Bank fragility and risk management*

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Abstract

Shocks to a bank's ability to raise liquidity at short notice can trigger depositor panics. Why don't banks take a more active role in managing these risks? We study contingent risk management (hedging) in a standard global-games model of a bank run. Banks fail to hedge precisely when the exposure to a shock is most severe, just when risk management would have the biggest impact. Higher bank capital and broader deposit-insurance coverage crowd out hedging, yet encourage more banks to establish risk management desks in the first place. The model also yields testable implications for hedging incentives and policy design.

Keywords: Bank runs, liquidity risk, hedging, interim asset valuation.

JEL Classification: G01, G21, G23.

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1 Introduction

The recent collapse of several financial institutions and the precarious positions of others provides a stark reminder of how issues related to financial instability and depositor behavior are still of paramount concern, even after a decade of higher capital ratios, broader deposit-insurance coverage, and tighter supervision following the Great Financial Crisis (GFC). For instance, Silicon Valley Bank (SVB), a large regional lender in California, suffered unexpectedly large withdrawals in March 2023. These outflows, exacerbated by the declining market value of assets due to rising interest rates, triggered a severe liquidity crisis. While there appear to have been many reasons for SVB's difficulties (Fed Board, 2023), the ensuing panics occurred despite little evidence of long-term insolvency for SVB. Similarly, large deposit outflows were observed at other medium-sized banks (Caglio et al., 2024; Choi et al., 2024), creating a crisis of confidence within the banking sector.

For many institutions affected by the Regional Banking Crisis of 2023, the primary concern appeared to revolve around depressed asset valuations upon early liquidation rather than accumulating loan losses or other asset impairments. These vulnerabilities could have been mitigated through standard risk management tools, such as interest rate swaps to hedge duration risk in a rising rate environment. Yet many banks, especially smaller and mid-sized ones, appeared to engage in little risk management, with the most vulnerable banks even decreasing existing hedges (McPhail et al., 2023; Granja et al., 2024).

Against this backdrop, our paper poses three questions. First, anticipating the possibility of depositor panics driven by reduced asset valuations before maturity, why do banks not take a more active role in reducing their exposure to these risks? Second, how do banks' incentives to manage risk depend on the degree of exposure to runs that banks face? Third, how do capital structure and deposit insurance, two key elements in the policy toolkit, influence a bank's incentives to manage risk?

To address these questions, we develop a two-period global-games model of bank runs in the tradition of Goldstein and Pauzner (2005). A monopolistic bank funds itself with deposits and equity to invest in a risky long-term project. Depositors receive noisy private signals about the bank's fundamentals and decide whether to withdraw early. To this framework, we add two additional important features. First, the bank faces the risk of a negative shock to the interim value of its assets (e.g., Allen and Gale (2007); Vives (2014); Liu (2023)).[1] This makes the early liquidation of assets costly for the bank, as well

[1]This can capture several reasons, such as a poor management of the asset, changes in the availability of the secondary market for assets ("market freezes") or in its competitiveness (e.g., Shleifer and Vishny, 1986), and changes in market interest rates. When part of the bank's portfolio are fixed-income assets, such

as possibly for depositors when many demand early repayment. Second, the bank has access to a contingent risk management tool to hedge against such shocks. We consider a parsimonious specification in which the bank can take a costly action to increase the interim asset value upon a negative shock.[2] We use this framework to analyze how shock severity–and the resulting bank fragility–affects ex-ante incentives to manage risk.

We first analyze the interim withdrawal stage and show that, as is standard in the literature, our model features a unique equilibrium in which all depositors choose to withdraw early–they *run*–if the signal about the fundamental is unfavorable. When the interim asset value of the bank is low, some of these runs are pure panics, driven by a coordination failure among depositors in withdrawal choices. The fundamental cutoff, or depositor run threshold, below which runs occur depends crucially on the realized interim asset value. Consistent with other work (e.g., Vives 2014; Liu 2023), the realization of a negative shock to the interim asset value leads to a greater probability of depositor runs.[3] However, greater bank capital reduces coordination failure and bank fragility.

Given the heightened run risk associated with negative shocks to the interim value of assets, we then examine whether banks have incentives to hedge this risk by using a contingent risk management tool and explore how these incentives vary with the severity of shocks. We find that while banks have an incentive to manage much of their risk when the degree of exposure is relatively small, they discontinue risk management altogether as exposure increases. Strikingly, the cessation of risk management occurs when fragility is most severe–precisely when hedging would be most effective in terms of reducing depositors' incentives to run. This choice constitutes a risk management *failure*: banks choose not to hedge the risk of negative shocks in situations where a constrained planner would choose to do so, and the bank hedges less than the planner. The identification of when and why banks fail to manage risk is the main contribution of this paper.

The intuition for this result is that as the negative shock to the interim value of assets grows, bank fragility increases and expected profits decline. Eventually, the bank's

as corporate loans or government bonds, their market value declines substantially when interest rates rise sharply, perhaps due to a rapid tightening of monetary policy.

[2]We interpret the hedging tool broadly as any action that reduces the exposure of the bank's assets to factors that may lower their interim value. This includes activities such as monitoring asset quality, improving marketability through search for more liquid resale markets, or identifying reliable counterparties. The tool can also be understood as a derivative contract–such as a swap–that pays off in the event of a negative shock. We explore this interpretation in more detail in an extension in Section 5.3.

[3]We primarily focus on shocks that affect financial *fragility*, but that do not necessarily affect the long-term value of investments. Such shocks are destabilizing but would have no real consequences if the projects were held to maturity. This approach isolates effects stemming from changes in depositor behavior, rather that from investment profitability. We extend our analysis to shocks to the long-term return in Section 5.4.

benefit from survival, which is inversely related to the probability of a run, becomes too small relative to the cost of establishing risk management capabilities.This mechanism bears resemblance to the classic debt overhang problem (Myers, 1977), where efficient investments are forgone because their benefits accrue primarily to existing creditors under high leverage. However, whereas debt overhang reflects misaligned financial incentives, our model emphasizes the role of a coordination failure among depositors in shaping risk management incentives. Specifically, once the shock is sufficiently severe, the cost of effectively reducing the probability of runs falls short of the bank's benefit, making it privately unprofitable to hedge precisely when fragility is highest. This distinction not only separates our mechanism from the debt overhang literature but also implies markedly different comparative statics, as we describe below.

Our result on risk management failure yields various testable implications. First, our framework highlights that small banks are less likely to engage in risk management activities than large banks because the overall cost of risk management is higher for them on a per-dollar of assets basis. This implication resonates with the 2023 U.S. Regional Banking Crisis, in which several mid-sized banks failed to hedge rising interest rate risk despite growing exposures. Put differently, large banks engage in risk management for a broader range of shocks than small banks.

Our analysis also highlights the role that core features of modern banking systems, such as bank capital and deposit insurance (or other government guarantees for bank deposits) play in shaping risk management incentives. Specifically, we show that better capitalized banks and those with a higher share of insured deposits choose to hedge less on the intensive margin, as both capital and deposit insurance reduce exposure to runs and thus lower the marginal benefit of hedging. This crowding-out effect stands in contrast to the classic debt overhang problem, where more capital strengthens risk management incentives, highlighting the distinct strategic mechanism in our framework. At the same time, both capital and deposit insurance increase the likelihood that a bank engages in risk management at all, making them complements to the establishment of a risk management desk. These differential effects on the intensive and extensive margin of risk management yield a second set of testable implications that are unique to our model and can inform future empirical work. Banks with a more stable funding base (because of better capitalization or a higher share of insured deposits) are more inclined to do at least some risk management, but the level of risk management that is optimal for them is lower than for banks with a less stable funding base. To our knowledge, these aspects have not been studied theoretically before, but are consistent with empirical findings in

Bianchi et al. (2025), who show that banks with less capital and with a lower degree of deposit funding exhibit a greater intensity of hedging in response to increased interest rate risk exposure.

Finally, we consider several extensions and robustness checks. First, while we initially keep deposit rates fixed in the model, we show that our main results also obtain when we endogenize the terms of the deposit contract so that consumers choose whether to accept the deposit contract offered by the bank. Second, we endogenize the bank's capital structure and show that banks generally have an incentive to raise equity capital, thus validating our focus on the role of capital for bank's risk management choices. This occurs because bank capital serves as a *non-contingent* tool for managing risk, reducing fragility in all states of the world. Third, we micro-found the hedge used by the bank and show that the bank, as before, chooses not to engage in risk management for a severe enough shock to its interim asset value. Hence, our results are robust to alternative ways of modeling risk management. Fourth, we broaden our focus from liquidity risk to credit risk and show that the same forces can give rise to the cessation of risk management in that context. Fifth, we show that bank risk-management decisions are indeed inefficient, justifying our label of risk management *failures*. Due to its lack of commitment, banks do less risk management (on both the intensive and extensive margins) than what would be chosen by a constrained social planner with the power to commit to future risk management when raising deposits.

Our results have normative implications. As the bank's capital structure decision is not subject to the commitment problem associated with risk management, the bank's privately optimal choice of capital is constrained efficient. This implies that there is no scope for prudential regulation of bank capital in our framework. As a result, our model emphasizes and isolates a failure of bank risk management separately from any concerns about bank capital. Even when the private and social incentives for bank capital are fully aligned, risk management failures can occur. Our result, therefore, suggests that bank capital regulation is not a substitute for the prudential regulation of bank risk management.

Our contribution is to provide a rational framework analyzing how banks' hedging decisions endogenously affect the probability of depositor runs, while shocks simultaneously impact both the run thresholds and hedging incentives—a feedback mechanism absent in existing risk management literature. Studying this interaction reveals not only that risk management fails when financial stability concerns are most acute, but we can also separate the extensive margin (establishing risk management) from the intensive margin (hedging intensity). In particular, we show that standard regulatory tools have opposing effects across these margins: while they reduce the need to hedge intensely, they encourage more banks to establish some degree of risk management operations. To the extent that there may also exist information asymmetries or behavioral factors that can in practice drive depositor behavior, we believe that those would magnify the effects we identify, but they are not necessary to generate the failures of risk management that we establish. Institutions' reduced incentives to manage risk, which emerge precisely as such risk increases, arises as a natural consequence of the maximization problem banks face.

Literature. Our paper contributes to the literature on bank fragility and related policy responses. Panic-driven depositor runs, whereby crises are self-fulfilling phenomena due to strategic complementarities in withdrawal decisions, have been extensively studied since Diamond and Dybvig (1983). The global games framework (Carlsson and van Damme, 1993; Morris and Shin, 2003; Vives, 2005) links depositors' incentives to economic fundamentals (Rochet and Vives, 2004; Goldstein and Pauzner, 2005), allowing the probability of a run to depend endogenously on bank characteristics and policy. Examples include work on the implications for fragility of information disclosure (Bouvard et al., 2015), debt maturity (Eisenbach, 2017), the level of debt collateralization (Ahnert et al., 2019), government guarantees (Allen et al., 2018; Carletti et al., 2023a), bank capital and portfolio liquidity (Kashyap et al., 2023), bank resolution (Schilling, 2023), and interventions (Shen and Zou, 2024). Our paper uses similar methods to analyze how risk management affects the probability of a run.

In our framework, risk management is motivated by the increased fragility brought about by a negative shock to the interim value of assets. Prior work has established a link between interim values and bank fragility (Vives, 2014; Liu, 2023). Building on this, we analyze the extent to which risk management can mitigate such fragility. Crucially, we also study how the severity of the shock, and the associated fragility, affects banks' incentives to manage risk in the first place, an aspect that to our knowledge has not previously been studied but which is important for the debate on the stability of the banking sector.

Our main result is that banks fail to engage in risk management precisely when shocks are severe and runs are likely. By focusing on incentives to hedge against liquidity risk, our analysis complements existing work on risk management failures, which mostly centers on credit risk. Like us, Bouvard and Lee (2020) model coordination failures, but focus on underinvestment in information acquisition, whereas we study effort to mitigate identified risks. Rampini and Viswanathan (2010, 2013) show that financially constrained firms are less likely to hedge—an implication that aligns with our result on bank size. However, the mechanism differs: we emphasize depositor coordination failures, not limited pledgeability. This distinction enables us to analyze both contingent and non-contingent policy tools. In their dynamic model, Rampini and Viswanathan (2013) also find that firms neglect risk management when shocks accumulate. Despite the differences in focus and modelling approach, our paper shares with theirs the implication that banks fail to engage in risk management when it would be most needed.

Our analysis also relates to recent work on the 2023 U.S. regional banking crisis, which featured panic-induced runs unfolding at unprecedented speed in a digitized, social media-driven environment (Cookson et al., 2023). Granja et al. (2024) emphasize SVB's large asset losses following monetary tightening and its heavy reliance on uninsured deposits as key factors behind a solvency-driven run. Complementing these perspectives, Cipriani et al. (2024) provide empirical evidence that coordination failures, rather than fundamentals alone, played a crucial role in the 2023 bank runs. Drechsler et al. (2023) attribute the runs to a coordination failure over the value of a bank's deposit franchise (Drechsler et al., 2017), which is highly sensitive to withdrawals in a high-rate environment. In contrast, we focus on how anticipated losses in interim asset values can destabilize banks at the time of withdrawals, and why banks fail to implement adequate risk management strategies in response. In this respect, our view aligns with Dursun-de Neef et al. (2023), Metrick (2024), and others, who argue that SVB's collapse was driven by losses from liquidating long-term assets, thus highlighting the fragility stemming from inadequate ex-ante risk mitigation.

Our result on the relationship between the use of contingent risk management tools and bank capital, which dampens the strategic complementarity in depositors' withdrawal decisions and so limits the occurrence of runs, speaks to the emerging discussion on the implications of the Regional Banking Crisis for banking regulation (Acharya et al., 2023). Since bank incentives to put in place risk management operations increase in bank capital, our analysis identifies a novel complementarity between bank capital and contingent risk management that underscores the value of capital beyond its well-known stabilizing role.

Structure. The paper is organized as follows. Section 2 describes the model and characterizes depositors' withdrawal decisions. Section 3 analyzes the incentives for risk management. Section 4 studies how these incentives are shaped by bank capital and deposit insurance. Section 5 discusses robustness and extensions, where we endogenize the deposit rate and bank capital, consider an alternative approach to modelling risk management, extend the result to credit risk, and finally examine an efficiency benchmark. Finally, Section 6 concludes. All proofs are in the Appendix.

2 Model

2.1 Environment

We build on Goldstein and Pauzner (2005) and Carletti et al. (2023b) to examine the risk management incentives of a bank. The economy extends over three dates $t = 0, 1, 2$, and is populated by a monopolistic bank and a unit continuum of consumers indexed by $i \in [0, 1]$. All agents are risk neutral, do not discount the future, and use a single divisible good for consumption and investment. Consumers are indifferent between consuming at either date. At $t = 0$, the bank is endowed with $k \in (0, 1)$ units (bank capital) and consumers are endowed with $1 - k$ units each, which they deposit in exchange for a deposit contract allowing them to withdraw at par at $t = 1$ and a promised repayment $r_2 > 1$ at $t = 2$.[4]

The bank has access to a profitable but risky long-term investment technology, such as corporate loans, that requires an investment of one unit at $t = 0$. It returns ℓ if liquidated prematurely at $t = 1$ and $R\theta$ upon maturity at $t = 2$, where $\theta \sim U[0, 1]$ represents the fundamentals of the economy and R is a constant that reflects the return from financial intermediation, which is assumed throughout to be high enough to ensure bank viability.[5]

The bank is exposed to the possibility that the interim value of its asset ℓ may suffer a negative shock and turn out to be lower. Accordingly, we assume that the interim value of bank assets is stochastic (as in e.g., Allen and Gale, 2004, 2007; Eisenbach, 2017), and its realization is publicly observed at the beginning of $t = 1$:

$$\ell = \begin{cases} \ell_L & w.p.\ p \\ \ell_H & w.p.\ 1 - p, \end{cases} \tag{1}$$

where $0 < \ell_L < \ell_H \leq 1$ and $p \in (0, 1)$.[6] There are many reasons why such a shock may arise. For instance, mismanagement of assets could reduce their value in outside use. Alternatively, part of the bank's portfolio may be fixed-income assets whose market value declines substantially when interest rates rise, perhaps due to a rapid tightening of monetary policy. A third possibility is that secondary markets for bank assets may suddenly

[4]Demand deposits are a common source of commercial bank funding. In the United States, for example, three quarters are deposits, half of which are uninsured (Egan et al., 2017). Carletti et al. (2023a) show that demand deposits arise as the ex-ante optimal arrangement in a closely related model (without risk management choice). Other motives for demandable debt include liquidity needs (Diamond and Dybvig, 1983) or resolving an agency conflict (Calomiris and Kahn, 1991; Diamond and Rajan, 2001).

[5]In Appendix A.6 we allow for a more general distribution and show that our main result of zero risk management effort continues to hold under weaker conditions.

[6]A low enough probability p ensures that the bank is viable ex ante even if ℓ_L becomes arbitrarily small.

and unexpectedly dry up ("market freezes"). In all of these instances, the realized value of bank assets if liquidated early may be lower than initially anticipated.

We envision risk management as a way to hedge against the realization of such a negative shock. We incorporate this by assuming that the bank can hedge at $t = 0$ against the shock of a low interim value $\ell = \ell_L$. In the main text, we use a parsimonious specification of an observable but non-verifiable effort choice subject to non-pecuniary costs. The bank exerts risk management effort $z \geq 0$ at $t = 0$ after deposits are raised to maximize expected profits. Risk management improves the interim asset value after the negative shock to:[7]

$$\ell_L + z. \tag{2}$$

The cost of risk management is non-pecuniary and is given by:

$$c\frac{z^2}{2} + F, \tag{3}$$

where the variable part of the cost, parameterized by $c > 0$, may represent the bank's search effort in finding an appropriate hedging instrument or partner, collateral and margins that the seller might need to pledge against the derivative position, or counterparty risk. It may also reflect the cost of diligently monitoring the bank's assets or of identifying resale partners and opportunities, thus increasing secondary market liquidity. The fixed part, parameterized by $F \geq 0$, may be interpreted as the cost of establishing a risk management desk, identifying the risk to be hedged, or understanding the bank's exposure.

Two remarks on this cost specification are in order. First, since risk management is non-verifiable, the bank cannot credibly commit to its future choice when raising deposits. However, depositors observe the realized interim value of bank assets when deciding whether to withdraw at the interim date. Second, the assumption of a quadratic cost of risk management is clearly stylized. For example, these costs may be convex but not exactly quadratic, or maybe these costs are not even convex to start with. The point of our assumption is to stack the odds against us. That is, it should be easy for the bank to engage in *some* risk management (because of zero marginal costs at $z = 0$).

Our notion of risk management in the main text is narrow on purpose, targeted to mitigate the exact risk that the bank faces: a low interim value of its assets. This isolates the bank's risk management function from other possible motives related to increasing bank profits even when there is no bank instability (i.e., a depositor run). To allow for a

[7]Jackson and Pennacchi (2021) model a bank that can exert costly effort to improve the value of its assets in a default state. Their focus is on the creation of safe assets by private intermediaries and the public sector.

broader notion of risk management and to micro-found our cost structure, we consider a specific hedging tool–a swap contract–in Section 5.3, which leads to a payout in the event of a negative shock irrespective of whether assets are actually liquidated. We also consider broader sources of risk that may similarly affect financial fragility, such as shocks to R, in Section 5.4.

Depositors can simultaneously withdraw their funds before the bank's investment matures. At $t = 1$, each depositor receives a noisy private signal about the fundamental:

$$s_i = \theta + \varepsilon_i, \tag{4}$$

where $\varepsilon_i \sim U\left[-\epsilon, +\epsilon\right]$ are identically and independently distributed. Following much of the global-games literature, we assume vanishing noise to simplify the analysis, $\epsilon \to 0$.[8]

The bank satisfies early withdrawals, denoted by n, by liquidating its investment. If withdrawal demand exceeds the interim value of the bank's assets (i.e., if $n(1-k) > \ell$), each depositor receives an equal share of the proceeds at $t = 1$. If the remaining investment proceeds are not enough to satisfy depositors upon maturity at $t = 2$, the bank is bankrupt. In this case, the bank makes zero profits by limited liability and depositors receive nothing because of bankruptcy costs (we assume full bankruptcy costs for simplicity).[9]

Date 0	**Date 1**	**Date 2**
1. Bank raises funds for risky investment	3. The interim asset value is observed	6. Investment matures
	4. The fundamental θ is realized but unobserved; depositors receive a noisy private signal s_i and may withdraw	7. Residual depositors are paid
2. Bank chooses its risk management z		8. Consumption
	5. Bank liquidates to meet withdrawals	

Table 1: Timeline of events.

Table 1 shows the timeline. We solve the model backwards, starting with the withdrawal game at $t = 1$ in Section 2.2, which summarizes results known in the global-games literature. Equipped with this, we turn to our main interest of the bank's risk manage-

[8]Vanishing private noise simplifies the analysis of the bank's ex-ante choices of the deposit contract and its risk management. Vives (2014) studies the properties of equilibria in a global-games bank-run setup when this assumption is relaxed. See Ahnert and Kakhbod (2017) for information acquisition by investors.

[9]There is evidence of large bankruptcy costs in practice. For example, James (1991) measures the losses associated with bank failure as the difference between the book value of assets and the recovery value less direct costs associated with failure. These losses amount to about 30% of failed banks' assets.

ment choice in Section 3. In these sections, we take the deposit contract and bank capital structure as given, but we will endogenize them in Sections 5.1 and 5.2, respectively.

2.2 Depositors' withdrawal choices

We start with the withdrawal choices for a given deposit contract $\{1, r_2\}$ and risk management z. With a slight abuse of notation, let ℓ denote the *effective* interim asset value, which is either ℓ_H or $\ell_L + z$. We focus on the case of $z < \ell_H - \ell_L$, so risk management does not fully compensate for the lower interim value upon the negative shock. As shown in Section 3, this inequality arises naturally when the variable cost parameter c is not too small.

