)^{-1}$ follow from market clearing. Then the first-order condition to the constrained planner problem is:

$$
\left\{\begin{array}{c} \lambda\left[\bar{R}*\left(-1+P\left(y\right)+y\frac{\partial P(y)}{\partial y}\right)\right] \\ +(1-\lambda)\left[\begin{array}{c} \alpha\left(-\frac{\overline{ER^{M}}}{P(y)}-\frac{\overline{ER^{M}}}{(P(y))^{2}}\frac{\partial P(y)}{\partial y}*(1-y)+\overline{ER^{M}}\right) \\ +(1-\alpha)\left(-\frac{\overline{ER^{M}}}{P(y)}-\frac{\overline{ER^{M}}}{(P(y))^{2}}\frac{\partial P(y)}{\partial y}*(1-y)+R^{H}\right) \end{array}\right] \end{array}\right\}+\mu_1-\mu_2=0. \tag{30}
$$

The term in curly brackets is always negative and, hence, $\mu_1>0$. To see this recall that $P=\frac{(1-\lambda)(1-y)}{\lambda y}<1$ and $\frac{\partial P(y)}{\partial y}=-\frac{(1-\lambda)}{\lambda y^{2}}$, which enables me to show that:

$$
-\lambda\bar{R}\left[y\frac{\partial P\left(y\right)}{\partial y}\right]=-\bar{R}\frac{1-\lambda}{y}<(1-\lambda)\left[\frac{\overline{ER^{M}}}{\left(P\left(y\right)\right)^{2}}\frac{\partial P\left(y\right)}{\partial y}*(1-y)\right]=-\frac{\overline{ER^{M}}}{P\left(y\right)}\frac{1-\lambda}{y},
$$

since $P\left(y\right)\geq\widehat{P^{H}}=R^{H}/\bar{R}$. Moreover, when evaluating the expression in curly brackets at the highest permissible price $P = \overline{ER^M}$, it takes on its highest value at $y = \widehat{y^{ER}}$ but is still negative:

$$\left\{\lambda\left(\overline{ER^M} - \bar{R}\right) + (1-\lambda)\left[\alpha\left(\overline{ER^M} - \bar{R}\right) + (1-\alpha)\left(R^H - \bar{R}\right)\right]\right\} < 0.$$

Next, evaluate equation (30) at values of y in the interval $0 \leq y < \widehat{y^{ER}}$, with μ_1 and μ_2 being the Lagrange multipliers on the inequality constraints $0 \leq y$ and $y < \widehat{y^{ER}}$, respectively. Here I have that $P(y) = \overline{ER^M}$ and $\frac{\partial P(y)}{\partial y} = 0$. Consequently, the sign of the term in curly brackets is ambiguous.

Recall equation (28), given which there exists an equilibrium with no trade and $y^* = 0$:

$$I(P)\Big|_{P=\overline{ER^M}} = \left\{\lambda\bar{R}\left(\overline{ER^M} - 1\right) + (1-\lambda)\left[\alpha\overline{ER^M} - 1 + (1-\alpha)R^H\right]\right\}\Big|_{P=\overline{ER^M}} \leq 0.$$

If equation (28) holds with strict inequality (which requires λ to be relatively high and $\overline{ER^M}$ to be relatively low), then this implies that $\mu_1 > 0$ which in turn implies that $y^*_{planner} = 0$. If, instead, equation (28) is violated, then I find that $y^*_{planner} = \widehat{y^{ER}}$. *This proves the first set of results in Proposition 2.*

Step 2: From Proposition 1 and the pricing kernel under pooling (as stated in the results of Appendix section A.1), the following statement can be confirmed. If $I\left(\overline{ER^M}\right) > 0$, then it has to be true in every pooling equilibrium that $y^* > (1-\lambda) = y^*_{planner}$. This implies that there is inefficient over-investment in equilibrium, as there is, a direct mapping between the allocation and social welfare. On the other hand, if $I\left(\overline{ER^M}\right) < 0$, then the equilibrium is characterized by $y^* = 0$. However, $I\left(\overline{ER^M}\right) < 0$ also implies that the constrained planner chooses $y^*_{planner} = 0$. In conclusion the equilibrium is efficient. Finally, the same holds when $I\left(\overline{ER^M}\right) = 0$, which can be shown following a similar argument. *Results (i) and (ii) of Proposition 2 are proved. (q.e.d.)*