This subgame is a fairly standard bank-run global game and we start by establishing dominance bounds. For a low fundamental, withdrawing is a dominant action for depositors. This arises for $\theta < \underline{\theta}$, where the lower dominance bound $\underline{\theta}$ solves $R\underline{\theta} - (1-k)r_2 \equiv 0$. In the lower dominance region $[0, \underline{\theta}]$, the bank is insolvent at $t = 2$ even if no depositor withdraws at $t = 1$. Thus, each depositor receives nothing at $t = 2$ (due to costly bankruptcy) and so finds it optimal to withdraw early irrespective of what other depositors do. Following Goldstein and Pauzner (2005), we assume an exogenous upper dominance bound $\overline{\theta} \in (0, 1)$ above which the bank always has enough resources to settle interim claims. Specifically, we assume $\ell = 1$ for $\theta \geq \overline{\theta}$. Therefore, waiting until $t = 2$ is a dominant action for a depositor for $\theta \geq \overline{\theta}$: the interim value fully covers the promised payment to all withdrawing depositors, so early withdrawals do not impose any loss on depositors who wait. Accordingly, these depositors prefer the payment of r_2 at $t = 2$ over the lower unit payment at $t = 1$. The upper dominance region $[\overline{\theta}, 1]$ can be arbitrarily small, and we assume $\overline{\theta} \to 1$. (For all $\theta < \overline{\theta}$, the interim value remains as specified in Equation (1).)

For the intermediate range $(\underline{\theta}, \overline{\theta})$, the withdrawal choice of a depositor depends on the withdrawal choices of other depositors. The following lemma describes these choices and characterizes the bank failure threshold as well as its comparative statics. It also describes the nature of runs (that is, fundamental-driven or also due to panics).[10]

Lemma 1. *All depositors withdraw their funds at $t = 1$ and the bank fails if the fundamental θ falls below a threshold θ^*:*

[10]The run thresholds below are derived under the assumption that the contract is incentive compatible, in the sense that depositors do not always run, so the bank accrues positive expected profits. For this to be the case, the repayment r_2 promised to depositors must be sufficiently high relative to what they can obtain if they withdraw early. Formally, this requires that $r_2 > \widehat{r}_2 > 1$, with the cutoff $\widehat{r}_2$ derived in the Appendix A.1. This condition is always satisfied in equilibrium when the bank chooses r_2 optimally, as in Section 5.1.

(i) For $\ell \geq 1-k$, the threshold equals the lower dominance bound (only fundamental runs exist):

$$\theta^* = \underline{\theta} \equiv \frac{(1-k)r_2}{R}. \tag{5}$$

The bank failure threshold decreases in bank capital, $\partial \underline{\theta} / \partial k < 0$.

(ii) For $1-k > \ell$, there are also panic runs and the threshold is:

$$\theta^* = \frac{(1-k)r_2\left(1-\frac{\alpha}{r_2}\right)}{R\left(1-\frac{1-k}{\ell}\frac{\alpha}{r_2}\right)} = \underline{\theta}\frac{r_2-\alpha}{r_2-\frac{\alpha(1-k)}{\ell}} > \underline{\theta}, \tag{6}$$

where $\alpha \equiv \int_0^{\overline{n}} dn + \int_{\overline{n}}^1 \frac{1}{n} dn$ is a depositor's expected payoff from withdrawing early and $\overline{n} \equiv \ell/(1-k)$ is the maximal level of withdrawals the bank can serve in full at the interim.

The run threshold decreases in bank capital and the interim value at a diminishing rate, $\partial\theta^*/\partial k < 0$, $\partial^2\theta^*/\partial k^2 > 0$, and $\partial\theta^*/\partial\ell < 0$, $\partial^2\theta^*/\partial\ell^2 > 0$. The effects of capital and the interim value on the run threshold are substitutes, $\partial^2\theta^*/\partial\ell\partial k > 0$. The panic-run range decreases in capital and the interim value, $\partial(\theta^* - \underline{\theta})/\partial k < 0$ and $\partial(\theta^* - \underline{\theta})/\partial\ell < 0$.

Proof: See Appendix A.1. □

Lemma 1 establishes the existence of a unique equilibrium of the withdrawal subgame in which depositors use threshold strategies and choose to withdraw upon unfavorable signals, $s_i < s^*$. For vanishing private noise, this signal threshold converges to a fundamental threshold, $s^* \to \theta^*$. This implies that, in equilibrium, all depositors behave alike: all withdraw for $\theta < \theta^*$ and nobody withdraws for $\theta > \theta^*$. The failure threshold θ^* fully summarizes the ex-ante probability of a bank run, which is our measure of bank stability.

When the interim asset value ℓ covers the repayment to all depositors $1-k$, there is no strategic complementarity among depositors and runs only occur when the investment return is low (fundamental runs for $\theta \leq \underline{\theta}$). Conversely, for $\ell < 1-k$, panic runs arise for $\underline{\theta} < \theta < \theta^*$ because the bank's ability to repay withdrawing depositors in full depends on the volume of withdrawals. Specifically, depositors fear that others may withdraw and that the bank will not have enough interim resources to repay them. A negative shock to the interim asset value exacerbates the coordination failure among depositors and increases both the likelihood of bank failure and the range of panic runs.[11]

[11] These findings are consistent with and analogous to the result in Vives (2014), who considers comparative statics to the interim value of the bank's assets and shows that lower interim values lead to a greater risk of panic runs. Although that setting is slightly different, and does not consider directly the withdrawal

In what follows, we focus on the case where $\ell_H \leq 1-k$, so that panic runs can arise and the relevant failure thresholds are given by Equation (6) when evaluated at $\ell = \ell_L + z$ and $\ell = \ell_H$, respectively. A panic run features the liquidation of profitable projects and is therefore always inefficient. This may create an incentive for the bank to reduce the ex-ante probability of a run via risk management, which we turn to next.

3 Risk management incentives

We turn to the bank's choice of risk management. Throughout the paper we focus on a high enough variable cost parameter to ensure that $\ell_L + z < \ell_H$, so the bank is more fragile upon the negative shock to the interim value of bank assets, $\theta_L^* > \theta_H^* \equiv \theta^*(\ell_H, k)$ and z is indeed a tool to manage the increased fragility brought about by the fall in ℓ. As done in the previous section, here again we keep the deposit contract r_2 fixed in order to isolate the direct effect from changes in the interim value ℓ_L on the bank's risk management incentives. This makes the analysis more tractable and the economics at work more transparent. Of course, changes in the interim value also affect depositors' expected return, which then may require the bank to adjust the deposit rate to ensure depositors' participation. We consider this indirect effect in Section 5.1.

The bank chooses risk management z to maximize its expected profits Π. The bank receives zero upon a bank run, and makes positive profits absent a run (which occurs for $\theta \geq \theta_L^*$ and $\theta \geq \theta_H^*$, respectively). In these cases, the bank's investment returns $R\theta$, so that after repaying depositors the bank's payoff is $R\theta - (1-k)r_2$. Integrating over the possible realizations of the fundamentals and accounting for the cost of risk management yields:

$$\Pi(z) \equiv p \int_{\theta_L^*}^{1} [R\theta - (1-k)r_2] d\theta + (1-p) \int_{\theta_H^*}^{1} [R\theta - (1-k)r_2] d\theta - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}. \tag{7}$$

Differentiating (7) with respect to z gives us the first-order condition (that abstracts from the fixed cost):

$$\frac{d\Pi}{dz} = p[R\theta_L^* - (1-k)r_2]\left(-\frac{d\theta_L^*}{dz}\right) - cz = 0, \tag{8}$$

where $d\theta_L^*/dz = d\theta_L^*/d\ell$ because of the substitutability of ℓ_L and z, with the latter derivative given in the proof of Lemma 1. The first term in Equation (8) is the marginal benefit of risk management: higher z reduces the failure threshold upon a shock, θ_L^*, which

incentives of depositors, the mechanism here is similar: the reduced interim value worsens the coordination failure among depositors (or fund managers in Rochet and Vives, 2004; Vives, 2014).

benefits the bank through the increase in residual profits around the failure threshold, $R\theta_L^* - (1-k)r_2 > 0$. The second term in (8) is the marginal cost of risk management.

Let $\widehat{z}$ solve Equation (8). We have the following result about the bank's risk management.

Proposition 1. *For all ℓ_L such that $\theta_L^*(\widehat{z}, \ell_L) < \overline{\theta}$, a negative shock to the interim asset value increases the incentives to manage risk at the intensive margin: $d\widehat{z}/d\ell_L < 0$. Moreover, risk management incentives decrease as bank capital and the variable cost parameter c increase, while they increase as the probability of the negative shock increases: $d\widehat{z}/dk < 0$, $d\widehat{z}/dc < 0$, $d\widehat{z}/dp > 0$.*

Proof: See Appendix A.2. □

Proposition 1 establishes that a lower interim asset value increase the banker's incentives for risk management, $d\widehat{z}/d\ell_L < 0$. This result is driven by the increased sensitivity of the run probability for any $\theta_L^* < \overline{\theta}$, which increases the marginal benefit of risk management.

Whether the bank actually does any risk management depends on whether it is sufficiently profitable. In other words, whether engaging in any risk management is optimal (on the extensive margin) depends on the benefit of risk management compared to the overall cost of setting up risk management operations, that is $\Pi(\widehat{z}(\ell_L)|\ell_L) - \Pi(0|\ell_L)$.

A crucial ingredient for our result is that fragility increases as the shock to the interim asset value is more severe, even when considering the bank's optimal risk management choice. We show this formally in Lemma 2 below: while the incentives for risk management $\widehat{z} = \widehat{z}(\ell_L)$ increase in shock severity, this never fully offsets the negative shock.

Lemma 2. *For all ℓ_L such that $\theta_L^*(\widehat{z}, \ell_L) < \overline{\theta}$, the run threshold decreases in ℓ_L, $\frac{d\theta_L^*(\widehat{z}, \ell_L)}{d\ell_L} < 0$.*

Proof: See Appendix A.3. □

Due to higher bank fragility upon a more severe shock to the interim asset value, bank profits in the low state decrease. This happens up to the point where the benefits of risk management are insufficient to justify the costs, as summarized in Proposition 2.

Proposition 2. *For any $F \geq 0$ and $c > \widehat{c}$, where $\widehat{c}$ is characterized in the appendix, the bank stops doing risk management when the interim asset value falls below a unique cutoff $\widetilde{\ell}_L = \widetilde{\ell}_L(F, c) \in (0, \overline{\ell})$, where $\overline{\ell}$ solves $\theta_L^*(0, \overline{\ell}) = \overline{\theta}$. That is, $z^* = 0$ whenever $\ell_L < \widetilde{\ell}_L$.*

Proof: See Appendix A.4. □

For sufficiently severe shocks, the bank does not set up any risk management operations at all. This occurs in a range of parameters where the cost of doing risk management, as captured by the fixed and variable cost parameters F and c, are such that in the presence of less severe shocks (i.e., larger values of ℓ_L) the bank would choose to manage risk.[12]

The result of zero risk management choice occurs even though the marginal impact of hedging on the run probability increases as ℓ_L decreases, that is $\partial\theta_L^*/\partial z$ decreases in ℓ_L. Yet, the differential profit the bank can accrue from risk management shrinks as the negative shock becomes extreme. In this circumstance, two forces are at play. First, the bank's profits when doing risk management decrease, which is driven by the higher run threshold (Lemma 2). Second, upon extreme negative shocks, the probability of a run without risk management reaches its upper bound, thus becoming insensitive to changes in the interim asset value. Hence, even in the absence of the fixed cost (that is, for $F = 0$), the gains to the bank from risk management monotonically decrease in the severity of the shock (lower ℓ_L).[13] The cutoff $\widehat{c}$ is the level of the variable cost parameter at which the bank cannot avoid a run when the shock to the interim asset value is most extreme (for $\ell_L \to 0$). In other words, it ensures that, for an extreme shock, the level of risk management necessary for the bank to reduce the run probability is too costly–even with a zero fixed cost, $F = 0$.[14]

The results of Proposition 1 and 2 are shown in the left panel of Figure 1, which shows how the optimal level of risk management effort z^* changes with the interim asset value. For low interim values, $\ell_L < \widetilde{\ell}_L$, the bank chooses not to set up a risk management desk, so $z^* = 0$. Conversely, the bank sets up a risk management desk when the negative shock to the interim value is less severe. In this case, the bank's optimal risk management effort is $z^* = \widehat{z}$, and these incentives are higher for lower levels of the interim asset value ℓ_L. The right panel of Figure 1 shows how the bank failure threshold θ_L^* depends on the interim asset value ℓ_L, taking into account the optimal risk management choice z^*. For $\ell_L > \widetilde{\ell}_L$, the bank chooses to set up a risk management desk and the bank failure threshold is lowered from the gray dashed line to the black solid line due to the stabilizing effect of risk management. The right panel also illustrates the result of Lemma 2: the probability

[12]Note that this assumes that the fixed cost parameter F is not too large. Otherwise, and trivially, even for very mild shocks the bank would not find it optimal to engage in risk management.

[13]The key mechanism driving the risk management failure result arises even without relying on the upper dominance region when we allow for a thin-tailed distribution of the economic fundamental θ, as we show in Appendix A.6. There, we consider a standard symmetric Beta distribution. We show that risk management failures arise before the fundamental threshold reaches the upper dominance bound. The main reason why this occurs is that, with a "thin tail" for realizations of the fundamental θ, the net benefit to doing risk management decreases as the negative shocks become sufficiently severe.

[14]A strictly positive fixed costs associated with risk management seems natural because of the nature of the activity, which involves tasking specific employees with identifying risk exposures and finding ways of hedging that risk. Our argument is merely that, for the purposes of the theoretical results, a positive fixed cost is not needed, even though they are likely important in practice.

Risk management choice

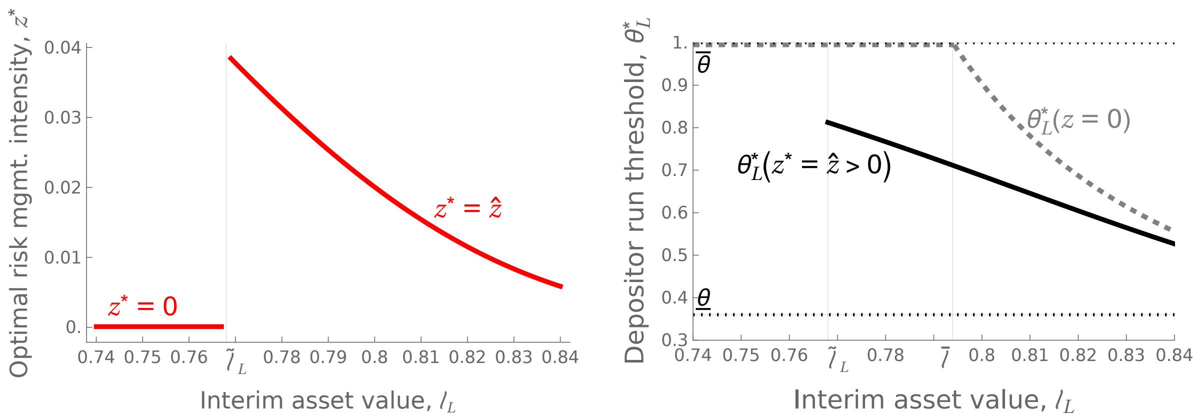

Figure 1: The left panel shows the risk management intensity z^* as a function of ℓ_L (red solid line) and the right panel shows the equilibrium run threshold θ_L^* as a function of ℓ_L for the case when $z = 0$ (gray dashed line) and for the case when $z^* = \widehat{z} > 0$ (black solid line). This numerical illustration uses the parameters: $R = 3$, $p = 1/4$, $k = 1/10$, $c = 50$, $F = 0$, and $r_2 = 6/5$.

of a bank run increases monotonically in the severity of the shock (as ℓ_L decreases).

As indicated by the cutoff $\widetilde{\ell}_L(F, c)$ in Proposition 2, the realized shock to the interim value ℓ_L and the magnitude of the fixed cost F matter jointly for the bank's decision to engage in risk management. In other words, banks with different fixed cost F have a different tolerance to shocks and, in turn, incentive to engage in risk management. Corollary 1 (below) establishes an equivalent interpretation: for any given shock ℓ_L, there must be a threshold fixed cost F associated with setting up risk management operations, whereby banks with higher fixed cost prefer not to do so. Denote such a cutoff value as $\widehat{F}$ (as defined in Appendix A.5), which is strictly positive for $\ell_L \in (\widetilde{\ell}_L(0, c), \overline{\ell})$.

Corollary 1. *The cutoff $\widehat{F}$ increases in ℓ_L: $\partial \widehat{F} / \partial \ell_L > 0$.*

Proof: See Appendix A.5. □

With a higher interim value of bank assets, the bank can achieve a positive profit from risk management upon a negative shock even for a higher fixed cost of setting up risk management operations. This result suggests an empirical link between bank size and the choice to set up risk management operations. To the extent that for larger banks the fixed cost F represents a smaller fraction of their total assets and revenue, we expect that larger banks find it optimal to set up risk management operations even when faced with the possibility of more extreme negative shocks to the interim value of their assets. By contrast, smaller banks engage in no risk management even for smaller possible shocks, recognizing that the benefit of managing this risk is insufficient to cover the costs in setting up such operations.[15] This testable implication may inform future empirical work.

4 The role of bank capital and deposit insurance

We next study how the level of bank capital (Section 4.1) and deposit insurance coverage (Section 4.2) affect the risk management effort at the intensive margin as well as the profitability of setting up risk management operations at the extensive margin. This analysis contributes to a nuanced understanding of how traditional tools of financial regulation interact with the contingent risk management tool at the heart of this paper.

4.1 Bank capital and risk management

In Section 3 we showed that a higher level of bank capital (e.g., due to higher regulatory bank capital requirements) crowds out risk management effort if the solution to the risk management problem is interior (Proposition 1). In other words, k and z are substitutes along the *intensive margin*, reflecting that more capitalized banks are less subject to run risk and financial instability and, hence, have less need to manage this risk. In essence, bank capital acts as a *non-contingent* tool for risk management, reducing the bank's exposure to depositor runs for any interim asset value, regardless of whether a shock is realized or not. However, as we show below, k and z are not substitutes along the *extensive margin*.

The following result states that higher bank capital is a catalyst for using *contingent* risk management, whereby it facilitates the choice of a positive z^*. In other words, bank capital is complementary to establishing a risk management desk, even if it mutes the bank's marginal incentives to exert effort. To show this complementarity along the *extensive margin*, we study how the threshold of the fixed cost is affected by bank capital.

Proposition 3. *The fixed cost threshold $\widehat{F}$ increases in bank capital, $d\widehat{F}/dk > 0$.*

Proof: See Appendix A.7. □

The fixed cost threshold below which the bank chooses to set up risk management operations and, hence, chooses a strictly positive amount of risk management, $z^* > 0$,

[15] It is straightforward to see that if all bank operations are scaled by a variable S, then the cutoff value of the fixed cost F above which banks do not engage in any risk management becomes $\widehat{F}/S$. Hence, for larger S, a given bank is more likely to have risk management operations in place, whereas smaller banks avoid such costs and instead risk collapse in the event of a sufficiently large negative shock.

increases in bank capital. As bank capital increases, a larger set of banks finds it optimal to hedge their exposures. Therefore, a positive and *complementary* relationship between bank capital and setting up a risk management desk arises, which can be viewed as the effect of capital on the extensive margin of risk management. Intuitively, more bank capital increases bank profits when it survives until the final date since it reduces the probability of runs in both states (that is, it lowers θ_L^* and θ_H^*) as well as the repayment obligation to depositors, $(1-k)r_2$. As a result, the bank is more willing to bear the cost of setting up a risk management desk since it has more to gain by doing so.[16]

Bank capital and risk management

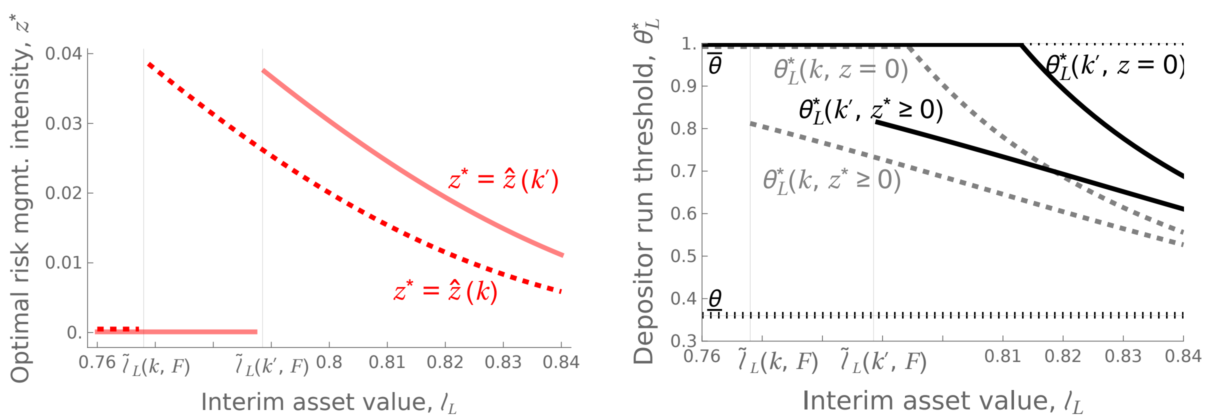

Figure 2: The left panel again shows z^* as a function of ℓ_L, but now for a lower level of bank capital $k' = 0.08$ (red solid line) and for the higher level of bank capital $k = 0.10$ previously shown in Figure 1 (red dotted line). The right panel shows the bank failure thresholds for both levels of capital, $\theta_L^*(k',z)$ and $\theta_L^*(k,z)$, as a function of ℓ_L. The black solid and gray dashed lines depict the run thresholds for the case when $z = 0$ and $z^* = \widehat{z} > 0$ for the lower level of capital, k', and for the higher level of bank capital, k, respectively. All other parameters are the same as in Figure 1.

Figure 2 shows the impact of changes to bank capital. The left panel shows the optimal level of risk management effort z^* and the right panel shows the bank failure threshold as a function of the interim value for two different levels of bank capital. We use the baseline level of bank capital $k = 0.10$ from Figure 1 and add a 20% lower level of bank capital $k' = 0.08$. The higher level of bank capital is associated with a lower risk management effort, $\widehat{z}(k) < \widehat{z}(k')$, conditional on setting up a risk management desk. That is, a bank with more capital does less risk management at the intensive margin, implying that bank capital crowds out risk management effort, meaning that the two are substitutes. As discussed above, this result is due to the stabilizing role of bank capital, which is reflected

[16]Our results on the extensive margin of risk management complement existing work that argues that non-contingent risk management tools such as bank capital and liquidity buffers discipline risk-taking (see, for example, Furlong and Keeley (1989), Admati et al. (2013), and Calomiris et al. (2023)).

in a lower run threshold for a higher levels of capital, i.e., $\theta_L^*(k, z = 0) < \theta_L^*(k', z = 0)$, as shown in Lemma 1 and represented by upper gray dashed and black solid lines. Also, $\theta_L^*(k, z^*) < \theta_L^*(k', z^*)$, as seen by comparing the lower gray dashed and black solid lines.

However, the complementary relationship between bank capital and the establishment of a risk management desk can also be seen in Figure 2, and is illustrated by the interval $(\widetilde{\ell}_L(k, F), \widetilde{\ell}_L(k', F))$ of interim values where a higher level of capital is a catalyst for the establishment of a risk management desk. The figure thus illustrates how a bank with higher capital will engage in risk management activities even at lower levels of interim value or, equivalently, at higher levels of fixed costs, compared to a bank with less capital.

To show that banks indeed may want to raise capital as a source of financing even if, arguably, it represents a more expensive form of financing, we endogenize bank capital in Section 5.2. There we show that banks have incentives to use equity capital in equilibrium.

4.2 Deposit insurance

In the previous sections, we have assumed that all deposits are uninsured, as is common in the literature on financial fragility. In practice, however, many deposits are often insured, and this insurance is likely to affect depositor behavior and thus, by extension, how banks respond. In this section, we extend the analysis to consider the role that deposit insurance (DI) plays for the bank's risk management choice. To do so, we assume that a fraction $\sigma \in [0, 1]$ of depositors is insured. Insured depositors are certain to receive the promised repayment and thus have no incentive to withdraw early, regardless of the signal s_i.[17] For simplicity, we assume that the outside option of consumers is $\rho_D = 1$ in this subsection.

We have two findings. First, and consistent with the banking literature, DI has a stabilizing effect on bank fragility. Second, and our main insight, we link this beneficial stabilizing effect of DI to the bank's risk management choice and find that it reduces the bank's marginal incentives to manage risk along the intensive margin. This crowding out result is much alike the other non-contingent tool we studied earlier, namely capital. At the same time, however, DI acts as a catalyst for establishing a risk management desk. In other words, even though the degree of risk management decreases conditional on the bank actually spending resources on managing such risk, the extensive margin effect of deposit insurance is to promote the creation of a risk management desk. Taken together,

[17] This specification resembles the sleepy (or inactive) depositor specification in Chen et al. (2010), although we assume that depositors' type is known, so the bank offers two deposit rates. Alternative approaches to modelling deposit insurance include Allen et al. (2018) and Dávila and Goldstein (2023).

deposit insurance coverage acts very similarly to bank capital, as analyzed in Section 4.1.

Let r_2^I and r_2^U denote the promised time 2 repayment for insured and uninsured depositors, respectively. As before, we solve the model by working backwards, starting with withdrawal decisions of the fraction $1-\sigma$ of uninsured depositors, for a given deposit contract $\{1, r_2^I, r_2^U\}$, capital structure k, and risk management choice z. To simplify the exposition, we introduce the subscript σ for all thresholds and define the average deposit rate as $\bar{r}_{2,\sigma} \equiv \sigma r_2^I + (1-\sigma) r_2^U$. As in the baseline model, we describe depositors' withdrawal decisions for a generic ℓ because its realization is known at $t=1$.

In the baseline model, panic-driven runs exist only if there is strategic complementarity in depositors' withdrawal decisions, which occurs when the interim value ℓ is insufficient to cover the repayment of all depositors at time $t=1$, $\ell < 1-k$. Generalizing this bound by noting that a fraction σ of depositors is insured and never withdraws, strategic complementarity in depositors' withdrawal decisions arises only for interim values below a threshold $\check{\ell}_\sigma(k) = \left(1-\sigma+\sigma/r_2^U\right)(1-k)$, which we derive in Appendix A.8. Note that $\check{\ell}$ equals $1-k$ for $\sigma=0$, as in the baseline model, and $d\check{\ell}_\sigma/d\sigma < 0$. Lemma 3 extends our results in Lemma 1 on the existence of a unique equilibrium of the withdrawal game.

Lemma 3. *Uninsured depositors withdraw at $t=1$ only if the fundamental of the economy θ is below a critical cutoff. This threshold is given by $\underline{\theta}_\sigma$ whenever $\ell \geq \check{\ell}_\sigma$ and by θ^*_σ for $\ell < \check{\ell}_\sigma$, with $\theta^*_\sigma \geq \underline{\theta}_\sigma$. Both thresholds, defined in the Appendix, decrease in σ: $\partial\underline{\theta}_\sigma/\partial\sigma < 0$ and $\partial\theta^*_\sigma/\partial\sigma < 0$.*

Proof: See Appendix A.8. □

As in the baseline model, a unique threshold equilibrium emerges in the characterization of depositors' withdrawal decisions. The exact threshold and the type of runs depend on the level of the interim value relative to the amount of bank capital and on the degree of DI coverage, summarized in the cutoff $\check{\ell}_\sigma(k)$. When the interim value, the fraction of insured depositors, and bank capital are high, so that $\ell \geq \check{\ell}_\sigma$, only fundamental runs occur, while panic runs emerge in equilibrium otherwise. Importantly, DI increases financial stability along two dimensions: it reduces the range of parameters for which panic runs occur, and it reduces the strategic complementarity in depositors' withdrawal decisions. To focus on situations in which financial fragility is of real concern, we again restrict attention to the case of $\ell < \check{\ell}_\sigma$ henceforth. This is equivalent to assuming that the degree of deposit insurance coverage, σ, is not so large that all fragility is eliminated.

Having characterized uninsured depositors' withdrawal decisions, we move to the bank's choice of risk management. We first restate the expression for expected bank profits. Relative to (7), we need to replace θ^* with the one characterized in Lemma 3. We also account for the two types of depositors. Expected profits are then given by:

$$\Pi_\sigma(z) \equiv p\int_{\theta^*_{L,\sigma}}^1 [R\theta - (1-k)\bar{r}_{2,\sigma}]d\theta + (1-p)\int_{\theta^*_{H,\sigma}}^1 [R\theta - (1-k)\bar{r}_{2,\sigma}]d\theta - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}.$$

Again ignoring the fixed cost, the optimal level of risk management is $\widehat{z}_\sigma$. Proposition 4 complements Proposition 1 by examining how the bank's incentives are affected by DI.

Proposition 4. *Let $\ell_L < \check{\ell}_\sigma$. The incentives to manage risk along the intensive margin increase as the interim value in the event of a shock decreases, $d\widehat{z}_\sigma / d\ell_L < 0$, and the incentives decrease as the fraction of insured depositors increases, $d\widehat{z}_\sigma / d\sigma < 0$.*

Proof: See Appendix A.9. □

The first result is identical to that in Proposition 1, suggesting that the key mechanics with deposit insurance are similar to the baseline model. The second result about the effect of DI coverage mirrors a similar result obtained for bank capital: having more insured deposits makes the bank more stable, reducing the need to engage in risk management along the intensive margin.

Next, we show that the result of zero risk management choice (as characterized in Proposition 2) similarly holds in this setting, with the bank deciding not to manage risk as the shock to the interim asset value becomes more severe. What is more, DI can be a catalyst for setting up a risk management desk, again similar to bank capital.

Proposition 5. *For any $F \geq 0$ and $c > \widehat{c}_\sigma$, the bank stops doing risk management when the interim asset value falls below a unique cutoff $\widetilde{\ell}_\sigma = \widetilde{\ell}_\sigma(F, c) \in (0, \bar{\ell}_\sigma)$, where $\bar{\ell}_\sigma$ solves $\theta^*_{L,\sigma}(\bar{\ell}_\sigma) = \bar{\theta}$. That is, $z^*_\sigma = 0$ whenever $\ell_L < \widetilde{\ell}_\sigma$ (or, equivalently, $F < \widehat{F}_\sigma$). This threshold increases in DI coverage, $d\widehat{F}_\sigma / d\sigma > 0$.*

Proof: See Appendix A.10. □

Intuitively, the *complementary* relationship between deposit insurance and setting up a risk management desk–the extensive margin effect of deposit insurance on risk management–arises because of the stabilizing effect of DI, which increases the expected profits of the bank from risk management by reducing the probability of a run.

5 Discussion of robustness and extensions

We extend our analysis in various ways to establish robustness and to incorporate greater realism. First, we show that our finding of a zero risk management choice is robust to the endogenization of deposit rates in Section 5.1. We then extend our model by endogenizing the bank capital structure in Section 5.2. Next, we consider an alternative, and perhaps more realistic, modelling approach to risk management in Section 5.3 and show that our main results are robust. Thereafter, we consider a different source of risk (credit risk as opposed to liquidity risk) in Section 5.4. We show that our main mechanism leading to zero risk management choices (via compressed bank profits as adverse shocks become more severe) applies more generally. Finally, Section 5.5 solves a constrained planner problem and shows that there is a wedge between the private and social incentives for risk management, confirming our argument that there are failures in risk management.

5.1 Endogenous deposit rates

To clearly illustrate the main forces at work, we have so far held the promised repayment to depositors r_2 constant when varying the interim asset value. However, the equilibrium repayment clearly depends on the interim value since changes in the distribution of ℓ affect the expected value of the deposit claim and hence shift depositors' participation constraint. Here, we demonstrate that our main result on zero risk management choice continues to hold when the indirect effects arising from the need to ensure depositors' participation are taken into account. Let the outside option of consumers yield a (gross) return of $\rho_D \geq 1$. The bank raises funds from consumers in exchange for a deposit contract and sets the deposit rate r_2 to maximize expected profits subject to consumer participation.

As a first step, the following lemma establishes how the interim value ℓ_L affects r_2 in the baseline model (without deposit insurance) and no risk management, $z = 0$.

Lemma 4. *Lower interim values are associated with a higher deposit repayment in equilibrium:* $dr_2^*/d\ell_L < 0$ *and* $dr_2^*/d\ell_H < 0$*.*

Proof. See Online Appendix A.11. □

Lemma 4 shows that the possibility of a larger negative shock leads the bank to offer a higher repayment r_2 to depositors. In the next lemma we study the implication for the overall effect of a change in the level of the interim value ℓ_L on the run threshold.

Lemma 5. *The run threshold increases upon a more severe shock to the interim value:* $d\theta_L^*/d\ell_L < 0$.

Proof. See Online Appendix A.12. □

Lemma 5 establishes that similar comparative statics to those obtained in Lemma 1 (for the case when r_2 is held fixed) also hold when r_2 is endogenous and pinned down by the binding participation constraint of investors.[18] This occurs even though, as shown in Lemma 4, r_2 must increase whenever ℓ_L decreases in order to compensate depositors for the potentially larger drop in the interim value of the bank's assets. The increase in r_2, however, reduces depositors' incentives to run, thus lowering θ^*, and at least partially offsets the effect of the drop in ℓ_L. Nevertheless, Lemma 5 establishes that the overall equilibrium effect of a change in ℓ_L is unchanged, because the indirect effect coming from the change in r_2 does not fully offset the direct effect. The implication is that, as before, a larger negative shock to the interim value of bank assets is destabilizing.

Having characterized how the probability of a run changes with the interim asset value, the following proposition restates our main result in the context where $r_2(\ell_L)$ is endogenous and is set to satisfy depositors' participation constraint.

Proposition 6. *For any* $F \geq 0$ *and* $c > \check{c}$*, the bank stops doing risk management when the interim asset value falls below a unique cutoff* $\widetilde{\ell}_{r_2} \in (0, \check{\ell})$*, where* $\check{\ell}$ *solves* $\theta_L^*(0, \check{\ell}) = \overline{\theta}$*. That is,* $z^* = 0$ *whenever* $\ell_L < \widetilde{\ell}_{r_2}$.

Proof. See Online Appendix A.13. □

Proposition 6 shows that our main result on the lack of risk management incentives when the shock to the interim asset value becomes severe continues to hold when r_2 is endogenized. As before, the constraint that c is not too small is there only to guarantee that risk management is not so cheap that the bank can simply fully offset any shock, no matter how negative. Hence, the bank's decision not to manage risk obtained earlier did not arise from the assumption that deposit interest rates were fixed, but rather stems from how bank incentives are altered as shocks become sufficiently large.

[18] The lemma focuses on the deposit rate corresponding to the solution to the binding participation constraint. In principle, the bank may find it optimal to increase r_2 further in this setting, leaving some slack in depositors' participation constraint. Such strategy could be optimal since a higher deposit rate reduces the run threshold and so may have a beneficial effect on the bank's expected profits. As shown in Ahnert et al. (2023), parametric restrictions on the investment profitability R ensure that the participation constraint binds in equilibrium.

5.2 Endogenous bank capital

So far, we have treated bank capital as exogenous, so that the comparative statics with respect to k could be interpreted as the response to a change in binding capital requirements, for instance. In this section, we endogenize the bank's capital structure. We show that the bank finds it optimal to raise a strictly positive amount of capital as long as the cost of bank capital is not too high relative to the cost of deposit funding. Intuitively, the bank values the role of bank capital as a non-contingent risk management tool that can reduce the probability of bank runs in all states.

To study this issue, suppose that the banker raises capital and deposits at the beginning of $t = 0$, so that k and r_2 are jointly determined. Assume also that the outside options of bank equity holders is $\rho_E \geq \rho_D$.[19] To ease the exposition, we focus on a parameter space where depositors' participation constraint binds. Thus, the banker's problem is:

$$\max_{k,r_2} \Pi = p \int_{\theta_L^*}^{1} [R\theta - (1-k)r_2] d\theta + (1-p) \int_{\theta_H^*}^{1} [R\theta - (1-k)r_2] d\theta - \rho_E k - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}, \quad (9)$$

where the term $\rho_E k$ represents the cost of equity for the bank. This maximization problem is subject to the participation constraints of the banker, $\Pi \geq 0$, and depositors:

$$V \equiv p_\ell \left[\int_0^{\theta_L^*} \frac{\ell_L + z}{1-k} d\theta + \int_{\theta_L^*}^{1} r_2 \, d\theta \right] + (1-p_\ell) \left[\int_0^{\theta_H^*} \frac{\ell_H}{1-k} d\theta + \int_{\theta_H^*}^{1} r_2 \, d\theta \right] = \rho_D, \quad (10)$$

where we used the fact that depositors receive an equal share of liquidation proceeds at time 1. Note that risk management effort is determined at the end of time 0, so it depends on the prevailing level of bank capital and the deposit rate $z = z^*(k, r_2)$.

We can now characterizes the bank's optimal capital structure.

Proposition 7. *The bank is funded with equity and debt, $k^* \in (0,1)$, if the cost of equity, ρ_E, is not too high relative to the cost of debt, ρ_D.*

Proof: See Online Appendix A.14. □

This proposition states that a positive but not too high equity premium results in a pos-

[19] While it is widely accepted that the cost of bank equity is higher than the cost of debt, due to factors such as taxes (Modigliani and Miller 1958, 1963), bankruptcy costs (Myers 1977), asymmetric information (Myers and Majluf 1984), as well as a cost of contingencies and limited financial market participation (Barberis et al. 2006; Guiso et al. 2008; Guiso and Sodini 2013), the magnitude of the equity premium is debated (Admati et al. 2013). See Allen et al. (2015) for a general equilibrium framework in which the cost of equity endogenously emerges as higher than the cost of deposits.

itive and interior choice of bank capital, $k^* \in (0,1)$. The intuition stems from recognizing that panic runs, which are the result of coordination failures among depositors, destroy welfare. As a result, the use of instruments or tools that can reduce the probability of a run will, all else equal, lead to a Pareto improvement as both depositors and the bank benefit from this reduction. Therefore, as long as the cost of equity capital, ρ_E, is not too much higher than the cost of debt, ρ_D, raising at least some amount of equity will be optimal for the bank. Ceteris paribus, depositors also benefit from capital because (i) it reduces the run threshold θ^*, thus making it more likely that they receive the promised payment at $t = 2$; and (ii) it increases the repayment upon a run. In equilibrium, this then allows the bank to reduce the date 2 repayment, r_2^*, and increases its profits.

The privately optimal choice of capital characterized in Proposition 7 is constrained efficient. That is, a social planner facing depositor runs and the bank's risk management decision would also choose k^*, trading off the higher cost of capital against the benefits in terms of lower run probability. As a result, there is no scope for prudential regulation of capital in our framework. However, the result in Section 4.1 highlights the effect of the level of bank capital on risk management, both on the intensive and extensive margins, pointing to an intricate relationship between capital and risk management. If the planner were to choose both capital and risk management, this interaction should lead to the use of both of these tools for managing financial stability and maximizing welfare.

5.3 Alternative modelling of risk-management: a swap

In the baseline model, we considered the benefit of risk management to be narrowly targeted to address the specific risk to which the bank is exposed to, namely the possibility that the interim value of assets turns out to be low. As a result, our notion of risk management was geared towards raising the interim value from ℓ_L to $\ell_L + z$ in the state with the negative shock. In this section, we broaden the analysis by considering an alternative modelling approach to risk management that resembles a swap. For example, interpreting the shock to the interim asset value as the result of monetary tightening, banks can use interest rate derivatives (e.g., an interest rate swap) to hedge against the drop in the value of their assets. In the context of our model the bank can buy a swap to obtain extra resources z in state L in exchange for a cash payment in state H.[20]

[20]An interest rate swap allows two counterparties to exchange cash payments based on changes in the interest rate. The swap seller receives a net payment when the interest rate falls, while its counterparty, the swap buyer, receives a net payment when the interest rate rises. This simplified modelling abstracts from payments of margin calls when the value of the derivative position worsens. Accounting for those would

This extension serves two purposes. First, it shows that our main result (the bank does not engage in risk management anticipating a severe shock) is robust to allowing for more general, and perhaps also more realistic, tools for managing risk. Second, it micro-founds the assumption of a convex cost of risk management used in the previous sections, showing that this cost structure can arise endogenously with other risk management tools.

For simplicity, we assume that the swap contract is free from counterparty risk and senior to other claims on the bank (see Dasgupta 2004 for a similar assumption). This could be because of collateral set aside for the transaction, which is then exempt from the resources of the bank depositors have access to when running (see, e.g., Ahnert et al. 2019). Also for simplicity, we focus on the symmetric case $p = 1/2$, which makes the design of an actuarially fair swap particularly simple: the bank receives z in state L and pays z in state H, for which it has to liquidate some investment. Importantly, the transfer of resources is independent of whether the bank liquidates any assets (i.e., independent of whether a run occurs) and does not require funding at $t = 0$ beyond setting up the risk management desk at non-pecuniary fixed cost $F \geq 0$, consistent with Bretscher et al. (2018). While all results in this section hold for $F = 0$, we allow a for a positive fixed cost for consistency with previous sections.

Since our focus is on the bank's exposure to and management of risk, we restrict the transfer z such that the total resources available to the bank in state L (upon the negative shock) are never greater than in state H. That is, the negative state remains the less favorable one even after optimally managing risk (or not) by the bank. Formally, this means that the bank's resources in the low state, $\ell_L + z$ in the event of liquidation, are not greater than in the high state, $(\ell_H - z)/\ell_H$. This restriction on z arises naturally for many parameter constellations given the increasing (endogenous) cost of risk management, but we assume it exogenously to make the analysis more tractable.[21]

The analysis proceeds as in the baseline model. First, we characterize depositors' withdrawal decisions. Then, we move to analyze the bank's incentives to manage risk.

Lemma 6. *Depositors withdraw at the interim date in state L and H if the fundamental of the economy θ falls below the cutoffs $\theta^*_{L,S}$ and $\theta^*_{H,S}$, respectively. The threshold $\theta^*_{L,S}$ decreases in z,*

increase the outflow of funds from the counterparty, who is having the net outflow of funds, thus affecting the cost of the swap. Abstracting from these additional considerations biases against our desired result.

[21] This constraint on the transfer of resources from state H to state L also serves to rule out the possibility that the bank is unable to repay depositors in state H even if the realization of the fundamental is most favorable. Hence, the assumption is primarily about preserving intermediation. In practice, larger swap positions are prohibited by financial regulators: to reduce the risk of under-capitalization stemming from complex or concentrated derivatives exposures, the Basel III reforms require that all material risk exposures—including contingent derivatives positions—be fully captured by capital requirements. See e.g., BCBS (2017).

while $\theta^*_{H,S}$ increases in it.

Proof: See Online Appendix A.15. □

Having characterized depositors' withdrawal decisions, expected bank profits are:

$$\Pi_s(z) \equiv \frac{1}{2}\int_{\theta^*_{L,S}}^{1} [R\theta + z - (1-k)r_2]d\theta + \frac{1}{2}\int_{\theta^*_{H,S}}^{1} \left[R\theta\left(1 - \frac{z}{\ell_H}\right) - (1-k)r_2\right]d\theta - F\mathbb{1}_{\{z>0\}}, \quad (11)$$

where the first term captures the profits the bank accrues in state L, $\Pi_{S,L}$, while the second one represents the profits accrued in state H, $\Pi_{S,H}$. The effect of the risk management is twofold. First, it affects the bank's exposure to runs (via changes in both failure thresholds). Second, it affects the amount of profits the bank accrues conditional on surviving a run. Compared to the baseline model, the effect on the failure threshold in the high state and the direct effects on profitability are novel channels.