A.4 Feedback effect and the new critical price thresholds

Intuitively, the relaxation of the borrowing constraint is key. To see this, consider the borrowing constraint for the good type deficit agents (given $yP + \theta\bar{R}r^{-1} < y$ and $\bar{R} > r$):

$$\overline{b^H_D(f^H)} = l^H * \theta\bar{R}r^{-1} = \left(1 - y + f^H_D * yP + b^H_D\right) * \theta\bar{R}r^{-1}, \tag{31}$$

where f^H_D is the fraction of assets sold. Equation (31) can be solved for:

$$\overline{b^H_D(f^H)} = \frac{1 - y + f^H_D * yP}{r - \theta\bar{R}}\theta\bar{R}.$$

Next, consider how the total payoff of a good type deficit agents changes if she decides to sell more assets to finance the liquidity provision to the internal project:

$$\begin{aligned} &\tfrac{\partial}{\partial f^H_D}\left(\left(1 - y + f^H_D * yP + \overline{b^H_D\left(f^H_D\right)}\right)\bar{R} + yR^H * \left(1 - f^H_D\right) - r\overline{b^H_D\left(f^H_D\right)}\right) \\ &= y\left(P\bar{R} - R^H\right) + \tfrac{yP\theta\bar{R}}{r-\theta\bar{R}}\left(\bar{R} - r\right). \end{aligned} \tag{32}$$

Given $yP + \theta\bar{R}r^{-1} < y$ and $\bar{R} > r$, expression (32) is positive when evaluated at $\widehat{P^H}$. It shows that the critical threshold below which a good type deficit agents refuses to sell her asset is now lower and given by:

$$\widehat{P^H}\frac{ER^M}{(1-\theta)\,ER^M+\theta R^H}\geq\widehat{P^H}.$$

Since ER^M is endogenous, I proceed with an argument that helps us to determine the critical threshold for the model with borrowing as a function of exogenous parameters. Define the new critical threshold as $\widetilde{P^H}$. If the good type deficit agents are willing to sell all their assets at a price above $\widetilde{P^H}$, then the bad type deficit agents are also be willing to sell all their lemons at this price, as the return on providing additional liquidity to the internal project exceeds the cost of financing. The price threshold in equation (6) follows.

A.5 The full schedule for the decision rules over the relevant price range

As before, we can construct the full schedule for the selling decision of type $k = L$ over the relevant price range (which is now a function of θ):

$$f_D^L\left(y,P\right)=\begin{cases}1 & \text{if } P>0\\ \in[0,1] & \text{otherwise.}\end{cases}$$

The borrowing decision now depends crucially on whether the liquidity need can be fully or only partially met, i.e. if $yP+\theta\bar{R}r^{-1}\geq y$ or $yP+\theta\bar{R}r^{-1}<y$. This is because of the financing asymmetry discussed before. The simplified schedule is given by:

$$b_D^L\left(y,P,ER^M\right)=\begin{cases}\frac{1-y+yP}{r-\theta\bar{R}}\theta\bar{R} & \text{if } yP+\theta\bar{R}r^{-1}<y\\ max\left\{0,y\left(1-P\right)\right\} & \text{otherwise.}\end{cases}$$

Where I used several facts about the possible relations of P and r. Notice that from the discussion of the feedback effect $P>\widetilde{P^H}\left(\theta,\overline{ER^M}\right)$ implies $\bar{R}>r$ as long as $\theta<1$ due to the no-arbitrage condition.