The right panel in Figure 3 illustrates the differential effect of z on the bank's profits, which is driven by its differential effect on the failure thresholds established in Lemma 6. The figure shows the case where the bank finds it optimal to engage in some risk management. Entering into the swap contract benefits the bank in state L, but entails a cost in state H. The figure illustrates that the magnitude of benefits and costs, as captured by the different slope of the two curves, changes with the risk management effort. When effort is small, a marginal change in z leads to an increase in state L profits larger than the decrease in profits in state H, so the bank engages in risk management. On the contrary, when z is high, an increase in z has a stronger impact on the profits in state H than in state L, so that the endogenous cost of additional risk management at that point exceeds its benefits.

The next proposition proves that, as in the baseline model, the bank does not engage in risk management when the shock to the interim asset is sufficiently severe. The formal proof relies on the condition that ℓ_H is low enough (the precise condition is stated in the appendix). While this sufficient condition may be seen as restrictive, numerical examples (including the one reported here) show that the result arises much more generally.

Proposition 8. *For $F \geq 0$ and ℓ_H low enough, the bank does not do any risk management when the interim asset value falls below some cutoff $\widetilde{\ell}_{L,S} > 0$.*

Proof. See Online Appendix A.16 □

Proposition 8 confirms that our results on the lack of risk management incentives when the shock to the interim asset value becomes severe, illustrated in the left panel of Figure 3, is robust to allowing for more general tools of risk management with an endogenous cost. Consistent with the baseline model, the choice of a positive level of risk management, denoted as $z^* \equiv \widehat{z}_S$, decreases in the interim value ℓ_L and drops to zero when the shock to the interim value is severe. Additional features that are likely important in practice, such as margin calls that would rise the cost of risk management further, would reinforce the result.

Risk management choice and expected bank profits

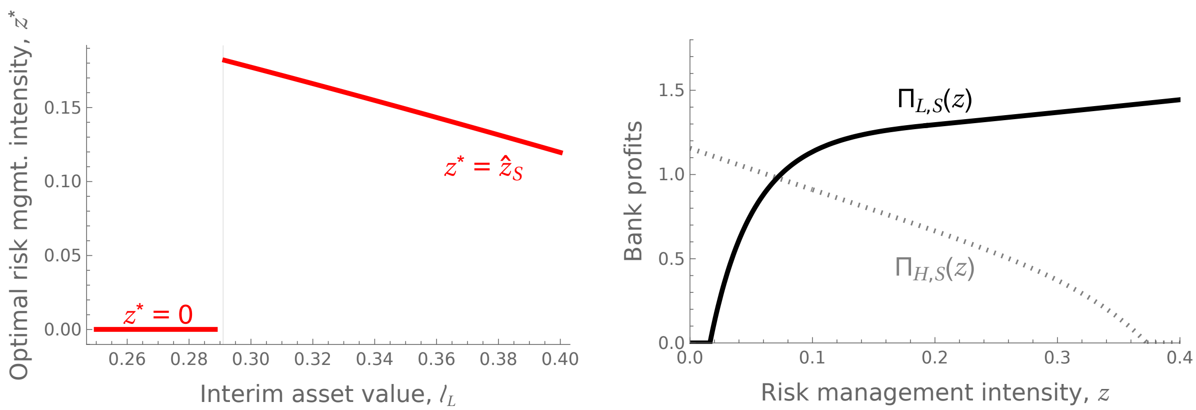

Figure 3: The left panel shows the equilibrium level of risk management z^* as a function of the realized interim value ℓ_L. The right panel shows bank profits $\Pi_{L,S}$ (solid line) and $\Pi_{H,S}$ (dotted line) as a function of z. Parameters: $R = 5$, $k = 1/5$, $\ell_L = 2/5$, $\ell_H = 9/10$, $r_2 = 2$ and $F = 2/5$.

To complete the analysis, we establish in Proposition 9 that the endogenous cost of risk management, which in this setup is given by the decline in bank profits in the high state as a result of the transfer z, shares similar characteristics to the reduced-form cost function we assumed for the baseline analysis. In particular, we demonstrate that the cost of taking the swap, denoted by C, is generally convex. While the formal result below uses a (somewhat restrictive) sufficient condition related to the interim asset value in the high state, numerical examples show that this convexity holds more broadly and arises in all examples we have constructed.

Proposition 9. *If $\ell_H \leq (1-k)/2$, then the endogenous cost of risk management is convex, $d^2C/dz^2 > 0$.*

Proof: See Online Appendix A.17. □

5.4 Credit risk management

So far, we focused only on liquidity risk and established a zero risk management result. A natural question is to what extent our results extend to other sources of risk, and in particular to credit risk, a primary type of risk that banks are regularly exposed to. While our framework up to now has endeavored to isolate the effect of shocks to only financial stability, here we allow for shocks that directly affect the long-term return of the bank's project and hence constitute credit risk. Specifically, suppose now that $\ell \in (0,1)$ is a constant and a credit risk shock is represented by the long-term return R taking one of two possible values, R_L or R_H, with p as the probability of the low state. Credit risk management (CRM) is again a state-contingent tool that improves the payoff in the low state from R_L to $R_L + z$. For simplicity, the bank's cost structure is as in the main model.

A shock to the bank's return (i.e., lower values of R_L, which represent greater credit risk) increases bank fragility. That is, the probability of a depositor run increases, as we show in the proof of the result below. Formally, $\theta^*(R_L)$ decreases in R_L, paralleling the effect of shocks to the interim asset value, ℓ_L. As in the the baseline model, the key to our result is that a shock increases bank fragility (even after accounting for the optimal risk management effort). Let z^*_{CRM} be the bank's optimal choice of (credit) risk management.

Proposition 10. *For all $c > \widehat{c}_{CRM}$ (defined in the appendix), there exists a threshold value $\widetilde{R}_L > 0$ such that the bank stops doing risk management when R_L falls below $\widetilde{R}_L$: $z^*_{CRM} = 0$ for all $R_L \leq \widetilde{R}_L$.*

Proof: See Online Appendix A.18. □

This result mirrors the result in Proposition 2, showing that zero risk management also arises when credit risk is the main source of risk. Thus, our main mechanism naturally extends to other sources of risk, including to settings where the possible shocks directly affect the value of the project, rather than only indirectly through its effect on financial stability. Analogous to our baseline model, the bank's expected profits in the low state are reduced, eventually resulting in zero risk management. The condition on the variable cost parameter c is also the analog of the one included in Proposition 2, guaranteeing that the bank cannot avoid a run when the shock is at its most extreme value, that is for $R_L \to 0$.

5.5 Constrained inefficiency

In this section we compare the solution to a constrained-planner problem to the one obtained privately by a bank. Our focus is on the main model analyzed in Section 3 with the shock to the interim asset value and an exogenous cost structure of risk management effort. This analysis establishes that the private incentives for risk management effort are insufficient and that our main result is indeed one of a *failure* in risk management.

The key friction that the planner can resolve is that it can commit to a risk management choice when raising deposit funding. This allows the planner P to internalize how the deposit rate, r_2, demanded by consumers at the funding stage depends on the committed risk management effort, z_P. By contrast, the bank has no such commitment since risk management actions are not verifiable. Formally, we consider a constrained planner who maximizes utilitarian welfare and takes the incomplete information in the economy and, in particular, the depositors' withdrawal choices as given. Thus, the planner takes the failure threshold θ^* specified in Lemma 1 as given, as does the bank. We focus again on the case where bank leverage is relatively high for the same reason as before: $\ell_H \leq 1-k$, so there are panic runs. We also assume that the participation constraint of investors binds, which pins down the equilibrium deposit rate. We have the following results.

Proposition 11. *There are risk management failures along two dimensions:*

(i) The planner engages in risk management for a larger range of parameters, $\widehat{F}_P > \widehat{F}$.

(ii) Along the intensive margin, the planner can increase welfare by commiting to a higher level of risk management than the bank, $\widehat{z}_P > \widehat{z}$.

Proof: See Online Appendix A.19. □

The intuition for these results is as follows. The commitment power of the planner implies that the deposit rates are responsive to future risk management choices, reducing the funding cost of the bank. On the intensive margin, i.e. when the bank engages in risk management, the planner engages in more risk management than the bank. On the extensive margin, it also sets up a risk management desk for a larger range of fixed costs. In this sense, the privately optimal risk management failures are *excessive*. A direct implication of these results is that the bank is excessively fragile (because it engages in too little risk management), which is socially costly.

6 Conclusion

Runs on banks have been pervasive historically, and recent episodes demonstrate that they continue to be a concern for bank shareholders, depositors, and regulators. Moreover, the failure of a number of financial institutions in 2023 proved once again that the conditions at which banks can obtain funds at short notice to meet withdrawals play a crucial role for their stability. Specifically, negative shocks to the interim value of bank assets, even if anticipated, can lead to greater instability and trigger a bank run. These channels of fragility and their implications for risk management have been largely ignored so far.

Incorporating such risk into a canonical global-games model of bank runs, we consider the question of what incentives a bank may have to hedge these risks. We show that even if a bank's marginal incentives to manage risk may increase as the severity of the possible negative shock increases, the bank's profitability nevertheless declines. For sufficiently large negative shocks, the bank may not find it profitable to set up any risk management operations. As a result, risk management operations are abandoned (or never entertained in the first place) precisely when managing those risks would have the largest impact in terms of improving financial stability.

Our framework helps to understand how private sector incentives to manage risk may differ from social incentives, and which bank characteristics play a role in shaping those incentives (e.g., bank size, capitalization, stability of the funding base). Hence, our analysis also helps understand the benefit of different mitigating tools available either to the banker or to a regulator. Because the model endogenizes both the extensive margin (whether a risk-management desk is set up) and the intensive margin (how much hedging effort is exerted), it clarifies how non-contingent instruments such as bank capital and deposit-insurance coverage reshape bankers' trade-offs and the regulator's toolkit.

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A Appendix

A.1 Proof of Lemma 1

Low interim asset value relative to depositor base: $\ell < 1-k$. The proof builds closely on Carletti et al. (2023b), who adapt the proof in Goldstein and Pauzner (2005) to establish the existence and uniqueness of a monotone equilibrium when banks maximize profits.

For $\theta \in (\underline{\theta}, \overline{\theta})$, depositor withdrawal choices determine whether the bank is illiquid or insolvent. When the liquidation proceeds at $t = 1$ are insufficient to meet withdrawals, $n > \overline{n} = \ell/(1-k) < 1$, the bank is illiquid. Otherwise, it continues to operate until $t = 2$. If the bank cannot meet remaining withdrawals, $n > \widehat{n}$, it is bankrupt due to insolvency at $t = 2$, where $\widehat{n}$ arises from:

$$R\theta\left(1 - \frac{(1-k)n}{\ell}\right) - (1-k)(1-n)r_2 = 0 \qquad \Rightarrow \widehat{n} \equiv \frac{R\theta - (1-k)r_2}{\frac{R\theta(1-k)}{\ell} - (1-k)r_2} < 1. \tag{12}$$

The first term of the solvency condition in Equation (12) on the left is the return on the part of the project not liquidated at $t = 1$ and the second term represents remaining withdrawals at $t = 2$. Table 2 shows the payoffs of depositors associated with the two withdrawal actions. Depositors' payoffs depend on the realized economic fundamental θ and the proportion n of depositors withdrawing.

aggregate action *individual action*	$n \leq \hat{n}$ bank is solvent	$\hat{n} < n \leq \overline{n}$ bank is insolvent but liquid at $t = 1$	$n > \overline{n}$ bank is insolvent and illiquid at $t = 1$
Withdraw	1	1	$\ell/(1-k)n$
Don't withdraw	r_2	0	0

Table 2: Payoffs in the withdrawal game for intermediate realized fundamentals, $\theta \in (\underline{\theta}, \overline{\theta})$.

For vanishing noise, we can write the indifference condition of the marginal depositor as:

$$\int_0^{\hat{n}} r_2 dn = \int_0^{\overline{n}} dn + \int_{\overline{n}}^1 \frac{\ell}{(1-k)n} dn = \alpha. \tag{13}$$

Using the marginal depositor's expected payoff from withdrawing, $\alpha = \frac{\ell}{1-k}\left[1 - \ln\left(\frac{\ell}{1-k}\right)\right]$, we obtain the fundamental threshold from Lemma 1. Throughout we focus on a high enough deposit rate r_2, for which $r_2 > \alpha(1-k)/\ell$. For lower values of r_2, it is a dominant strategy for depositors to withdraw at time 1, so $\theta^* = \overline{\theta}$. This arises if Condition (13) holds as a strict inequality, which yields: $r_2 < \widehat{r}_2 \equiv \frac{\left(\frac{\overline{\theta}}{\underline{\theta}} - \frac{\ell}{1-k}\right)\left(1 - \ln\left(\frac{\ell}{1-k}\right)\right)}{\frac{\overline{\theta}}{\underline{\theta}} - 1}$, with $\widehat{r}_2 > \frac{\alpha(1-k)}{\ell}$.

High interim value relative to depositor base: $\ell \geq 1-k$**.** For $\ell = 1-k$, we have $\bar{n} = 1$ and $\alpha = 1$. Furthermore, the bank's solvency condition, characterized in (12), can be rearranged to $[R\theta - (1-k)r_2](1-n) = 0$, which is positive for $\theta > \underline{\theta}$ and negative for $\theta < \underline{\theta}$. It follows that a depositor's expected payoff from withdrawing at $t=1$ and $t=2$ is independent of n, and so the relevant bank failure threshold is $\underline{\theta}$ because $r_2 > 1$. Intuitively, the withdrawals do not impose a loss on other depositors when $\ell = 1-k$. The fundamental-run threshold in (5) decreases in bank capital, $\frac{\partial \underline{\theta}}{\partial k} = -r_2/R < 0$. Hence, the solvency condition in (12) becomes less binding for any n when ℓ exceeds $1-k$. Intuitively, a high interim value relative to the deposit base means that withdrawals by other depositors increases a given depositor's incentive *not* to withdraw (strategic substitutability). It follows that, for any $\ell > 1-k$, the relevant threshold is still $\underline{\theta}$. Finally, we have $\partial^2\underline{\theta}/\partial k^2 = 0$ and $\partial\underline{\theta}/\partial\ell = 0 = \partial^2\underline{\theta}/\partial\ell^2 = \partial^2\underline{\theta}/\partial k\partial\ell$.

Comparative statics of the failure threshold for $\ell < 1-k$**.** Next, we analyze how the bank failure threshold with panics, θ^*, changes with bank capital, the interim value of the bank asset, and both of them. Since ℓ is used to generically denote both $\ell_L + z$ and ℓ_H, and since z and ℓ_L are perfect substitutes, it follows that the comparative statics with respect to ℓ below can be used for the comparative statics with respect to ℓ_L, ℓ_H, and z. The interim value and bank capital only enter into the failure threshold in Equation (6) as their ratio, $\ell/(1-k) \equiv x$. It is easy to see that $dx/d\ell = (1-k)^{-1} > 0$, $dx/dk = \ell(1-k)^{-2} > 0$, and $d^2x/dkd\ell = (1-k)^{-2} > 0$. Using this definition, we can express the marginal depositor's expected payoff from withdrawing as $\alpha = x(1-\ln(x))$, with $d\alpha/dx = -\ln(x) > 0$ because $x < 1$. Inserting the definitions of α and x into (6) yields $\theta^* = \underline{\theta}\frac{r_2 - x(1-\ln(x))}{r_2-(1-\ln(x))} > \underline{\theta}$. Define β as $\theta^* \equiv \underline{\theta}\beta$, so $\beta > 1$ for panic runs. We can write the difference as $\theta^* - \underline{\theta} = \underline{\theta}\frac{(1-x)(1-\ln(x))}{r_2-(1-\ln(x))} = \underline{\theta}\frac{1-x}{\frac{r_2}{1-\ln(x)}-1}$, which decreases in x because both effects of x reduce the left-hand side: $d(\theta^* - \underline{\theta})/dx < 0$. This implies that $d(\theta^* - \underline{\theta})/d\ell < 0$ and $d(\theta^* - \underline{\theta})/dk < 0$. Note that $d\theta^*/d\ell = (d\theta^*/dx)(dx/d\ell)$ because $d\underline{\theta}/d\ell = 0$. Moreover, $d\theta^*/dk = \beta d\underline{\theta}/dk + (d\theta^*/dx)(dx/dk)$. Thus, it is critical to determine the derivative w.r.t. x: $\frac{d\theta^*}{dx} = \underline{\theta}\frac{(1-\ln(x))^2 - r_2(1/x-\ln(x))}{[r_2-(1-\ln(x))]^2} < 0$, because $x < 1$ and $r_2 \geq \widehat{r_2} > \alpha/x$ in any equilibrium. The numerator is negative even at $r_2 = \alpha/x$, and it decreases in r_2. Since $d\theta^*/dx < 0$, we obtain $d\theta^*/dk < 0$ and $d\theta^*/d\ell < 0$.

Turning to second derivatives, note that $d^2\theta^*/d\ell^2 = (d^2\theta^*/dx^2)(dx/d\ell)$ because $d^2x/d\ell^2 = 0$. Thus, $d^2\theta^*/d\ell^2 > 0$ iff $d^2\theta^*/dx^2 > 0$. Moreover, $d^2\theta^*/d\ell dk = (d\theta^*/dx)(d^2x/dkd\ell) + (d^2\theta^*/dx^2)(dx/d\ell)$. Hence, a sufficient condition for $d^2\theta^*/d\ell dk > 0$ is also $d^2\theta^*/dx^2 > 0$. Next, $\frac{d^2\theta^*}{dx^2} = \frac{\underline{\theta}r_2}{x^2[r_2-(1-\ln(x))]^3}(2(1-x) + (1+x)[r_2 - (1-\ln(x))]) > 0$. Hence, $d^2\theta^*/d\ell^2 > 0$ and $d^2\theta^*/d\ell dk > 0$. To obtain the sign of $d^2\theta^*/dk^2$, we differentiate θ^*:

$$\frac{d\theta^*}{dk} = \frac{r_2^2(x - r_2 - \ln(x))}{R(r_2-(1-\ln(x)))^2} < 0, \quad \frac{d^2\theta^*}{dk^2} = \frac{d\theta^*}{dk}\frac{2(r_2-(x-\ln(x))) + (x-1)(r_2-(1-\ln(x)))}{(1-k)(x-r_2-\ln(x))(r_2-(1-\ln(x)))} > 0. \tag{14}$$

A.2 Proof of Proposition 1

The stated comparative statics of $\widehat{z}$, the interior solution to (8), with respect to ℓ_L, k, and p_ℓ are obtained using the implicit function theorem. The second-order conditions are:

$$\frac{d^2\Pi}{dz^2} = -pR\left(\frac{d\theta_L^*}{dz}\right)^2 - p[R\theta_L^* - (1-k)r_2]\frac{d^2\theta_L^*}{dz^2} - c < 0, \quad \frac{d^2\Pi}{dzd\ell_L} = -pR\frac{d\theta_L^*}{dz}\frac{d\theta_L^*}{d\ell_L} - p[R\theta_L^* - (1-k)r_2]\frac{d^2\theta_L^*}{dzd\ell_L} < 0$$

$$\frac{d^2\Pi}{dzdk} = -p\left[R\frac{d\theta_L^*}{dk} + r_2\right]\frac{d\theta_L^*}{dz} - p[R\theta_L^* - (1-k)r_2]\frac{d^2\theta_L^*}{dzdk} < 0, \qquad \frac{d^2\Pi}{dzdp} = [R\theta_L^* - (1-k)r_2]\left(-\frac{d\theta_L^*}{dz}\right) > 0,$$

where we used $d^2\theta_L^*/dzdk = d^2\theta_L^*/d\ell dk > 0$ as shown in the proof of Lemma 1 and $Rd\theta_L^*/dk + r_2 < 0$. To see this, we use the expression for $d\theta_L^*/dk$ in (14) and, after a manipulation, we obtain:

$$R\frac{d\theta_L^*}{dk} + r_2 = -r_2\left[\frac{r_2}{r_2-(1-ln(x))}\frac{r_2-(x-ln(x))}{r_2-(1-ln(x))} - 1\right] < 0, \tag{15}$$

because $\frac{r_2}{r_2-(1-ln(x))} > 1$ and $x = \frac{\ell}{1-k} < 1$ so that also $\frac{r_2-(x-ln(x))}{r_2-(1-ln(x))} > 1$. Therefore, $d\widehat{z}/d\ell_L < 0$ $d\widehat{z}/dk < 0$, $d\widehat{z}/dc < 0$ and $d\widehat{z}/dp > 0$ and the proposition follows. □

A.3 Proof of Lemma 2

Proof by contradiction. Recall that $d\widehat{z}/d\ell_L < 0$ from Proposition 1 and $\partial\theta_L^*/\partial z \equiv \partial\theta_L^*/\partial\ell_L < 0$. Consider two different values for ℓ_L, ℓ_L' and ℓ_L'', with $\ell_L' < \ell_L''$, and suppose that, contrary to the claim, $\theta_L^*(z^*(\ell_L), \ell_L)$ actually *increases* in ℓ_L, so that $\theta_L^*(z'(\ell_L'), \ell_L') < \theta_L^*(z''(\ell_L''), \ell_L'')$, where z' and z'' represent the bank's optimal choices of z in each case. Since $\theta_L^*(\widehat{z}(\ell_L), \ell_L)$ decreases in ℓ_L for a given z, we can only have $\theta_L^*(z'(\ell_L'), \ell_L') < \theta_L^*(z''(\ell_L''), \ell_L'')$ if $z' >> z''$. Moreover, $\ell_L' + z' > \ell_L'' + z''$ must hold. Consider the FOC for the case of $\ell_L = \ell_L''$. To be explicit, we rewrite it with ℓ_L'' and z'':

$$-p\frac{\partial\theta_L^*(z''(\ell_L''), \ell_L'')}{\partial z}\left(R\theta_L^*(z''(\ell_L''), \ell_L'') - (1-k)r_2\right) - cz'' = 0.$$

Suppose the solution to the risk management problem is interior and denote by $\check{z}$ the value of z that would give the same run threshold if instead $\ell_L = \ell_L'$: $\theta_L^*(\check{z}(\ell_L'), \ell_L') = \theta_L^*(z''(\ell_L''), \ell_L'')$, and note that $\check{z} \in (z'', z')$. Now consider the derivative of bank profits, evaluated at $z = \check{z}$ and $\ell_L = \ell_L'$:

$$-p\frac{\partial\theta_L^*(\check{z}(\ell_L'), \ell_L')}{\partial z}\left(R\theta_L^*(\check{z}(\ell_L'), \ell_L') - (1-k)r_2\right) - c\check{z}.$$

Since $\theta_L^*(\check{z}(\ell_L'),\ell_L') = \theta_L^*(z''(\ell_L''),\ell_L'')$ and $\ell_L' + \check{z} = \ell_L'' + z''$, this can be rewritten as:

$$-p\frac{\partial \theta_L^*(z''(\ell_L''),\ell_L'')}{\partial z}\left(R\theta_L^*(z''(\ell_L''),\ell_L'') - (1-k)r_2\right) - c\check{z}.$$

The expression above is the same as the FOC for the case where $\ell_L = \ell_L''$, except for the last term, $c\check{z}$. However, we know that at equilibrium the derivative of bank profits equals zero when evaluated at $z = z''$. Hence, this derivative must be negative for $z = \check{z}$. Hence, contrary to the supposition, it cannot be optimal for the bank to choose a value of z such that $\ell_L' + z' > \ell_L'' + z''$. Then, implies that $\theta_L^*(z'(\ell_L'),\ell_L') < \theta_L^*(z''(\ell_L''),\ell_L'')$ cannot hold. □

A.4 Proof of Proposition 2

Denote ast $\widehat{z}(\ell,c)$ the interior solution to the first-order condition in (8) for a given interim asset interim value ℓ and variable cost parameter c. This solution exists whenever the run threshold is responsive to risk management, that is when $\theta^*(\widehat{z},\ell_L) < \overline{\theta}$. Conversely, risk management becomes ineffective at altering the run threshold for some $z \geq 0$ when $\theta^*(\widehat{z},\ell_L) = \overline{\theta}$, resulting in a corner solution of $z = 0$. The proof considers both possibilities.