On the one hand, the good type deficit agents only fully liquidate their asset portfolio, i.e. $f_D^H=1$, if the remaining liquidity need after borrowing up to the constraint requires it. Otherwise, good type deficit agents choose optimally to sell as few assets as possible. The full schedule for the selling decision of type $k = H$ over the relevant price range now becomes:

$$f_D^H\left(y,P\right)=\begin{cases}min\left\{\frac{y-b_D^H\left(y,P,ER^M\right)}{yP},1\right\} & \text{if } R^H>P>\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\\ \in\left[0,min\left\{\frac{y-b_D^H\left(y,P,ER^M\right)}{yP},1\right\}\right] & \text{if } P=\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\\ 0 & \text{otherwise.}\end{cases}\tag{33}$$

The borrowing decision is represented by:

$$b_D^H\left(y,P,ER^M\right)=\begin{cases}\frac{1-y+yP}{r-\theta\bar{R}}\theta\bar{R} & \text{if } P\geq\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } yP+\frac{\theta\bar{R}}{r}<y\\ min\left\{y,\frac{\theta\bar{R}}{r}\right\} & \text{if } P\geq\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } yP+\frac{\theta\bar{R}}{r}\geq y\\ \in\left[0,\frac{1-y+yP}{r-\theta\bar{R}}\theta\bar{R}\right] & \text{if } P=\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } yP+\frac{\theta\bar{R}}{r}<y\\ \in\left[0,min\left\{y,\frac{\theta\bar{R}}{r}\right\}\right] & \text{if } P=\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } yP+\frac{\theta\bar{R}}{r}\geq y\\ \frac{1-y}{r-\theta\bar{R}}\theta\bar{R} & \text{if } P<\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } \bar{R}>r\text{ and }\frac{\theta\bar{R}}{r}<y\\ y & \text{if } P<\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } \bar{R}>r\text{ and }\frac{\theta\bar{R}}{r}\geq y\\ \in\left[0,min\left\{\frac{1-y}{r-\theta\bar{R}}\theta\bar{R},y\right\}\right] & \text{if } P<\widetilde{P^H}\left(\theta,\overline{ER^M}\right)\text{ and } \bar{R}=r\\ 0 & \text{otherwise,}\end{cases}\tag{34}$$

where I use the additional fact that $P=\widehat{P^H}$ implies $\bar{R}=r$. The demand for collateralized borrowing of the good type deficit agents is more complicated because, when facing discounted asset prices, she prefers to borrow up to her constraint whenever $\bar{R}>r$. This is not true for the bad type deficit agent.

Finally, the optimal supply of cash by deficit agents can be computed in the same way as in the special case with $\theta=0$:

$$c_D^k\left(y,P,ER^M\right)=\begin{cases}1-y+f_D^k(P)\,yP+b_D^k\left(y,P,ER^M\right)-l^k\left(y,P,ER^M\right) & \text{if }\frac{ER^M}{P}>1\\ \in\left[0,1-y+f_D^k(P)\,yP+b_D^k\left(y,P,ER^M\right)-l^k\left(y,P,ER^M\right)\right] & \text{if }\frac{ER^M}{P}=1\\ 0 & \text{otherwise.}\end{cases}$$

For the same reason as before we have that $c_D^H\left(y,P,ER^M\right)=0$.

A.6 Proof of Lemma 3

To prove both results of the lemma, I first derive $\frac{\partial P^{C1}(\theta)}{\partial\theta}$ by using the implicit function theorem and then use equation (11) to obtain the derivative $\frac{\partial y^{C1}(\theta)}{\partial\theta}$. The first result of the lemma follows immediately when considering the limit $\theta\to 0$ as demonstrated in step 1 below. Instead the second result derived in step 2 requires computing an upper bound on θ^{P1} and then using equation (14) to evaluate the signs of $\frac{\partial P^{C1}(\theta)}{\partial\theta}$ and $\frac{\partial y^{C1}(\theta)}{\partial\theta}$.

Step 1: Recall $III(y;\theta)$. By using the implicit function theorem we can compute how the equilibrium price changes with θ:

$$\left.\frac{\partial P}{\partial\theta}\right|_{P=P^{C1}(\theta)}=-\frac{\frac{\partial III(.)}{\partial\theta}}{\frac{\partial III(.)}{\partial P}}.$$

The sign of the nominator turns out to be negative and the sign of the denominator is ambiguous:

$$\frac{\partial III(.)}{\partial P}=\frac{\overline{ER^M}-\theta\bar{R}}{\left(\overline{ER^M}-\theta\bar{R}P\right)^2}\lambda\left(1-\theta\right)\bar{R}\overline{ER^M}+\left(1-\lambda\right)\frac{\overline{ER^M}}{P^2}\lesseqgtr 0.\tag{35}$$

Next, the variation of the optimal portfolio choice in θ can be derived as:

$$\frac{\partial y^{C1}(\theta)}{\partial\theta}=\frac{-\left(P^{C1}(\theta)+\theta\frac{\partial P^{C1}(\theta)}{\partial\theta}\right)\lambda\overline{ER^M}\bar{R}P^{C1}(\theta)}{\left[\left(1-\lambda\right)\overline{ER^M}+\left(\lambda\overline{ER^M}-\theta\bar{R}\right)P^{C1}(\theta)\right]^2}.\tag{36}$$

Notice that in the limit if $\theta \to 0$ expression (35) is found to be strictly positive and finite. Hence we have that:

$$\left.\frac{\partial P^{C1}(\theta)}{\partial \theta}\right|_{lim_{\theta\to 0}} > 0 \quad \text{and} \quad \left.\frac{\partial y^{C1}(\theta)}{\partial \theta}\right|_{lim_{\theta\to 0}} < 0.$$

Step 2: The sign of equation (35) can be determined under a "relatively mild" condition on the relative returns of the long-term technologies, namely the condition in equation (14). To see this, notice that equation (35) tends to be negative for high values of θ. For this reason the strategy of the proof is to derive an upper bound for θ^{P1}. Then I show that $\overline{ER^M} - \theta\bar{R} \geq 0$ when evaluated at the upper bound for θ^{P1} if equation (14) holds. The second result of the lemma follows immediately.

To determine θ^{P1}, recall equation (12) and observe that θ^{P1} can be obtained by solving:

$$T(\theta) \equiv y^{C1}(\theta)\left(P^{C1}(\theta) - 1\right) + \theta\bar{R}\overline{ER^M}^{-1}P^{C1}(\theta) = 0. \tag{37}$$

After plugging in for $y^{C1}(\theta)$ from equation (11), we can define θ^{P1} implicitly as the solution to:

$$\theta = -\frac{(1-\lambda)\,\overline{ER^M}\left(P^{C1}(\theta) - 1\right)}{\bar{R}P^{C1}(\theta)}. \tag{38}$$

Again, supposing there exists a unique threshold, we can define an upper bound that is implicitly given by:

$$\overline{\theta^{P1}} \equiv \left.\frac{(1-\lambda)\,\overline{ER^M}\,(1-P)}{\bar{R}P}\right|_{P=\widetilde{P^H}\left(\overline{\theta^{P1}},\overline{ER^M}\right)}. \tag{39}$$

Evaluating $\overline{ER^M} - \theta\bar{R}$ at $\overline{\theta^{P1}}$ leads to:

$$(2-\lambda)\,\overline{ER^M} - (1-\lambda)\frac{\overline{ER^M}}{\widetilde{P^H}\left(\overline{\theta^{P1}},\overline{ER^M}\right)} = (2-\lambda)\,\overline{ER^M} - (1-\lambda)\frac{\bar{R}}{R^H}\left[\left(1-\overline{\theta^{P1}}\right)\overline{ER^M} + \overline{\theta^{P1}}R^H\right].$$

The last equation is positive given the sufficient condition $(2-\lambda)\,\overline{ER^M} \geq (1-\lambda)\,\bar{R}$. In conclusion, we have that $\frac{\partial P^{C1}(\theta)}{\partial\theta} > 0$ and $\frac{\partial y^{C1}(\theta)}{\partial\theta} < 0$ for all $\theta \in \left[0, \theta^{P1}\right)$ if the *sufficient condition* $\bar{R} \leq \frac{2-\lambda}{1-\lambda}\overline{ER^M}$ holds. *(q.e.d.)*