We begin the proof by establishing some preliminary results in the following two Lemmas. Lemma 7 builds on Lemma 2 and characterizes expected bank profits as the interim asset value changes. Lemma 8 describes the solutions to Equation (8) and defines the variable cost threshold $\widehat{c}$ in Equation (16), which we assume to be consistent with $\ell_L + \widehat{z} < \ell_H$. Note that $c > \widehat{c}$ is a sufficient condition for the cutoff $\widetilde{\ell_L}$ to be strictly positive.

Lemma 7. *For all ℓ_L such that $\theta_L^*(\widehat{z},\ell_L) < \overline{\theta}$, the bank's expected profits $\Pi(\widehat{z},\ell_L)$ increase in ℓ_L.*

Proof. We prove Lemma 7 by using the Envelope theorem. Denote as $\Pi = \Pi(\widehat{z},\ell_L)$ the bank's expected profits evaluated at the $\widehat{z}$ obtained from the bank's FOC with respect to z:

$$\Pi(\widehat{z},\ell_L) \quad = \quad (1-p)\int_{\theta_H^*}^{1}(R\theta - (1-k)r_2)\,d\theta + p\int_{\theta_L^*(\widehat{z},\ell_L)}^{1}(R\theta - (1-k)r_2)\,d\theta - \frac{c}{2}\widehat{z}^2 - F\mathbb{1}_{\{\widehat{z}>0\}}.$$

We can compute $\frac{d\Pi(\widehat{z},\ell_L)}{d\ell_L} = \frac{\partial\Pi}{\partial z}\frac{d\widehat{z}}{d\ell_L} + \frac{\partial\Pi}{\partial\ell_L}$, where we use $\partial\Pi/\partial z = 0$ from the FOC with respect to z for an interior solution. The overall sign then just equals the sign of $\partial\Pi/\partial\ell_L$, where $\frac{d\Pi}{d\ell_L} = \frac{\partial\Pi}{\partial\ell_L} = -p\frac{\partial\theta_L^*(\widehat{z})}{\partial\ell_L}\left(R\theta_L^*(\widehat{z}) - (1-k)r_2\right) > 0$. Expected bank profits increase in ℓ_L.
□

Lemma 8. *Consider a cutoff $\widehat{c}$ for the variable cost parameter, defined as the positive and finite* *value that solves the following equation evaluated at $z = \overline{\ell}$ and $\ell_L \to 0$:*

$$p[R\theta_L^* - (1-k)r_2]\left(-\frac{d\theta_L^*}{dz}\right) \equiv \widehat{c}\overline{\ell}, \tag{16}$$

where the interim asset value threshold $\overline{\ell}$ solves $\theta_L^*(0,\overline{\ell}) = \overline{\theta}$. For any $c > \widehat{c}$, we have:

1. *for $\ell_L > \overline{\ell}$, the interior solution $\widehat{z}(\ell_L, c) > 0$ is the only solution to Equation (8);*
2. *for $\ell_L \in (\underline{\ell}(c), \overline{\ell}]$, there are two solutions, $\widehat{z}(\ell_L, c) > 0$ and the corner solution $z = 0$;*
3. *for $\ell_L \leq \underline{\ell}(c)$, the corner solution $z = 0$ is the only solution,*

where the interim asset value threshold $\underline{\ell}(c)$ and the associated $\widehat{z}(\underline{\ell}, c)$ jointly solve:

$$\lim_{\ell_L \to \underline{\ell}}\left(p[R\theta_L^* - (1-k)r_2]\left(-\frac{d\theta_L^*}{dz}\right)|_{\ell_L + \widehat{z}(\ell_L, c)} - c\widehat{z}(\ell_L, c)\right) = 0, \tag{17}$$

where $\theta_L^*(\underline{\ell} + \widehat{z}(\underline{\ell}, c)) = \overline{\theta}$.

Proof. The proof of Lemma 8 builds on the previous lemmas. From Proposition 1, the interior solution to Equation (8) for a given interim asset value and cost parameter, $\widehat{z}(\ell_L, c)$, is continuous and decreasing in ℓ_L and in c. In addition, there can be the corner solution $z = 0$ when risk management becomes ineffective at altering the run threshold for some $z \geq 0$, because $\theta^*(\ell_L) = \overline{\theta} \approx 1$.

The first-order condition has one or two solutions. For $c > \widehat{c}$, where $\widehat{c}$ is defined in Equation (16), we can distinguish between three different cases depending on the level of the interim asset value ℓ_L. For $\ell_L > \overline{\ell}$ the interior solution to the first-order condition, $\widehat{z}(\ell_L, c) > 0$, is the only solution to Equation (8), because the first-order condition is strictly positive when evaluated at $z = 0$. The result follows directly from our specification, which is designed to make risk management as easy as possible: the marginal benefit at $z = 0$ is strictly positive while the marginal cost is zero.

For $\ell_L \in (\underline{\ell}, \overline{\ell}]$ there are two solutions to the first-order condition, the interior solution $\widehat{z}(\ell_L, c) > 0$ and the corner solution $z = 0$, where $\underline{\ell}(c)$ is defined in Equation (17). To see that for any $c > \widehat{c}$ there there exists a unique $\underline{\ell}$ that solves the system, recall that $\widehat{z}(\ell_L, c)$ decreases in c and that $\ell_L + \widehat{z}(\ell_L, c)$ increases in ℓ_L, which means that $\theta_L^*(\ell_L + \widehat{z}(\ell_L, c))$ decreases in ℓ_L, as shown in Lemma 2. Hence, there must be a point where we enter the upper dominance region as ℓ_L decreases, because $\widehat{z}(\ell_L, c) < \overline{\ell}$ for $c > \widehat{c}$. This point is reached at $\ell_L = \underline{\ell}(c) < \overline{\ell}$. Specifically, for $\ell_L > \underline{\ell}$ there exists a sufficiently large $\widehat{z}(\ell_L, c)$ that solves the first-order condition, but there is a discontinuity at $\ell_L = \underline{\ell}$ where the derivative of the run threshold drops to zero and the interior solution ceases to exist. The argument relies on the observation that $d\theta_L^*/dz$ is finite for any ℓ_L. Finally, for $\ell_L \leq \underline{\ell}(c)$ the corner solution $z = 0$ is the only solution to the first-order condition. □

Based on Lemmas 2, 7 and 8, we can prove Proposition 2. From Lemma 8 it immediately follows that for $\ell_L \leq \underline{\ell}(c)$ zero risk management is optimal for any $F \geq 0$ and the bank fails with probability one. Instead, for $\ell_L > \overline{\ell}$, the bank's probability of failure is strictly less than one, and it depends on the level of the fixed cost whether the bank optimally engages in risk management or not. Therefore, we need to compare expected profits with and without risk management to determine the bank's optimal risk management strategy.

We start with a discussion of the intermediate region $\ell_L \in (\underline{\ell}(c), \overline{\ell}]$. It is optimal for the bank to engage in risk management as long as its expected profits upon the negative shock net of the fixed cost F are higher than its expected profits in the absence of risk management: $\Pi(\widehat{z}(\ell_L)|\ell_L) - \Pi(0|\ell_L) \geq 0$. To obtain the cutoff $\widetilde{\ell}_L(F,c)$ in the proposition, denote $\widehat{F}(\widetilde{\ell}_L, c) = F$ as the solution to:

$$F = p \int_{\theta_L^*(\widetilde{\ell}_L + \widehat{z}(\widetilde{\ell}_L, c))}^{\theta_L^*(\widetilde{\ell}_L < \overline{\ell}) = \overline{\theta} \approx 1} [R\theta - (1-k)r_2] d\theta - \frac{c(\widehat{z}(\widetilde{\ell}_L, c))^2}{2}, \tag{18}$$

with $\widetilde{\ell}_L + \widehat{z}(\widetilde{\ell}_L, c) > \overline{\ell}$ and $d\widetilde{\ell}_L/dc < 0$. To see this, recall that $c > 0$ and $\widehat{z}(\ell_L, c) > 0$. Thus, $z^* = 0$ for all $\ell_L < \widetilde{\ell}_L$ and for any $F \geq 0$. The RHS in (18) increases in ℓ_L, so $d\widetilde{\ell}_L/dF > 0$.

Next, we turn to the case $\ell_L > \overline{\ell}$. Now the upper bound of the integral in Equation (18) is $\theta_L^*(\ell_L, 0) < \overline{\theta}$ and we can show by contradiction that there does not exist another interim asset value threshold, say $\widetilde{\ell}_L' > \overline{\ell}$, such that the bank stops doing risk management for all $\ell_L < \widetilde{\ell}_L'$. In other words, the cutoff $\widetilde{\ell}_L$ is unique. The result of Proposition 2 follows. □

A.5 Proof of Corollary 1

Take $\widehat{F}(\ell_L, c)$ from Appendix A.4. It is strictly positive for $\ell_L \in (\widetilde{\ell}_L(0,c), \overline{\ell})$. This cutoff is unique and increases in ℓ_L since both $\widehat{z}(.)$ and $\theta^*(\ell_L, \widehat{z}(.))$ decrease in ℓ_L. The result follows.

A.6 Alternative fundamental and signal distribution

Here, we present an alternative specification that shows that our main result (the bank chooses not to do any risk management when the shock becomes sufficiently severe) need not rely on the upper dominance region. All that is necessary is that the value of risk management decrease as shocks become more extreme, as shown below. The result holds for any arbitrarily small (but strictly positive) fixed cost. As argued before, some fixed cost seems natural for the application and we do not view this as a limitation of our result.

Model. Relative to the baseline model, we make two modifications. First, the fundamental θ at $t = 2$ is drawn from a Beta(δ, δ) distribution with $\delta > 1$, whose density is denoted by h and satisfies $h(0) = h(1) = 0$. Second, depositors' private signals at $t = 1$ are given by:

$$s_i = \theta + \sigma\eta_i, \tag{19}$$

where $\sigma > 0$ is arbitrarily small and η_i is i.i.d. across depositors with density g and zero mean, following Morris and Shin (2003). We assume g is symmetric and satisfies the monotone likelihood ratio property (MLRP). A sufficiently precise private signal ensures a unique cutoff equilibrium in which depositors follow a threshold strategy (Morris and Shin, 2003). Henceforth, we focus on the limit of vanishing private signal, $\sigma \to 0$.

Withdrawal incentives. Letting G denote the CDF of η_i, then for vanishing private signal noise, the marginal depositor receiving s_A expects a fraction $q = \lim_{\sigma\to 0} \Pr(s_i < s_A | \theta = s_A) = G(0) \in (0, 1)$ of other depositors to run (for symmetric noise, $q = 1/2$). The marginal depositor is indifferent precisely when the expected withdrawals equal the bank's resources, $\widehat{n}(\theta_L^*, \ell) = q$. Solving this critical mass condition for θ_L^* yields the equilibrium run threshold:

$$\theta_L^*(\ell) = \frac{(1-k)(1-q)r_2}{R\left(1 - \frac{(1-k)q}{\ell}\right)} \qquad \text{for } \ell > (1-k)q. \tag{20}$$

Note that we recover the run threshold from the main text for $q = \alpha/r_2$. Similarly, one can see that when $\ell \leq (1-k)q$, $\theta_L^*(0, \ell_L) = \overline{\theta} \approx 1$ and, hence, there exists a $\underline{\ell}_A > (1-k)q$ that solves $\theta_L^*(0, \ell_L) = \overline{\theta}$. For all $\ell > \underline{\ell}_A$, $\theta_L^*(0, \ell_L)$ is strictly below $\overline{\theta}$. Moreover, observe that the prior distribution of θ, H, does not affect the equilibrium run thresholds. It affects, however, the marginal benefit of risk management, as we will show below. Note that the dependency of the run threshold on bank capital and the interim asset value are:

$$\frac{\partial \theta_L^*}{\partial \ell} = -\frac{q\theta_L^*}{\ell - (1-k)q}\frac{1-k}{\ell} < 0; \qquad \frac{\partial \theta_L^*}{\partial k} = -\frac{\frac{\partial \theta_L^*}{\partial \ell}}{\theta_L^*} < 0;$$

$$\frac{\partial^2 \theta_L^*}{\partial \ell_L^2} = \frac{\partial^2 \theta_L^*}{\partial \ell_L \partial z} = \frac{\partial^2 \theta_L^*}{\partial z^2} = \frac{\left(\frac{\partial \theta_L^*}{\partial \ell}\right)^2}{\theta_L^*} + \frac{\theta_L^*(1-k)q(2\ell - (1-k)q)}{(\ell^2 - (1-k)q\ell)^2} > 0.$$

The qualitative results are identical to our baseline model. Furthermore, we can show that the sensitivity of the run threshold to changes in the interim asset value is bounded and finite. First, recall that $\partial\theta_L^*/\partial \ell_L = 0$ for $\ell_L \leq \underline{\ell}_A$. Second, we calculate the derivative at the point where ℓ_L decreases toward $\underline{\ell}_A$, $\left.\frac{\partial\theta_L^*}{\partial z}\right|_{\ell_L\to\underline{\ell}_A,z=0} = -\frac{Rq}{(1-q)r_2\underline{\ell}_A^2} > -\infty$.

Risk management incentives. The FOC of the risk management problem is

$$\frac{d\Pi}{dz} = \underbrace{p[R\theta_L^* - (1-k)r_2]h(\theta_L^*)\left(-\frac{d\theta_L^*}{dz}\right)}_{\equiv\Omega(\ell_L,z)} - cz = 0, \tag{21}$$

which is the same as in the main text, up to a factor of the density $h(\theta_L^*)$ instead of one in the marginal benefit of risk management. Differentiating (21) with respect to z:

$$\frac{d^2\Pi}{dz^2} = -pR\left(h(\theta_L^*) + \left[\theta_L^* - \frac{(1-k)r_2}{R}\right]\frac{dh(\theta_L^*)}{d\theta_L^*}\right)\left(\frac{d\theta_L^*}{dz}\right)^2 - p[R\theta_L^* - (1-k)r_2]h(\theta_L^*)\frac{d^2\theta_L^*}{dz^2} - c, \tag{22}$$

which is negative for a sufficiently large c, which we assume henceforth. So an interior solution for risk-management exists, as in the baseline model, and we again denote it as $\widehat{z}$.

We can now establish a parallel result as in the main text for when the bank chooses to do zero risk management. This happens when $\Pi(\widehat{z}(\ell_L),\ell_L) - \Pi(0,\ell_L) \le 0$, which simplifies to:

$$p\int_{\theta_L^*(\widehat{z}(\ell_L),\ell_L)}^{\theta_L^*(0,\ell_L)} [R\theta - (1-k)r_2]\,h(\theta) - c\frac{(\widehat{z}(\ell_L))^2}{2} \le F. \tag{23}$$

The expression in (23) can be rearranged as,

$$p\int_0^{\widehat{z}}\left[-\frac{\partial\theta_L^*(z,\ell_L)}{\partial z}(R\theta^*(z,\ell_L) - (1-k)r_2)h(\theta^*(z,\ell_L)) - cz\right]dz \le F. \tag{24}$$

Denote the LHS in (24) as $\Delta\Pi$. Differentiating it with respect to ℓ_L, we obtain:

$$\frac{d\Delta\Pi}{d\ell_L} = p\frac{\partial\widehat{z}}{\partial\ell_L}\left[\Omega(\widehat{z},\ell_L) - c\widehat{z}\right] - p\int_0^{\widehat{z}}\left[\frac{\partial^2\theta_L^*}{\partial z\partial\ell_L}\Phi h(\theta_L^*) + \frac{\partial\theta_L^*}{\partial z}R\frac{\partial\theta_L^*}{\partial\ell_L}h(\theta_L^*) + \frac{\partial\theta_L^*}{\partial z}h'(\theta_L^*)\frac{\partial\theta_L^*}{\partial\ell_L}\Phi\right]dz,$$

where $\Phi = R\theta_L^* - (1-k)r_2$, $\frac{\partial\theta_L^*}{\partial z} = \frac{\partial\theta_L^*}{\partial\ell_L} < 0$ and $\frac{\partial^2\theta_L^*}{\partial z\partial\ell_L} > 0$. Note that the first term is zero from the Envelope Theorem. Hence, the sign depends only on the second term. With the distribution considered, it is easy to see that $h'(\theta_L^*) < 0$ if $\theta_L^* \in (1/2,1)$ and positive otherwise. It follows immediately that the term in the square bracket is positive when $\theta_L^* < 1/2$, thus making $\frac{d\Delta\Pi}{d\ell_L} < 0$. Hence, a necessary condition for $\frac{d\Delta\Pi}{d\ell_L} > 0$ is that the shock is sufficiently severe so that $\theta_L^* > 1/2$. In other words, a necessary condition is that $\ell_L < \ell_{L,1/2}$, where $\ell_{L,1/2}$ is the value of ℓ_L at which $\theta_L^* = 1/2$. Consider the case $\ell_L < \ell_{L,1/2}$.

Using the expression derived earlier for the derivatives of the run thresholds, we can rewrite the integrand as $\frac{\partial^2 \theta_L^*}{\partial z^2}\left\{\frac{1}{\theta_L^*}+\frac{\theta_L^*(1-k)q(2\ell_L-(1-k)q)}{(\ell_L^2-(1-k)q\ell_L)^2}h(\theta^*)\Phi+Rh(\theta^*)+\Phi h'(\theta^*)\right\}$. The above is negative, as desired, if $-\frac{h'(\theta_L^*)}{h(\theta_L^*)}>\frac{1}{\theta_L^*}+\frac{\theta_L^*(1-k)q(2\ell_L-(1-k)q)}{(\ell_L^2-(1-k)q\ell_L)^2}+\frac{R}{\Phi}$. The ratio on the LHS simplifies to $(\delta-1)\frac{1-2\theta_L^*}{\theta_L^*(1-\theta_L^*)}$ so that we can rewrite the inequality above as:

$$\delta > 1+\left[\frac{1}{\theta_L^*}+\frac{\theta_L^*(1-k)q(2\ell_L-(1-k)q)}{(\ell_L^2-(1-k)q\ell_L)^2}+\frac{R}{\Phi}\right]\frac{\theta_L^*(1-\theta_L^*)}{1-2\theta_L^*}, \tag{25}$$

with the RHS being independent of δ and only a function of ℓ_L. It can be seen immediately that as $\ell_L \to \underline{\ell}_A$, so that $\theta_L^* \to 1$, the inequality above holds for any $\delta > 1$. Combining this with the fact that the slope of the profit gains (LHS in (24)) is positive when $\ell_L = \ell_{L,1/2}$, it must be the case that it becomes negative for some $\ell_L \in \{\underline{\ell}_A, \ell_{L,1/2}\}$, for a sufficiently large δ. In other words, the inequality above holds also for a larger ℓ_L so that $\theta_L^* < \overline{\theta} < 1$. This implies that, given a fixed cost F for which the bank may find it optimal to engage in risk management for smaller shocks to ℓ_L, the bank will instead find it optimal to choose $z^* = 0$ for ℓ_L small enough, but still strictly greater than $\underline{\ell}_A$.

The numerical example in Figure 4 helps to build intuition. The top panel shows the two forces that are at play as the severity of the negative shock increases (lower values of ℓ_L): *(1)* the run threshold becomes more sensitive to the interim asset value, as in the main text; and *(2)* the probability density gets smaller as the run threshold moves into the tail. Taken together, the net benefit of doing risk management can be hump-shaped: it first increases in the severity of the shock and then decreases in it. Importantly, the benefit to risk management upon a large negative shock is smaller than the cost required to move the run thresholds away from the tail. We find that *(a)* for a given level of fixed cost $F > 0$, the bank stops doing risk management when the shock is severe, and *(b)* that this result does not rely on the insensitivity of the run threshold in the upper dominance region. Moreover, the sufficient condition in (25) makes clear that a larger value of δ, which makes the tails thinner, allows the risk-management result to occur for a lower fixed cost.

Risk management choice

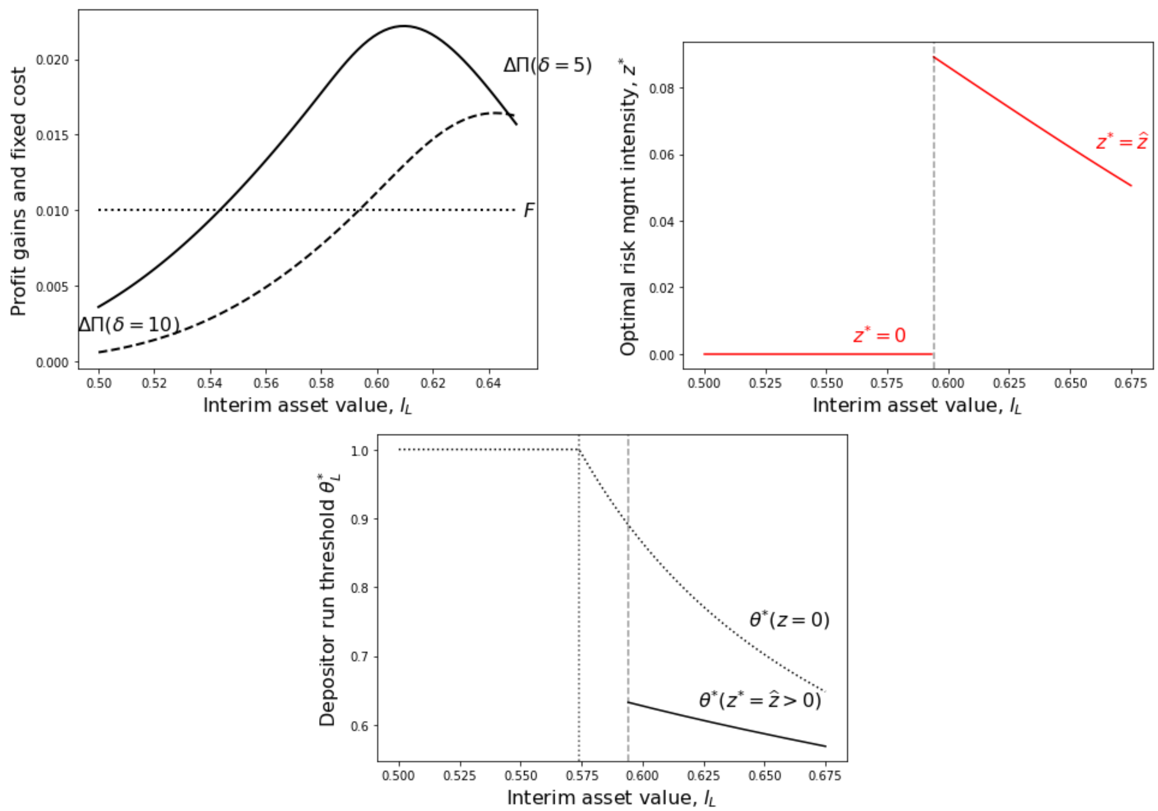

Figure 4: The the left-hand side of Equation (23) against the fixed cost F for two levels of the distribution parameter, $\delta \in \{5, 10\}$. For the prior with thinner tails ($\delta = 10$), the benefit of risk management starts decreasing in the size of the negative shock, and falls short of the fixed cost, for higher levels of ℓ_L, compared to the prior with fatter tails ($\delta = 5$). The left panel shows the risk management intensity z^* as a function of ℓ_L (red solid line). The right panel shows the equilibrium run threshold θ_L^* as a function of ℓ_L for $z = 0$ (dashed line) as well as $z^* = \hat{z} > 0$ (solid line). Parameters: $R = 5/2$, $p = 1/4$, $k = 1/10$, $c = 70$, $F = 1/100$, $p = 1/4$ $\delta = 10$ and $r_2 = 6/5$.

A.7 Proof of Proposition 3

Focus on the range $\ell < \bar{\ell}$ and consider the cutoff $\widehat{F}$ corresponding to the solution to (18) in Proposition 2. Taking the derivative with respect to bank capital gives:

$$\begin{aligned}\frac{d\widehat{F}}{dk} &= -p\left(\underbrace{\frac{\partial\theta_L^*}{\partial k}}_{<0}+\underbrace{\frac{\partial\theta_L^*}{\partial z}}_{<0}\underbrace{\frac{dz^*}{dk}}_{<0}\right)[R\theta_L^*-(1-k)r_2]+p\int_{\theta_L^*}^{1} r_2\,d\theta - cz\underbrace{\frac{dz^*}{dk}}_{<0}\\ &= -p\frac{\partial\theta_L^*}{\partial k}[R\theta_L^*-(1-k)r_2]+\frac{dz^*}{dk}\left(\underbrace{-p\frac{\partial\theta_L^*}{\partial z}[R\theta_L^*-(1-k)r_2]-cz}_{FOC_z}\right)+p\int_{\theta_L^*}^{1} r_2\,d\theta > 0.\end{aligned}$$

The first term is positive because $\partial\theta_L^*/\partial k < 0$, as is the third term. The second term is zero, as the term in brackets is the same as the FOC for z as given in (8). The result follows. □

A.8 Proof of Lemma 3

This proof is analogous to that of Lemma 1. We derive the thresholds for a generic ℓ, which takes one of two values depending on the realized shock. The lower dominance region is similar to the baseline model (because this bound assumes no withdrawals). However, here we have to average over deposit rates:

$$\underline{\theta}_\sigma \equiv \frac{1-k}{R}[\sigma r_2^I + (1-\sigma) r_2^U] = \frac{1-k}{R}\bar{r}_{2,\sigma}. \tag{26}$$

For any $\theta > \underline{\theta}_\sigma$, it is useful to distinguish between different levels of DI coverage σ:

Case *(i)*: High level of DI coverage, $1 \geq \sigma \geq r_2^U \frac{\frac{1-k}{\ell}-1}{\left(r_2^U-1\right)\frac{1-k}{\ell}} \Leftrightarrow \ell \geq \check{\ell}_\sigma(k)$

Case *(ii)*: Intermediate level of DI coverage, $1-\ell/(1-k) \leq \sigma < r_2^U \frac{\frac{1-k}{\ell}-1}{\left(r_2^U-1\right)\frac{1-k}{\ell}} \Leftrightarrow (1-\sigma)(1-k) \leq \ell < \check{\ell}_\sigma(k)$

Case *(iii)*: Low level of DI coverage, $0 \leq \sigma < 1-\ell/(1-k) \Leftrightarrow \ell \leq (1-\sigma)(1-k)$.

The equilibrium run threshold takes a different functional form for each case, and we consider the three cases in turn. We also formally derive the threshold $\check{\ell}_\sigma$. For low levels of deposit insurance coverage, $0 \leq \sigma < 1-\ell/(1-k)$, the bank may become illiquid at time 1 if a large fraction of uninsured depositors decide to withdraw, leading to *rationing* at the interim date, as in our baseline model. Given a proportion n of uninsured depositors running, the bank becomes insolvent whenever $n > \widehat{n}_\sigma(\theta)$, solves the following equation:

$$R\theta\left(1-\frac{n(1-k)(1-\sigma)}{\ell}\right) - (1-n)(1-k)(1-\sigma) r_2^U - (1-k)\sigma r_2^I = 0, \tag{27}$$

which gives $\widehat{n}_\sigma(\theta) = \frac{R\theta-(1-k)(1-\sigma)r_2^U-\sigma(1-k)r_2^I}{R\theta\frac{(1-k)(1-\sigma)}{\ell}-(1-k)(1-\sigma)r_2^U}$. It is easy to see that (27) decreases in n: $-R\theta\frac{(1-k)(1-\sigma)}{\ell} + (1-k)(1-\sigma) r_2^U < 0 \Leftrightarrow \frac{(1-k)(1-\sigma)}{\ell}\left[-R\theta + \ell r_2^U\right] < 0$, since $\ell < (1-k)$ and $\theta > \underline{\theta}_\sigma$, as we are now considering the intermediate region where $\theta \in \left(\underline{\theta}_\sigma, \overline{\theta}\right)$. Hence, the indifference condition of uninsured depositors is $\int_0^{\widehat{n}_\sigma(\theta)} r_2^U dn = \int_0^{\overline{n}_\sigma} dn + \int_{\overline{n}_\sigma}^1 \frac{\ell}{(1-k)(1-\sigma)n} dn$, where the liquidation needs are insufficient to meet withdrawals by uninsured depositors if $n > \overline{n}_\sigma \equiv \ell/((1-k)(1-\sigma)) > \overline{n}$. Note that $\overline{n}_\sigma \leq 1$ iff $0 \leq \sigma \leq 1-\ell/(1-k)$, which describes the parameter condition for Case *(iii)*, where the level of DI coverage is low.

Let $\alpha_{\sigma,r} \equiv \int_0^{\overline{n}} dn + \int_{\overline{n}}^1 \frac{\ell}{(1-k)(1-\sigma)n} dn$. Following the same steps as in the baseline model, we obtain:

$$r_2^U \widehat{n}_\sigma(\theta) = \alpha_{\sigma,r} \Leftrightarrow \theta^*_{\sigma,r} \equiv \underline{\theta}_\sigma \frac{r_2^U - \frac{r_2^U(1-\sigma)}{\bar{r}_{2,\sigma}}\alpha_{\sigma,r}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_{\sigma,r}}. \tag{28}$$

Since the first term is the fundamental run threshold, we can see that $\theta^*_{\sigma,r}$ is above it because $r_2^U - \frac{r_2^U(1-\sigma)}{\bar{r}_{2,\sigma}}\alpha_{\sigma,r} > r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_{\sigma,r}$, given that $r_2^U/\bar{r}_{2,\sigma} < (1-k)/\ell$ for $\sigma \leq 1 - \ell/(1-k)$. That is, Case *(iii)* features panic runs, as in our baseline model.

Next, we move to Case *(ii)*, where the level of deposit insurance coverage is in the intermediate range. The lower bound of the intermediate range follows from $\bar{n}_\sigma \geq 1$, which implies that the bank is never illiquid at time 1. In other words, there is *no rationing* at the interim date. We continue by deriving the equilibrium run threshold for Case *(ii)* and, thereafter, the upper bound of the intermediate range, which demarks the point when there are only fundamental and no panic runs. For Case *(ii)* the indifference condition of uninsured depositors is $\int_0^{\widehat{n}_\sigma(\theta)} r_2^U d\theta = 1$ because $\bar{n}_\sigma > 1$. Defining $\alpha_{\sigma,nr} \equiv 1$ and following the same steps as before, we obtain $r_2^U \widehat{n}_\sigma(\theta) = \alpha_{\sigma,nr} \Leftrightarrow \theta^*_{\sigma,nr} \equiv \underline{\theta}_\sigma \frac{r_2^U - \frac{r_2^U(1-\sigma)}{\bar{r}_{2,\sigma}}\alpha_{\sigma,nr}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_{\sigma,nr}}$. Also Case (ii) features panic runs despite the absence of rationing at time 1, since $r_2^U - \frac{r_2^U(1-\sigma)}{\bar{r}_{2,\sigma}} > r_2^U - \frac{(1-k)(1-\sigma)}{\ell}$, which holds if and only if the level of DI coverage is below the upper bound of the intermediate range, or equivalently, if the interim value falls below $\check{\ell}$:

$$\sigma < r_2^U \frac{\frac{1-k}{\ell} - 1}{\left(r_2^U - r_2^I\right)\frac{1-k}{\ell}} \Leftrightarrow \ell < \check{\ell}_\sigma(k) \equiv \left((1-\sigma) + \sigma\frac{r_2^I}{r_2^U}\right)(1-k) = \left(1 - \sigma + \frac{\sigma}{r_2^U}\right)(1-k), \tag{29}$$

where we used the result that $r_2^I = 1$. This arises from the run threshold increasing in r_2^I and bank profits if no run occurs decreases in it as well. Hence, the bank chooses the lowest possible value of r_2^I. This, combined with the fact that depositors receive 1 at time 1 when a run occurs, implies that, with $\rho_D = 1$, the lowest possible level is $r_2^I = 1$. We revisit this aspect in Section 5.1.

Finally, in Case *(i)* the level of insurance coverage is in the the upper range such that Inequality (29) holds. Importantly, by rearranging $\sigma \geq \left(\frac{1-k}{\ell} - 1\right) / \left(\left(r_2^U - 1\right)\frac{1-k}{\ell}\right)$ and expressing it as a function of ℓ, we obtain the cutoff $\check{\ell}$ of the proposition. In this range, the indifference condition does not apply for uninsured depositors, since $r_2^U \geq 1$, and they optimally choose to run if and only if $\theta < \underline{\theta}_\sigma$. To see this, observe that $\widehat{n}_\sigma(\theta)|_{\ell=\check{\ell}} = \frac{R\theta - (1-k)(1-\sigma)r_2^U - \sigma(1-k)r_2^I}{R\theta r_2^U \frac{1-\sigma}{\bar{r}_{2,\sigma}} - (1-k)(1-\sigma)r_2^U} = \frac{\bar{r}_{2,\sigma}}{(1-\sigma)r_2^U}$ and $\int_0^{\widehat{n}_\sigma|_{\ell=\check{\ell}}(\theta)} r_2^U d\theta = \frac{\bar{r}_{2,\sigma}}{1-\sigma} > 1$.

To complete the proof we compute $\partial\theta^*_\sigma/\partial\sigma$. Thereafter, we derive additional comparative static results that will be useful in the subsequent analysis. $\partial\theta^*_\sigma/\partial\ell$, $\partial^2\theta^*_\sigma/\partial\ell^2$,

$\partial^2\theta^*_\sigma/\partial\ell\partial\sigma$, $\partial\theta^*_\sigma/\partial r^I_2$ and $\partial\theta^*_\sigma/\partial r^U_2$. Differentiating (28) with respect to σ, we obtain:

$$\begin{aligned}\frac{\partial\theta^*_\sigma}{\partial\sigma} = {} & \frac{(1-k)\left(r^I_2 - r^U_2\right)}{R}\frac{r^U_2 - \frac{r^U_2(1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \frac{(1-k)\bar{r}_{2,\sigma}}{R}\frac{-\frac{\partial\alpha_\sigma}{\partial\sigma}\frac{(1-\sigma)r^U_2}{\bar{r}_{2,\sigma}} - \alpha_\sigma\frac{-r^U_2\bar{r}_{2,\sigma} - r^U_2(1-\sigma)\left(r^I_2 - r^U_2\right)}{\bar{r}^2_{2,\sigma}}}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ & - \frac{(1-k)\bar{r}_{2,\sigma}}{R}\frac{\left(r^U_2 - \frac{r^U_2(1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}\right)\left(-\frac{\partial\alpha_\sigma}{\partial\sigma}\frac{(1-k)(1-\sigma)}{\ell} + \alpha_\sigma\frac{1-k}{\ell}\right)}{\left(r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2} < 0,\end{aligned}$$

where $\alpha_\sigma = 1$ and $\frac{\partial\alpha_\sigma}{\partial\sigma} = 0$ in Case *(ii)* and $\frac{\partial\alpha_\sigma}{\partial\sigma} = \int_{\bar{n}}^1 \frac{\ell}{(1-k)(1-\sigma)^2 n}dn > 0$ and $\alpha_\sigma - \frac{\partial\alpha_\sigma}{\partial\sigma}(1-\sigma) = \int_0^{\bar{n}_\sigma} dn + \int_{\bar{n}_\sigma}^1 \frac{\ell}{(1-k)(1-\sigma)n}dn - \int_{\bar{n}_\sigma}^1 \frac{\ell}{(1-k)(1-\sigma)n}dn = \int_0^{\bar{n}_\sigma} dn > 0$ in Case *(ii)*. Next, the derivative with respect to ℓ is:

$$\frac{\partial\theta^*_\sigma}{\partial\ell} = \frac{(1-k)r^U_2}{R}\frac{-\frac{\partial\alpha_\sigma}{\partial\ell}(1-\sigma)}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} - \theta^*_\sigma\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} + \frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} < 0,$$

where $\alpha_\sigma = 1$ and $\frac{\partial\alpha_\sigma}{\partial\ell} = 0$ in Case *(ii)* and $\frac{\partial\alpha_\sigma}{\partial\ell} = \int_{\bar{n}_\sigma}^1 \frac{1}{(1-k)(1-\sigma)n}dn > 0$ and $-\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} + \alpha_\sigma\frac{(1-k)(1-\sigma)}{\ell^2} = \frac{(1-k)(1-\sigma)}{\ell^2}\left(-\ell\frac{\partial\alpha_\sigma}{\partial\ell} + \alpha_\sigma\right) = \frac{(1-k)(1-\sigma)}{\ell}\int_0^{\bar{n}_\sigma} dn > 0$ in Case *(ii)*.

The second derivative with respect to ℓ can be derived as:

$$\begin{aligned}\frac{\partial^2\theta^*_\sigma}{\partial\ell^2} = {} & \frac{(1-k)r^U_2(1-\sigma)}{R\left(r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2}\left\{-\frac{\partial^2\alpha_\sigma}{\partial\ell^2}\left(r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right) - \left(\frac{\partial\alpha_\sigma}{\partial\ell}\right)^2\frac{(1-k)(1-\sigma)}{\ell} + \frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma\right\} \\ & - \frac{\partial\theta^*_\sigma}{\partial\ell}\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} + \frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} - \theta^*_\sigma\frac{\frac{\partial^2\alpha_\sigma}{\partial\ell^2}\frac{(1-k)(1-\sigma)}{\ell} - \frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell^2} - 2\alpha_\sigma\frac{(1-k)(1-\sigma)}{\ell^3} + \frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell^2}}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ & + \theta^*_\sigma\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} + \alpha_\sigma\frac{(1-k)(1-\sigma)}{\ell^2}}{\left(r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2}\frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma > 0.\end{aligned}$$

Note that $\frac{\partial^2\alpha_\sigma}{\partial\ell^2} = 0$ in Case *(ii)* and $\frac{\partial^2\alpha_\sigma}{\partial\ell^2} = -\frac{\partial\bar{n}_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{(1-k)(1-\sigma)\ell} < 0$ in Case *(ii)*. The terms in the second line (curly bracket) sum up to a positive for Case *(iii)* and they are zero for Case *(ii)*. The third, forth and fifth lines are positive. Next, we derive the cross-partial as:

$$\begin{aligned}\frac{\partial\theta^*_\sigma}{\partial\ell\partial\sigma} = {} & \frac{(1-k)r^U_2}{R}\frac{-\frac{\partial^2\alpha_\sigma}{\partial\ell\partial\sigma}(1-\sigma) + \frac{\partial\alpha_\sigma}{\partial\ell}}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \frac{(1-k)r^U_2}{R}\frac{-\frac{\partial\alpha_\sigma}{\partial\ell}(1-\sigma)\left(\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} - \alpha_\sigma\frac{(1-k)}{\ell}\right)}{\left(r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2} \\ & - \frac{\partial\theta^*_\sigma}{\partial\ell}\frac{\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} + \frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} - \theta^*_\sigma\frac{\frac{\partial^2\alpha_\sigma}{\partial\ell\partial\sigma}\frac{(1-k)(1-\sigma)}{\ell} - \frac{\partial\alpha_\sigma}{\partial\ell}\frac{1-k}{\ell} - \frac{1-k}{\ell^2}\alpha_\sigma + \frac{(1-k)(1-\sigma)}{\ell^2}\frac{\partial\alpha_\sigma}{\partial\sigma}}{r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} \\ & + \theta^*_\sigma\frac{\left(+\frac{\partial\alpha_\sigma}{\partial\ell}\frac{(1-k)(1-\sigma)}{\ell} + \frac{(1-k)(1-\sigma)}{\ell^2}\alpha_\sigma\right)^2}{\left(r^U_2 - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2} > 0,\end{aligned}$$

where again $\frac{\partial^2\alpha_\sigma}{\partial\ell\partial\sigma} = 0$ in Case *(ii)* and $\frac{\partial^2\alpha_\sigma}{\partial\ell\partial\sigma} = -\frac{\partial\bar{n}_\sigma}{\partial\ell}/(1-\sigma) + \int_{\bar{n}_\sigma}^1 \frac{1}{(1-k)(1-\sigma)^2 n}dn = -\frac{1}{(1-k)(1-\sigma)^2} +$

$\int_{\bar{n}_\sigma}^{1} \frac{1}{(1-k)(1-\sigma)^2 n} dn < 0$ in Case *(iii)*. Recall that $\alpha_\sigma - \frac{\partial \alpha_\sigma}{\partial \sigma}(1-\sigma) = \int_0^{\bar{n}_\sigma} dn > 0$ and also $\frac{(1-k)(1-\sigma)}{\ell^2}\left(-\ell \frac{\partial \alpha_\sigma}{\partial \ell} + \alpha_\sigma\right) = \frac{(1-k)(1-\sigma)}{\ell}\int_0^{\bar{n}_\sigma} dn > 0$, as well as $\ell \frac{\partial^2 \alpha_\sigma}{\partial \ell \partial \sigma} + \frac{\partial \alpha_\sigma}{\partial \sigma} < 0$. Therefore, all terms are positive in Case *(iii)*, while the second, third and forth terms are positive in Case *(iii)*, and the other terms are zero.