A.7 Proof of Lemma 4

An implicit expression for the threshold θ^{P1} is given by equation (38). It can be shown in 3 steps that the solution is unique. ***Step 1:*** notice that $T(0) < 0$. ***Step 2:*** $T(\theta)$ is *continuously increasing* for all $\theta^{P1} \in \left[0, \overline{\theta^{P1}}\right)$ if $\frac{\partial P^{C1}(\theta)}{\partial\theta} > 0$, which is guaranteed by equation (14). In conclusion, the *single crossing property* is fulfilled and we have a *unique threshold* in the interval $\theta^{P1} \in \left[0, \overline{\theta^{P1}}\right)$. ***Step 3:*** Since there cannot be a $\theta \geq \overline{\theta^{P1}}$ such that $T(\theta) = 0$, we have that the threshold θ^{P1} is unique in the whole interval $0 \leq \theta \leq 1$. *(q.e.d.)*

A.8 Proof of Proposition 5

The characterization of the equilibrium follows from Lemmas 3 and 4. What needs to be proved is existence for all $0 \leq \theta < \theta^{P1}$, non-existence for all $\theta \geq \theta^{P1}$ and that $\lim\limits_{\theta\to\theta^{P1}_{-}} y^*(\theta) = (1-\lambda)$. In step 1, I derive *necessary and sufficient* conditions guaranteeing the existence of the described equilibrium for values of $\theta < \theta^{P1}$. Then, I show in step 2 that it does not exist for all $\theta \geq \theta^{P1}$. Third, I prove in step 3 that the first-best allocation is reached in the limit.

Step 1: I argued previously that equation (13) gives a necessary condition for this type of equilibrium to exist. Furthermore, a second necessary condition is given by equation (8). Given the results of Lemmas 3 and 4, we have that

$$T(\theta) < 0 \quad \text{for all } \theta < \theta^{P1}.$$

Hence, aggregate liquidity is indeed scarce in equilibrium and consistent with the conjecture used when solving $(P3)$. In conclusion, conditions (13) and (8) are not only *necessary* but also *sufficient* for existence.

Step 2: From Lemma 4 we know that the threshold θ^{P1} is the unique solution to $T(\theta) = 0$. Moreover $\frac{\partial P^{C1}(\theta)}{\partial \theta}$ and $\frac{\partial y^{C1}(\theta)}{\partial \theta}$ are continuous functions of θ and:

$$\frac{\partial T(\theta)}{\partial \theta} > 0 \quad \text{for all } \theta \in \left[0, \overline{\theta^{P1}}\right).$$

Accordingly, there cannot exist a pooling equilibrium characterized by scarce aggregate liquidity, i.e. with $T(\theta) < 0$, for any $\theta \geq \theta^{P1}$.

Step 3: $lim_{\theta \to \underline{\theta}^{P1}} T(\theta) = 0$ implies a demand for cash of λy units. Given that, market clearing yields $y^* = y^*_{FB} = (1-\lambda)$. In conclusion, $lim_{\theta \to \underline{\theta}^{P1}} T(\theta) = 0$ implies that $lim_{\theta \to \underline{\theta}^{P1}} y^{C1}(\theta) = y^*_{FB}$. *(q.e.d.)*

A.9 Proof of Proposition 6

First, I analyze in section A.9.1 the case with $y^* < (1-\lambda)$ and establish result (b) of Proposition 6. Then, in section A.9.2, I consider the case $y^* = (1-\lambda)$ and establish result (a). Finally, result (c) follows directly from the discussion in section A.9.2.

A.9.1 First case: $y^* < (1-\lambda)$

Suppose there exists a market equilibrium with pooling characterized by $y^* < (1-\lambda)$. I proceed in 2 steps. Step 1 gives a characterization of the candidate equilibrium and step 2 analyzes its existence.

Step 1: Recall that a pooling equilibrium with excessive aggregate liquidity requires that in equilibrium the following condition holds: $yP + \theta \bar{R} r^{-1} > y$. Moreover, the net demand for cash is given by λy and falls short of the net supply. Therefore, $\frac{ER^M(y,\theta)}{P(y,\theta)} = r(y,\theta) = 1$ has to hold in equilibrium. Using equation (15) and the fact that $P = ER^M$, the candidate equilibrium price can be derived as:

$$P^{C2} = \frac{-\left\{R^H - \lambda\alpha - (1-\lambda)\right\} + \sqrt{\left\{R^H - \lambda\alpha - (1-\lambda)\right\}^2 + 4\lambda(1-\alpha)R^H}}{2}. \tag{40}$$

The candidate equilibrium level of investment (y) can be derived by solving:

$$ER^M = P^{C2} = \frac{\lambda(1-\alpha)\left(1 - \frac{\theta\bar{R}}{y}\right)\frac{R^H}{P^{C2}}}{\alpha + \lambda(1-\alpha)\left(1 - \frac{\theta\bar{R}}{y}\right)\frac{1}{P^{C2}}},$$

which yields:

$$y^{C2}\left(\theta, P^{C2}\right) = \frac{\lambda\left(1-\alpha\right)\theta\bar{R}}{\lambda\left(1-\alpha\right) - \alpha\frac{P^{C2}}{\frac{R^H}{P^{C2}}-1}} > 0. \tag{41}$$

Notably, $y^{C2}\left(\theta, P^{C2}\right)$ is *continuous* and *increasing* in θ, with $\lim_{\theta\to 0} y^{C2}\left(\theta, P^{C2}\right) = 0$. Moreover, it is guaranteed that the good type deficit agents cannot fully refinance by borrowing against the internal project since expression (41) is strictly smaller than $\theta\bar{R}$.

Step 2: For the candidate equilibrium to exist, it must be that $P^{C2} \geq \widehat{P^H}$. This is guaranteed if:

$$\left\{\lambda\left[\alpha\widehat{P^H} + \left(1-\alpha\right)R^H\right]\left(1 - \left(\widehat{P^H}\right)^{-1}\right) + \left(1-\lambda\right)\left[\alpha\widehat{P^H} + \left(1-\alpha\right)R^H - 1\right]\right\} \leq 0. \tag{42}$$

First, recall that the feedback effect does not play a role if $y < (1-\lambda)$. For this reason the critical price threshold for the good type deficit agents stays the same as in the special case with $\theta = 0$. Second, notice that $\frac{\partial IV}{\partial P} > 0$. In conclusion, equation (2) gives a *necessary condition* for the existence of a pooling equilibrium which is weaker then equation (42).

Moreover, the candidate equilibrium can only exist if aggregate liquidity is indeed plentiful, that is if:

$$y^{C2}\left(\theta, P^{C2}\right) = \frac{\lambda\left(1-\alpha\right)\theta\bar{R}}{\lambda\left(1-\alpha\right) - \alpha\frac{P^{C2}}{\frac{R^H}{P^{C2}}-1}} \leq \left(1-\lambda\right).$$

The latter *necessary condition* can be solved for a critical threshold, say θ^{P2}:

$$\theta^{P2} \equiv \left(1-\lambda\right)\frac{\lambda\left(1-\alpha\right) - \alpha\frac{P^{C2}-R^L}{\frac{R^H}{P^{C2}}-1}}{\lambda\left(1-\alpha\right)\bar{R}}.$$

Finally, since $y^{C2}\left(\theta, P^{C2}\right)$ is continuously increasing in θ and $\lim_{\theta\to 0} y^{C2}\left(\theta, P^{C2}\right) = 0$, I can conclude that the *necessary and sufficient conditions* for the existence of the candidate equilibrium are given by equation (42) and $0 < \theta \leq \theta^{P2}$. *This concludes result (b) stated in Proposition 6.*

A.9.2 Second case: $y^* = (1-\lambda)$

If liquidity is "just about" sufficient, it has to be true that the good type deficit agents can just about raise full financing for their internal project. Hence, the following condition has to hold with equality in equilibrium:

$$f_D^H y^* P + \theta\bar{R}P\left(ER^M\right)^{-1} = y^*, \quad \text{where } y^* = \left(1-\lambda\right). \tag{43}$$

The proof of result *(b)* proceeds by analyzing equation (43) over the range of θs in four steps. Finally, result *(c)* of Proposition 6 is proved in step 5.