Finally, taking the derivative with respect to r_2^I and r_2^U, we obtain:

$$
\begin{aligned}
\frac{\partial \theta_\sigma^*}{\partial r_2^I} &= \frac{\partial \underline{\theta}_\sigma}{\partial r_2^I} \frac{r_2^U - \frac{r_2^U (1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \underline{\theta}_\sigma \frac{\frac{r_2^U(1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}^2}\sigma}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} > 0 \\
\frac{\partial \theta_\sigma^*}{\partial r_2^U} &= \frac{\partial \underline{\theta}_\sigma}{\partial r_2^U} \frac{r_2^U - \frac{r_2^U (1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} + \underline{\theta}_\sigma \frac{1 - \frac{1-\sigma}{\bar{r}_{2,\sigma}}\alpha_\sigma + \frac{r_2^U(1-\sigma)^2\alpha_\sigma}{\bar{r}_{2,\sigma}^2}}{r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma} - \underline{\theta}_\sigma \frac{r_2^U - \frac{r_2^U(1-\sigma)\alpha_\sigma}{\bar{r}_{2,\sigma}}}{\left(r_2^U - \frac{(1-k)(1-\sigma)}{\ell}\alpha_\sigma\right)^2},
\end{aligned}
$$

where the last derivative has again an ambiguous sign. This completes the proof. □

A.9 Proof of Proposition 4

To study risk management incentives, we differentiate $\Pi_\sigma(z; \ell_L, \ell_H)$ w.r.t. z, again ignoring F, to compute the level of risk management that maximizes profits. The FOC is:

$$\frac{d\Pi_\sigma(z; \ell_L, \ell_H)}{dz} = p[R\theta_L^* - (1-k)(\sigma r_2^I + (1-\sigma) r_2^U)]\left(-\frac{d\theta_L^*}{dz}\right) - cz = 0. \tag{30}$$

As in (8), the first term is the marginal benefit of risk management and the second term is the marginal cost. However, the solution to (30), denote as $\hat{z}_\sigma$, is now a function of σ.

We focus on panic runs, $\ell < \check{\ell}_\sigma$. Differentiating the FOC with respect to ℓ_L and σ gives:

$$
\begin{aligned}
\frac{d^2\Pi_\sigma}{dz d\ell_L} &= -p\left(\frac{d^2\theta_L^*}{dz d\ell_L}\right)[R\theta_L^* - (1-k)(\sigma r_2^I + (1-\sigma) r_2^U)] - p\frac{d\theta_L^*}{dz}\frac{d\theta_L^*}{d\ell_L} R < 0 \\
\frac{d^2\Pi_\sigma}{dz d\sigma} &= -p\left(\frac{d^2\theta_L^*}{dz d\sigma}\right)[R\theta_L^* - (1-k)(\sigma r_2^I + (1-\sigma) r_2^U)] - p\frac{d\theta_L^*}{dz}[R\frac{d\theta_L^*}{d\sigma} - (1-k)(r_2^I - r_2^U)] < 0,
\end{aligned}
$$

where $d\theta_L^*/d\ell = d\theta_L^*/dz < 0$, $d^2\theta_L^*/d\ell dz > 0$, $d\theta_L^*/d\sigma < 0$, $d^2\theta_L^*/d\ell d\sigma > 0$ and $R\frac{\partial \theta_L^*}{\partial \sigma} - (1-k)\left(r_2^I - r_2^U\right) < 0$, which together with $d^2\Pi_\sigma/dz^2 < 0$ leads to the results in Proposition 4, concluding the proof. □

A.10 Proof of Proposition 5

We follow the argument of the proof of Proposition 2. Lemmas 2, 7, and 8 continue to hold with minor modifications. Similar to Lemma 8, there exists a positive, finite value of the variable cost parameter $\widehat{c}_\sigma$. Take $c > \widehat{c}_\sigma$ and $\ell_L \to 0$, then $\widehat{z}_\sigma(0,c) < \overline{\ell}_\sigma$ from Lemma 8 and Inequality (31) is violated for any $F \geq 0$. Since $\theta^*_\sigma(\widehat{z}_\sigma(\ell_L), \ell_L)$ decreases in ℓ_L, bank profits increase in ℓ_L. Thus, for $c > \widehat{c}_\sigma$ there exists a strictly positive cutoff value $\widetilde{\ell}_L(F) \in (0, \overline{\ell}_\sigma)$ with associated optimal effort $\widehat{z}_\sigma(\widetilde{\ell}_L(F), c)$ that solves the differential profit condition with equality. The first result in Proposition 5 follows. Consider the role of DI coverage. As in the proof of Proposition 2, $\widehat{F}_\sigma$ solves:

$$\widehat{F}_\sigma = p_\ell \int_{\theta^*_{L,\sigma}(z^*_\sigma)}^1 [R\theta - (1-k)\overline{r}_2]\, d\theta - c\frac{(z^*_\sigma)^2}{2}, \tag{31}$$

where z^*_σ solves the $FOC_z = 0$, as defined in (30). The derivative of the RHS of (31) is:

$$\frac{\partial \widehat{F}_\sigma}{\partial \sigma} = -p_\ell \left(\underbrace{\frac{\partial \theta^*_{L,\sigma}(z^*_\sigma)}{\partial \sigma}}_{<0} + \underbrace{\frac{\partial \theta^*_{L,\sigma}(z^*_\sigma)}{\partial z}}_{<0} \underbrace{\frac{d\widehat{z}_\sigma}{d\sigma}}_{<0} \right) [R\theta^* - (1-k)\overline{r}_2] - p_\ell \int_{\theta^*_{L,\sigma}(z^*_\sigma)}^1 (1-k) \underbrace{(r_2^I - r_2^U)}_{<0} d\theta - cz^*_\sigma \underbrace{\frac{d\widehat{z}_\sigma}{d\sigma}}_{<0},$$

which yields $\frac{\partial \widehat{F}_\sigma}{\partial \sigma} = -p_\ell \frac{\partial \theta^*_{L,\sigma}}{\partial \sigma} [R\theta^* - (1-k)\overline{r}_2] + p_\ell \int_{\theta^*_{L,\sigma}}^1 (1-k)\left(r_2^I - r_2^U\right) d\theta$, because $\partial \theta^*_{L,\sigma}/\partial \sigma < 0$ and the second term is zero because z is chosen optimally. Hence, $d\widehat{F}_\sigma / d\sigma > 0$. □

A.11 Proof of Lemma 4

The bank chooses r_2 so as to maximize its expected profits as given in (4) subject to the participation constraint in Condition (10). Differentiating (4) with respect to r_2, we obtain:

$$FOC = -p_\ell \int_{\theta^*_L}^1 (1-k)\, d\theta - p_\ell \frac{\partial \theta^*_L}{\partial r_2} [R\theta^*_L - (1-k)\, r_2] - (1-p_\ell) \int_{\theta^*_H}^1 (1-k)\, d\theta + (1-p_\ell) \frac{\partial \theta^*_H}{\partial r_2} [R\theta^*_H - (1-k)\, r_2] = 0.$$

The equilibrium is $r_2^* = \max\left\{r_2^\Pi, r_2^V\right\}$, where r_2^Π is the solution to (32), while r_2^V is the solution to (10) holding with equality. We start with r_2^Π and compute $SOC \equiv \partial FOC / \partial r_2$ by differentiating (32) with respect to r_2. Hence, we obtain:

$$\begin{aligned} SOC = {} & +p_\ell (1-k) \frac{\partial \theta^*_L}{\partial r_2} - p_\ell \frac{\partial^2 \theta^*_L}{\partial r_2^2} [R\theta^*_L - (1-k)\, r_2] - p_\ell R \left(\frac{\partial \theta^*_L}{\partial r_2} \right)^2 + p_\ell (1-k) \frac{\partial \theta^*_L}{\partial r_2} + (1-p_\ell)(1-k) \frac{\partial \theta^*_H}{\partial r_2} \\ & -(1-p_\ell) \frac{\partial^2 \theta^*_H}{\partial r_2^2} [R\theta^*_H - (1-k)\, r_2] - (1-p_\ell) R \left(\frac{\partial \theta^*_H}{\partial r_2} \right)^2 + (1-p_\ell)(1-k) \frac{\partial \theta^*_H}{\partial r_2} < 0, \end{aligned}$$

because $\partial\theta_i^*/\partial r_2 < 0$ and $\partial^2\theta_i^*/\partial r_2^2 > 0$. The former derivative was derived in Lemma 1 and can be rearranged as $\frac{1-k}{R}\frac{r_2-\alpha}{r_2-\alpha\frac{(1-k)}{L}} + \frac{\underline{\theta}-\theta_i^*}{r_2-\alpha\frac{(1-k)}{L}}$. This must be negative as otherwise the FOC will be negative and so r_2^{Π} cannot be a solution. The latter derivative equals:

$$\frac{\partial^2\theta_i^*}{\partial r_2^2} = \frac{1-k}{R}\frac{-\alpha+\alpha\frac{(1-k)}{L}}{\left(r_2-\alpha\frac{(1-k)}{L}\right)^2} + \frac{(1-k)}{R}\frac{1}{r_2-\alpha\frac{(1-k)}{L}} - \frac{\partial\theta_i^*}{\partial r_2}\frac{1}{r_2-\alpha\frac{(1-k)}{L}} + \frac{\theta_i^*-\underline{\theta}}{r_2-\alpha\frac{(1-k)}{L}} > 0.$$

for $i=\{L,H\}$. Given $SOC<0$, using the implicit function theorem, the sign of $dr_2^{\Pi}/d\ell_L$ is equal to the sign of $\partial FOC/\partial\ell_L$, which equals:

$$p_\ell\frac{\partial\theta_L^*}{\partial\ell_L}(1-k) - p_\ell\frac{\partial^2\theta_L^*}{\partial r_2\partial\ell_L}\left[R\theta_L^*-(1-k)r_2\right] - p_\ell\frac{\partial\theta_L^*}{\partial r_2}R\frac{\partial\theta_L^*}{\partial\ell_L} < 0,$$

since $\partial\theta_L^*/\partial\ell_L < 0$, $\partial^2\theta_L^*/\partial r_2\partial\ell_L > 0$ and $\partial\theta_L^*/\partial r_2 < 0$, which is a necessary condition for $r_2^* = r_2^{\Pi}$. Thus, $dr_2^*/d\ell_L < 0$ if $r_2 = r_2^{\Pi}$.

Next, consider r_2^V. The effect of a change in ℓ_L on r_2^V can be computed using the IFT $\frac{dr_2^V}{d\ell_L} = -\frac{\partial V/\partial\ell_L}{\partial V/\partial r_2}$. The numerator is always positive; differentiating (10) w.r.t. ℓ_L gives:

$$p_\ell\int_0^{\theta_L^*}\frac{1}{1-k}d\theta - \frac{\partial\theta^*}{\partial\ell_L}\left(r_2-\frac{\ell_L+z}{1-k}\right) > 0.$$

To sign the denominator, we consider separately the case in which $\partial\theta^*/\partial r_2 < 0$ and when $\partial\theta^*/\partial r_2 > 0$. In the former case, we can immediatelly see that if r_2 increases V increases due to both an increase in the repayment if no run occurs and because the run thresholds decreases in r_2. When $\partial\theta^*/\partial r_2 > 0$, it is less straightforward as the increase in the run thresholds decreases the expected payoffs of depositors, which constitutes an opposing effect. We, next, prove that $\partial\theta^*/\partial r_2 > 0$ leads to a contradiction and can, thus, be excluded. Note first that $\partial\theta^*/\partial r_2 > 0$ is incompatible with a slack depositor participation constraint. To see this, observe that if the constraint were slack, then the bank would want to reduce r_2 in order to increase its profits, and would do so until the participation constraint becomes binding, i.e., $V=\rho_D$. Next, along the binding participation constraint, if r_2 is chosen such that $\partial\theta^*/\partial r_2 > 0$, then it must be that depositor expected payoffs are increasing in r_2: $dV/dr_2 > 0$. This is the case because otherwise the bank would instead prefer to reduce r_2 instead, since that would increase its profits, and would make depositors better off, which would be inconsistent with saying that r_2 has been chosen optimally. We arrive at a contradiction, because the depositor participation constraint cannot be slack. A similar argument can be made for changes in ℓ_H. The result in Lemma 4 follows. □

A.12 Proof of Lemma 5

To study the indirect effect via changes in the repayment of depositors, consider the bank's problem:

$$\max_{r_2} p\int_{\theta_L^*(r_2)}^{1}(R\theta-(1-k)r_2)\,d\theta+(1-p)\int_{\theta_H^*(r_2)}^{1}(R\theta-(1-k)r_2)\,d\theta-k\rho_E \;\; s.t.\, V(r_2,\ell_L,\ell_H)\geq\rho_D,$$

where we introduce a modified notation to highlight the dependence of the run thresholds on the repayment to depositors, i.e., $\theta^*_{L,H}(r_2)$. Next, assuming that the depositor participation constraint binds, we can rewrite the bank's problem as:

$$\max_{r_2} p\int_{0}^{\theta_L^*(r_2)}(\ell_L+z)d\theta+p\int_{\theta_L^*(r_2)}^{1}R\theta d\theta+(1-p)\int_{0}^{\theta_H^*(r_2)}\ell_H d\theta+(1-p)\int_{\theta_H^*(r_2)}^{1}R\theta d\theta-(1-k)\rho_D-k\rho_E.$$

Observe that the bank has one policy variable, r_2, which affects the equilibrium run thresholds in both states in a deterministic way. From the above, it is clear that the objective of the bank is to select r_2 so as to minimize the overall run probability.

For a given ℓ_L, we have an optimal solution r_2^* which pins down θ_L^* and θ_H^*. Now suppose ℓ_L decreases to some $\ell_L' < \ell_L$. From Lemma 2 we know that $\ell_L' + (z^*)' < \ell_L + z^*$ and $\theta_L^*(\ell_L' + (z^*)') > \theta_L^*(\ell_L + z^*)$ for a given r_2. Now the depositors' participation constraint would no longer be satisfied, both because ℓ_L went down, but also because all things equal the run risk would increase as well. So r_2 needs to increase in order to satisfy depositors' participation constraint.

Next, we argue that the fall in ℓ_L cannot lead to both θ_L^* and θ_H^* going down in equilibrium, after the adjustment in r_2. First, observe that the bank's profits are unambiguously decreasing in the level of fragility. Second, let $r_2(\ell_L')$ be the level of deposit repayment chosen when ℓ_L falls to ℓ_L'. If both run thresholds were to go down when $\ell_L = \ell_L'$ and $r_2 = r_2(\ell_L')$, then it would be the case that profits evaluated at the original ℓ_L and $r_2 = r_2(\ell_L')$ would be even higher. This means that it would have been optimal for the bank to choose this level of deposit repayment even before the drop in ℓ_L since depositors' participation constraint was for sure satisfied with $r_2 = r_2(\ell_L')$. Since the bank had chosen a lower one, it cannot be that $r_2(\ell_L')$ is associated with a lower level of fragility in both states. Given that θ_H^* does not depend on ℓ_L and $r_2(\ell_L') > r_2(\ell_L)$ from Lemma 4, it must be the case that $\theta_H^*(r_2(\ell_L')) < \theta_H^*(r_2(\ell_L))$ from Lemma 1 . Therefore, we must also have that $\theta_L^*(\ell_L') > \theta_L^*(\ell_L)$, as stated in the lemma. This argument holds taking into account the risk management choice. □

A.13 Proof of Proposition 6

As in the proof of Proposition 2, bank engages in risk management as long as it obtains a net gain in terms of expected profits from it in the event of a negative shock (i.e., when $\ell = \ell_L$). This is given by the differential profit condition in the proof of Proposition 2, which we restate here:

$$p_\ell \int_{\theta_L^*(z,\ell_L)}^{\theta_L^*(0,\ell_L)} \left[R\theta - (1-k)\, r_2\left(\ell_L\right)\right] d\theta - c\frac{z^2}{2} \geq F, \tag{32}$$

where $\theta_L^*(0,\ell_L)$ represents the run threshold without any risk management, $z^* = 0$. Hence, the two extremes of the integral only differ because of z. Consider the upper bound. The run threshold $\theta_L^*\left(0,\ell_L\right)$ converges to 1 when ℓ_L falls below some positive threshold $\check{\ell}$. Formally, we show that $\exists\, \check{\ell} \in (0, 1-k)$ such that $\theta_L^* \to \overline{\theta} \approx 1$ for $\ell_L \to \check{\ell} > 0$. To see this, it is useful to rewrite $\theta_L^*(0)$ as follows:

$$\theta_L^*(0,\ell_L) = \frac{(1-k)\, r_2}{R} \frac{r_2\left(\ell_L\right) - \frac{\ell_L}{1-k}\left(1 - \ln\left(\frac{\ell_L}{1-k}\right)\right)}{r_2\left(\ell_L\right) - \left(1 - \ln\left(\frac{\ell_L}{1-k}\right)\right)}.$$

First, notice that for θ_L^* to exist the following two conditions are necessary:

$$r_2(\ell_L) - \ell_L/(1-k)\left(1 - \ln\left(\ell_L/(1-k)\right)\right) \;>\; r_2(\ell_L) - \left(1 - \ln\left(\ell_L/(1-k)\right)\right) \tag{33}$$

$$r_2(\ell_L) - \left(1 - \ln\left(\ell_L/(1-k)\right)\right) \;>\; 0, \tag{34}$$

where the second inequality follows from depositor indifference. Both the numerator and the denominator of θ_L^* are monotonically decreasing in ℓ_L. Furthermore, the bank operates only with non-negative profits, which imposes an upper bound on $r_2\left(\ell_L\right)$, i.e. $r_2\left(\ell_L\right) < R/(1-k)$ independent on whether it is determined by (10) or (32).

As $\ell_L \to 0$, the left-hand-side of Inequality (33) goes to $r_2 < R/(1-k)$, while the right-hand side is strictly negative. Conversely, for $\ell_L \to 1-k$, both sides of Inequality (33) go to r_2. Thus, by continuity and monotonicity, $\exists\, \check{\ell} > \check{\check{\ell}}$ such that $\theta_L^*(0,\ell_L) = 1$ for all $\ell_L \in [0, \check{\ell}]$. Next, the cutoff $\widetilde{\ell}_{r_2} \in (0, \check{\ell})$ in the proposition can be obtained denoting $\widehat{F}(\widetilde{\ell}_{r_2}, c)$, as the unique solution to:

$$\widehat{F}(\widetilde{\ell}_{r_2}, c) = p \int_{\theta_L^*(\widetilde{\ell}_{r_2} + \widehat{z}(\widetilde{\ell}_{r_2}, c))}^{\theta_L^*(\widetilde{\ell}_{r_2} < \check{\ell}) = \overline{\theta} \approx 1} [R\theta - (1-k) r_2(\widetilde{\ell}_{r_2})] d\theta - \frac{c(\widehat{z}(\widetilde{\ell}_{r_2}, c))^2}{2},$$

where the RHS increases in $\widetilde{\ell}_{r_2}$, because $dr_2^*/d\ell_L < 0$ from Lemma 4. Following the same argument as in the proof of Proposition 2, we can show that zero risk management is optimal for all $\ell_L < \widetilde{\ell}_{r2}$ and for any $F \geq 0$ and $c > \check{c}$, where $\check{c}$ is defined in an analogous way. This completes the proof. □

A.14 Proof of Proposition 7

We assume that R is high enough to ensure that financial intermediation is profitable and the participation constraint of the banker is always slack, $\Pi \geq 0$. Since the participation constraint of depositors binds, it pins down r_2^* for any level of bank capital k. That is, $V(r_2^*, k) = \rho_D$ for all $k < 1 - \ell$. Multiplying the binding participation constraint by deposits $(1-k)$ and inserting into the bank's expected profits yields the following reduced problem:

$$\max_k \Pi = p\left(\int_0^{\theta_L^*} (\ell_L + z) d\theta + \int_{\theta_L^*}^1 R\theta d\theta\right) + (1-p)\left(\int_0^{\theta_H^*} \ell_H d\theta + \int_{\theta_H^*}^1 R\theta d\theta\right) - (1-k)\rho_D - k\rho_E - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}.$$

Let $z = z^*(k)$ be the optimal future choice of risk management. There are two cases: $z^* = \widehat{z}$ and $z^* = 0$, which we consider in turn. First, consider the interior solution $\widehat{z} > 0$ and invoking the envelope theorem and $d\Pi/dz = 0$, we have the following first-order condition for bank capital:

$$\frac{d\Pi}{dk} = p\left[R\theta_L^* - (\ell_L + z)\right]\left(-\frac{d\theta_L^*}{dk}\right) + (1-p)\left[R\theta_H^* - \ell_H\right]\left(-\frac{d\theta_H^*}{dk}\right) - (\rho_E - \rho_D) = 0, \qquad (35)$$

where the failure thresholds and their derivatives are evaluated at $z = \widehat{z}$. Equation (35) shows the trade-off associated with more bank capital. The first two terms are the endogenous expected marginal benefit of capital in terms of improving bank stability in both states of the world. The last term is the marginal cost of capital because capital is assumed to be a more expensive form of bank funding. Each of the first two terms are strictly positive, while the last term converges to zero as $\rho_E \to \rho_D$. Therefore, it is optimal for the bank to raise a strictly positive amount of bank capital as long as ρ_E is not too much larger than ρ_D.

Next, we consider the case of $z = 0$. Then, the first-order condition is:

$$\frac{d\Pi}{dk} = p\left[R\theta_L^* - \ell_L\right]\left(-\frac{d\theta_L^*}{dk}\right) + (1-p)\left[R\theta_H^* - \ell_H\right]\left(-\frac{d\theta_H^*}{dk}\right) - (\rho_E - \rho_D), \qquad (36)$$

where the failure thresholds and their derivatives are evaluated at $z = 0$. Again, the first two terms are positive, representing the marginal benefit of capital as a non-contingent tool, while the marginal cost converges to zero as $\rho_E \to \rho_D$. Therefore, $k^* > 0$.

As a final step, note that we held $z = 0$ constant as we changed the level of capital in the second case. While the optimal level of z may increase in response to such a change, here we show that this channel works against our desired result. Recall that bank capital and risk management effort are complements at the extensive margin, so $\frac{d\Pi}{dk}$ evaluated at $z = 0$ is a lower bound on $\frac{d\Pi}{dk}$. Therefore, allowing risk management effort to increase in response to a change in k would only strengthen our case, generating a higher level of optimal bank capital. This completes the proof. □

A.15 Proof of Lemma 6

To analyze the alternative model of risk management, we start by deriving the equilibrium run thresholds, focusing on the aspects that differ from the main specification. We start from the bad state, $\ell = \ell_L$. There are now three cases for interim withdrawals. We consider them in turn.