Step 1: For θ close to zero, equation (43) does not hold with equality because P has to be strictly smaller than 1 in any pooling equilibrium. Furthermore, evaluating equation (43) at the point θ^{P1} leads to the condition $P = P^{C1}\left(\theta^{P1}\right)$. However, it can be shown that $IV\left(P^{C1}\left(\theta^{P1}\right), \overline{ER^M}\right) > III\left(P^{C1}\left(\theta^{P1}\right), \overline{ER^M}; \theta^{P1}\right) = 0$, yielding the corner solution $y^* = (1-\lambda)$. Consequently, the pooling equilibrium characterized by $y^* = (1-\lambda)$, which exists for $\theta = \theta^{P1}$, has to exist at least for some $\theta > \theta^{P1}$.

Step 2: Now consider the whole interval $\theta \in \left[\theta^{P1}, \theta^{P2}\right]$. It shows that P^{C2}, as given in equation

(16) has to be strictly smaller than $P^{C1}\left(\theta^{P1}\right)$ because $IV\left(P^{C1}\left(\theta^{P1}\right), \overline{ER^M}\right) > 0$, while $IV\left(P^{C2}, ER^M\right) = 0$, $\frac{\partial IV}{\partial P} > 0$ and $\frac{\partial IV}{\partial ER^M} < 0$ where $ER^M = P^{C2}$. Furthermore, suppose that there exists a pooling equilibrium characterized by $y^* < (1-\lambda)$ and evaluate equation (43) at θ^{P2} and $\lim\limits_{\theta \to \theta^{P2}} y^{C2}\left(\theta, P^{C2}\right) = (1-\lambda)$:

$$f_D^H (1-\lambda) P^{C2} + \theta^{P2} \bar{R} P^{C2} \left(ER^M\right)^{-1} = (1-\lambda), \quad \text{where } P^{C2} = ER^M. \tag{44}$$

Now consider a decrease in θ. A lower θ leads to a higher f_D^H, which in turn, allows for ER^M and P to be higher. Since $\frac{\partial IV}{\partial P} > -\frac{\partial IV}{\partial ER^M}$, a one-to-one increase in ER^M and P to ER^{M}' and P' implies that $IV\left(P', ER^{M}{'}\right) > 0$. Hence, the solution takes on the corner $y^* = (1-\lambda)$. Together with the result from Step 1, it follows that the pooling equilibrium characterized by $y^* = (1-\lambda)$ does exist for all $\theta \in \left[\theta^{P1}, \theta^{P2}\right]$, as long as the pooling equilibrium characterized by $y^* < (1-\lambda)$ exists, that is if equation (42) holds.

Step 3: From equation (44), it can be shown that a pooling equilibrium characterized by $y^* = (1-\lambda)$ does not exist for all $\theta > \theta^{P2}$ because it is not possible to satisfy equation (44) and $IV\left(P, ER^M\right) \geq 0$ for values of θ larger than θ^{P2}. To see this, notice that the left-hand side of equation (44) cannot be lowered by increasing ER^M or decreasing P since $f_D^H = \frac{(1-\lambda) - \theta \bar{R} P / ER^M}{(1-\lambda) P}$.

Step 4: It remains to be shown that the interval $\theta \in \left[\theta^{P1}, \theta^{P2}\right]$ is non-empty. This follows from the results in Step 2 together with the result in Step 1 that there has to exist a pooling equilibrium characterized by $y^* = (1-\lambda)$ at least for some $\theta > \theta^{P1}$.

Step 5: The problem was solved under the presumption that there exists a pooling equilibrium characterized by $y^* < (1-\lambda)$, i.e. that equation (42) holds. If this is not the case, it follows immediately from Step 2 that the pooling equilibrium characterized by $y^* = (1-\lambda)$ can only exist for some values θ' in the interval $\theta^{P1} \leq \theta' < \theta^{P2}$. *This concludes result (a) and (c) of Proposition 6. (q.e.d.)*

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