Case 1: $n \leq z/(1-k) \equiv \underline{n}$**.** No liquidation is needed to serve withdrawals because the cash received from the risk management contract suffices. Thus, the bank is liquid at $t = 1$ and stores the remainder, $z - n(1-k)$, until time 2. The bank is solvent at $t = 2$ whenever $R\theta + z - n(1-k) \geq (1-k)(1-n)r_2$. Note that the lower dominance bound is also different relative to the baseline model and now solves $R\theta + z = (1-k)r_2$, so the fundamental threshold changes to:

$$\underline{\theta}_{L,S} \equiv \frac{(1-k)r_2 - z}{R},$$

where S stands for the modelling of risk management with swaps. We have the following ranking: $\underline{\theta}_{L,S} < \underline{\theta}$. Since $\theta \geq \underline{\theta}_{L,S}$ holds when establishing the bank failure threshold, the bank is always solvent at $t = 2$. To see this, $R\underline{\theta}_{L,S} + z - n(1-k) \geq (1-k)(1-n)r_2 \Leftrightarrow r_2 \geq 1$, which always holds.

Case 2: $z/(1-k) < n \leq \overline{n}_{L,S} \equiv (\ell_L + z)/(1-k)$**.** For intermediate levels of withdrawals, the bank is liquid at $t = 1$ and can meet all withdrawals, so depositors who withdraw early are repaid in full. To ensure this, some liquidation is required, namely the fraction $(1-k)(n - \underline{n}_{L,S})/\ell_L$ of investment. Thus, the bank is solvent at $t = 2$ if $R\theta\left[1 - \frac{(1-k)n - z}{\ell_L}\right] \geq (1-k)(1-n)r_2$, so $\widehat{n}_{L,S} \equiv \frac{R\theta\left(1+\frac{z}{\ell_L}\right) - (1-k)r_2}{R\theta\left(\frac{1-k}{\ell_L}\right) - (1-k)r_2}$. We focus on parameters such that z^* is low enough in order to ensure that $\widehat{n}_{L,S} \leq 1$.

Case 3: $\overline{n}_{L,S} < n$**.** Full liquidation occurs at $t = 1$ and withdrawing depositors receive a pro-rata share of liquidation proceeds. Depositors who wait until $t = 2$ receive nothing. For vanishing private noise, the usual Laplacian property holds. Thus, a marginal depositor's expected payoff from withdrawing early is $\alpha_{L,S} = \int_0^{\bar{n}_{L,S}} dn + \int_{\bar{n}_{L,S}}^1 \frac{\ell_L+z}{(1-k)n} dn$. As in the baseline model, risk management effort z increases depositors' expected payoff from withdrawing. The effect is twofold. First, it increases the pro-rata share upon bank failure at $t = 1$. Second, it makes bank failure at $t = 1$ less likely. Thus, $\partial \alpha_{L,S}/\partial z > 0$. The equilibrium failure threshold again solves $\int_0^{\widehat{n}_{L,S}} r_2 dn = \alpha_{L,S}$, so:

$$\theta^*_{L,S} = \frac{(1-k)r_2}{R} \frac{r_2 - \alpha_{L,S}}{\left(1 + \frac{z}{\ell_L}\right) r_2 - \frac{\alpha_{L,S}(1-k)}{\ell_L}}. \tag{37}$$

We turn to good state, $\ell = \ell_H$. The bank is illiquid at $t = 1$ if withdrawals and swap payment exhaust the interim value of investment: $(1-k)n + z = \ell_H$. Thus, $\bar{n}_{H,S} = \frac{\ell_H - z}{1-k} < 1$ because $\ell_H < 1-k$. Intuitively, a higher interest rate swap payment reduces the liquidity available to depositors at the interim date, $\frac{d\bar{n}_{H,S}}{dz} = -\frac{1}{1-k} < 0$. For $n \leq \bar{n}_{H,S}$, the bank continues until date 2 and fails due to insolvency if $n > \widehat{n}_{H,S} \equiv \frac{R\theta\left(1-\frac{z}{\ell_H}\right) - (1-k)r_2}{\frac{R\theta(1-k)}{\ell_H} - (1-k)r_2}$, where $\widehat{n}_{H,S}$ solves the insolvency condition, $R\theta\left[1 - \frac{z+(1-k)n}{\ell_H}\right] - (1-k)(1-n)r_2 = 0$. Moreover, $\frac{d\widehat{n}_{H,S}}{dz} < 0$. The lower dominance bound is:

$$\underline{\theta}_{H,S} = \frac{(1-k)r_2}{R\left(1 - \frac{z}{\ell_H}\right)} = \frac{r_2}{R} \frac{\ell_H}{\bar{n}_{H,S}}. \tag{38}$$

We can define $\alpha_{H,S} \equiv \frac{\ell_H - z}{1-k}\left[1 - \ln\left(\frac{\ell_H - z}{1-k}\right)\right] = \bar{n}_{H,S}\left[1 - \ln\left(\bar{n}_{H,S}\right)\right]$, so $\frac{d\alpha_{H,S}}{d\bar{n}_{H,S}} = -\ln(\bar{n}_{H,S}) > 0$. For future reference: $\frac{d\alpha_{H,S}}{dz} = \frac{\partial \alpha_{H,S}}{\partial z} = \frac{1}{1-k}\ln\left(\frac{\ell_H - z}{1-k}\right) = \frac{1}{1-k}\ln\left(\bar{n}_{H,S}\right) < 0$ from the chain rule, where the first inequality arises from $\frac{\partial \alpha_{H,S}}{\partial \bar{n}_S} = 0$. The indifference condition $r_2 \widehat{n}_{H,S} \equiv \alpha_{H,S}$ yields the failure threshold:

$$\theta^*_{H,S} \equiv \frac{r_2(1-k)}{R} \frac{r_2 - \alpha_{H,S}}{r_2\left(1 - \frac{z}{\ell_H}\right) - (1-k)\frac{\alpha_{H,S}}{\ell_H}} = \underline{\theta}_{H,S} \frac{r_2 - \alpha_{H,S}}{r_2 - \alpha_{H,S}\frac{1-k}{\ell_H - z}} > \underline{\theta}_{H,S}, \tag{39}$$

where the existence of panic runs arises from $1-k > \ell_H - z$. As it is useful later for the derivatives, we re-express the failure threshold in terms of $\bar{n}_S$ and α_S only: $\theta^*_{H,S} \equiv \frac{r_2}{R}\ell_H \frac{r_2 - \alpha_{H,S}}{r_2 \bar{n}_{H,S} - \alpha_{H,S}}$.

Next, we proceed to sign the effect of higher risk management on bank fragility. Let $x_{L,S} \equiv \frac{\ell_L + z}{1-k}$. Then, we can again express the expected payoff from withdrawing early compactly as $\alpha_{L,S} \equiv x_{L,S}(1-\ln(x_{L,S}))$. Using this expression for $\alpha_{L,S}$, we can express the failure threshold as:

$$\theta^*_{L,S} = \underline{\theta}\frac{\ell_L}{\ell_L+z}\frac{r_2 - x_{L,S}(1-\ln(x_{L,S}))}{r_2-(1-\ln(x_{L,S}))}.$$

It is immediate that $d\theta^*_{L,S}/dz < 0$. The first factor is independent of risk management, the second factor decreases in risk management, and the third factor has the same mathematical structure as before, so $d\theta^*_{L,S}/dx_{L,S} < 0$ and $dx_{L,S}/dz > 0$ (see also Appendix A.1).

Consider now state H. Differentiating (39) with respect to z, we obtain: $\frac{d\theta^*_{z,H}}{dz} = \frac{\theta^*_{z,H}}{\ell_H - z} - \frac{\theta^*_{z,H}\ln\left(\frac{\ell_H-z}{1-k}\right)}{(1-k)(r_2-\alpha_{z,H})} + \frac{\theta^*_{z,H}}{\ell_H-z}\frac{1}{r_2-\alpha_{z,H}\frac{1-k}{\ell_H-z}} > 0$, where we used $\frac{d\alpha_{z,H}}{dz}\frac{1-k}{\ell_H-z} + \alpha_{z,H}\frac{1-k}{(\ell_H-z)^2} = \frac{1}{\ell_H-z}$. This completes the proof. □

A.16 Proof of Proposition 8

The bank manages risk when it improves expected profits upon a negative shock. The differential profit condition is equivalent to that in the proof of Proposition A.4:

$$\frac{1}{2}\int_{\theta^*_{L,S}(z)}^{\theta^*_{L,S}(0)}[R\theta-(1-k)r_2]d\theta + \frac{1}{2}\int_{\theta^*_{L,S}(z)}^{1} z d\theta - \frac{1}{2}\int_{\theta^*_{H,S}(0)}^{\theta^*_{H,S}(z)}[R\theta-(1-k)r_2]d\theta - \frac{1}{2}\int_{\theta^*_{H,S}(z)}^{1}\frac{z}{\ell_H}d\theta \geq F. \quad (40)$$

First, note that because $\theta^*_{L,S}(z,\ell_L) = \frac{\ell_L}{\ell_L+z}\theta^*_L(z,\ell_L)$, a sufficient condition for $\lim_{\ell_L\to 0}\theta^*_{L,S}(z,\ell_L) = \overline{\theta}$ is given by $z \leq \overline{\ell}_S \equiv (1-k)e^{1-r_2}$. Next, define $\bar{z}_S$ as the solution to $\ell_L + \bar{z}_S = (\ell_H - \bar{z}_S)/\ell_H$ and note that:

$$\theta^*_{H,S}(\bar{z}_S,\ell_H) = \underline{\theta}\frac{1}{\ell_L+z}\frac{r_2-\alpha_{H,S}(\bar{z}_S)}{r_2-\frac{1-k}{\ell_H-z}\alpha_{H,S}(\bar{z}_S)} > \theta^*_{L,S}(\bar{z}_S,\ell_L) = \underline{\theta}\frac{\ell_L}{\ell_L+z}\frac{r_2-\alpha_{L,S}(\bar{z}_S)}{r_2-\frac{1-k}{\ell_L+z}\alpha_{L,S}(\bar{z}_S)}, \quad (41)$$

where $\alpha_{H,S}(\bar{z}_S) > \alpha_{L,S}(\bar{z}_S)$. Therefore, for $\ell_L \to 0$, a sufficient condition for $z^* = 0$ is given by $\theta^*_{S,H}(\bar{z}_S,\ell_H) \geq \overline{\theta}$ when $z = \bar{z}_S$. This is guaranteed by ℓ_H being small enough. Using the expression above, we can formally derive an upper bound for ℓ_H that is $\ell_H < \frac{(1-k)e^{1-r_2}}{1-(1-k)e^{1-r_2}}$. Next, the left-hand side of Inequality (40) is continuous in ℓ_L and negative under the sufficient condition for $\ell_L \to 0$, when the run threshold is at its upper bound. Furthermore, the first derivative of the left-hand side of Inequality (40) with respect to z is strictly positive when evaluated at $\ell_L \to \ell_H$. By continuity, there exists some $\widetilde{\ell}_{L,S} > 0$, which we define as the smallest solution that satisfies (40) with equality, such that the bank chooses not do risk management when the interim asset value falls below the cutoff $\widetilde{\ell}_{L,S}$. Note that such a value exists even for $F = 0$. This completes the proof. □

A.17 Proof of Proposition 9

Differentiating the second integral in Equation (11) with respect to z, we obtain the marginal costs of risk management, which can be expressed as:

$$C'(z) \equiv (1-p)\left[R\theta^*_{H,S}\left(1-\frac{z}{\ell_H}\right)-(1-k)r_2\right]\frac{d\theta^*_{H,S}}{dz}+\frac{1-p}{\ell_H}\int_{\theta^*_{H,S}}^{1} R\theta d\theta > 0,$$

because the marginal profit at the failure threshold is positive (due to panic runs). It follows from the expression above that the cost of risk management is convex if:

$$C''(z) \equiv (1-p)R\left(\frac{d\theta^*_{H,S}}{dz}\right)^2\left(1-\frac{z}{\ell_H}\right)+(1-p)[R\theta^*_{H,S}(z)-(1-k)r_2]\frac{d^2\theta^*_{H,S}}{dz^2}-2\frac{1-p}{\ell_H}R\theta^*_{H,S}\frac{d\theta^*_{H,S}}{dz} > 0.$$

Note that $\frac{d^2\theta^*_{H,S}}{dz^2} \geq 0$ and $\frac{\ell_H - z}{2}\frac{d\theta^*_{H,S}}{dz} \geq \theta^*_{H,S}$ are sufficient for convexity (where the second sufficient condition arises from combining the first and third term of the second derivative). We can then express the derivative of $\theta^*_{H,S}$ obtained in the proof of Proposition 6 as follows:

$$\frac{d\theta^*_{z,H}}{dz} = \theta^*_{z,H}\left[\frac{1}{\ell_H - z}-\frac{\ln\left(\frac{\ell_H - z}{1-k}\right)}{(1-k)(r_2-\alpha_{z,H})}+\frac{1}{\ell_H - z}\frac{1}{r_2-\alpha_{z,H}\frac{1-k}{\ell_H - z}}\right] \tag{42}$$

Denote the term in the square bracket as Φ. Then, we can write the second derivative as follows $\frac{d^2\theta^*_{z,H}}{dz^2} = \frac{d\theta^*_{z,H}}{dz}\Phi + \theta^*_{z,H}\frac{d\Phi}{dz}$. Hence, it follows immediately that $\frac{d^2\theta^*_{z,H}}{dz^2} > 0$ if $\frac{d\Phi}{dz} \geq 0$. Taking out the term $\frac{1}{1-k}$ from the expression for Φ, and substituting for $\overline{n}_S$, we can simplify this term as follows:

$$\Phi(\overline{n}_S) \equiv \frac{1}{1-k}\left[\frac{1}{\overline{n}_S}-\frac{\ln(\overline{n}_S)}{r_2-\alpha_S}+\frac{1}{r_2\overline{n}_S-\alpha_S}\right],$$

so $\frac{d\Phi}{dz} \geq 0$ whenever $\frac{d\Phi}{d\overline{n}_S} \leq 0$ (because $\frac{d\overline{n}_S}{dz} = -\frac{1}{1-k} < 0$). Computing this derivative, we have:

$$\frac{d\Phi}{d\overline{n}_S} = \frac{1}{1-k}\left[-\frac{1}{\overline{n}_S^2}-\frac{1}{\overline{n}_S(r_2-\alpha_S)}-\frac{\ln(\overline{n}_S)}{(r_2-\alpha_S)^2}\frac{d\alpha_S}{d\overline{n}_S}-\frac{1}{(r_2\overline{n}_S-\alpha_S)^2}\left(r_2-\frac{d\alpha_S}{d\overline{n}_S}\right)\right] < 0.$$

This (desired) sign arises for two reasons (and under a sufficient condition). First, consider the fourth term and note that $r_2 - \frac{d\alpha_S}{d\overline{n}_S} > 0 \Leftrightarrow r_2 > -\ln(\overline{n}_S)$ because $r_2 > 1-\ln(\overline{n}_S)$ from the definition of the failure threshold (a positive denominator). Thus, the fourth term has the desired sign. Second, the third term has the opposing sign. Combining the first and the third term, a sufficient condition for $\frac{d\Phi}{d\overline{n}_S} < 0$ is $-\frac{1}{\overline{n}_S^2}-\frac{\ln(\overline{n}_S)}{(r_2-\alpha_S)^2}\frac{d\alpha_S}{d\overline{n}_S} \leq 0 \Leftrightarrow \overline{n}_S^2\ln(\overline{n}_S)^2 \leq (r_2-\alpha_S)^2$.

A sufficient condition for the latter inequality is $r_2 - \alpha_S \geq -n_S \ln(\overline{n}_S)$. Since $r_2 > 1 - \ln(\overline{n}_S)$ and $-n_S \ln(\overline{n}_S) < n_S(1 - \ln(\overline{n}_S))$, a sufficient condition for the desired inequality is:

$$1 - \ln(\overline{n}_S) - \overline{n}_S(1 - \ln(\overline{n}_S)) \geq \overline{n}_S(1 - \ln(\overline{n}_S)) \Leftrightarrow 1 - \overline{n}_S \geq \overline{n}_S.$$

Since $z \geq 0$, a sufficient condition is $\ell_H \leq (1-k)/2$. This is quite restrictive, but our numerical example in the main text shows that the desired result holds more broadly. Taken together, under this condition, the failure threshold is convex in risk management effort. Using Equation (42), we can rewrite the second sufficient condition as $\frac{r_2 - \alpha_{z,H}}{r_2 - \frac{\alpha_{z,H}}{\overline{n}_S}} \geq \overline{n}_S \ln(\overline{n}_S)$, which always holds because the LHS is proportional to $\theta^*_{H,S}$ and thus positive, while the RHS is negative (because $\overline{n}_S < 1$). The proposition follows. □

A.18 Derivations of CRM model and Proof of Proposition 10

In the model with credit risk management (CRM) and exogenous capital and bank deposits, the bank failure threshold is:

$$\theta^*_{CRM} = \frac{(1-k)r_2}{R} \frac{r_2 - \alpha}{r_2 - \frac{\alpha(1-k)}{\ell}}, \tag{43}$$

where $R \in \{R_L + z, R_H\}$. Accordingly, we have $\frac{\partial \theta^*_{CRM}}{\partial R_L} = \frac{\partial \theta^*_{CRM}}{\partial z} = -\frac{\theta^*_{CRM}}{R_L + z} < 0$ and $\frac{\partial^2 \theta^*_{CRM}}{\partial R_L^2} = \frac{\partial^2 \theta^*_{CRM}}{\partial z^2} = \frac{\partial^2 \theta^*_{CRM}}{\partial R_L \partial z} = -\frac{\frac{\partial \theta^*_{CRM}}{\partial R_L}(R_L+z) - \theta^*_{CRM}}{(R_L+z)^2} = 2\frac{\theta^*_{CRM}}{(R_L+z)^2} > 0$. Optimal CRM, z^*_{CRM}, maximizes:

$$\Pi_{CRM}(z) \equiv p \int_{\theta^*_{L,CRM}}^{1} [(R_L + z)\theta - (1-k)r_2] d\theta + (1-p) \int_{\theta^*_{H,CRM}}^{1} [R_H \theta - (1-k)r_2] d\theta - c\frac{z^2}{2} - F\mathbb{1}_{\{z>0\}}, \tag{44}$$

which yields $\widehat{z}_{CRM}$ as the solution to:

$$I \equiv \frac{d\Pi_{CRM}}{dz} = -p\frac{\partial \theta^*_{L,CRM}}{\partial z}[(R_L + z)\theta^*_{L,CRM} - (1-k)r_2] + p\int_{\theta^*_{L,CRM}}^{1} \theta d\theta - cz = 0, \tag{45}$$

where a high enough c, which we maintain henceforth, ensures a unique solution $\widehat{z}_{CRM}$ because the second-order condition is $\frac{d^2\Pi_{CRM}}{dz^2} = p\left(1 + \frac{1}{R}\right)\frac{\partial \theta^*_{L,CRM}}{\partial z}[(R_L + z)\theta^*_{L,CRM} - (1-k)r_2] - 2p\frac{\partial \theta^*_{L,CRM}}{\partial z}\theta^*_{L,CRM} - c$ is negative for a high enough variable cost parameter c.

Turning to zero risk management on the extensive margin, the proof parallels that of Lemma 2 and Proposition 2 and is skipped for brevity. The important sufficient condition is again a high enough variable cost parameter c, where $\widehat{c}_{CRM}$ is the analog cutoff value.

In addition to the (indirect) effect of R_L on bank fragility, which is same as in Proposition 2, there is also a direct effect present in the model with CRM: a lower value of R_L enters the expected bank profit Π_{CRM} directly, so the objective function is pushed down even further as R_L decreases, making the desired result easier to establish. Thus, our previous conditions for the case of liquidity risk management are sufficient to establish the result for CRM. This completes the proof. □

A.19 Proof of Proposition 11

To see the effect of commitment on deposit rates, consider the participation constraint of investors, given in Condition (10). Using the implicit function theorem, higher risk management reduces the deposit rate, i.e. $\frac{dr_2^*}{dz} < 0$. To see this, note that a greater amount of risk management and a higher deposit rate increase the value of the deposit claim V:

$$\begin{aligned}
\frac{\partial V}{dz} &= p_\ell \left\{ \int_0^{\theta_L^*} \frac{1}{1-k} d\theta + \left[r_2 - \frac{\ell_L + z}{1-k} \right] \left(-\frac{d\theta_L^*}{dz} \right) \right\} > 0 \\
\frac{\partial V}{dr_2} &= p_\ell \int_{\theta_L^*}^1 d\theta + (1-p_\ell) \int_{\theta_H^*}^1 d\theta + \left[r_2 - \frac{\ell_L + z}{1-k} \right] \left(-\frac{d\theta_L^*}{dr_2} \right) + \left[r_2 - \frac{\ell_H}{1-k} \right] \left(-\frac{d\theta_H^*}{dr_2} \right),
\end{aligned}$$

because risk management reduces bank fragility upon a shock, $d\theta_L^*/dz < 0$. A higher deposit rate directly increases the value of the deposit claim and has an indirect effect via bank fragility. A sufficient condition for the value of the claim to increase in deposit rates, $\partial V/dr_2 > 0$, is that a higher deposit rate reduces fragility, which arises for a low equilibrium deposit rate r_2^* (Lemma 1). For example, a high enough return on investment R suffices for this to arise. Similarly, a higher deposit rate directly reduces bank profits and has an indirect effect via fragility:

$$\frac{\partial \Pi}{\partial r_2} = -(1-k) \left[p_\ell (1-\theta_L^*) + (1-p_\ell)(1-\theta_H^*) \right] - p_\ell \left[R\theta_L^* - (1-k) r_2 \right] \frac{\partial \theta_L^*}{\partial r_2} - (1-p_\ell) \left[R\theta_H^* - (1-k) r_2 \right] \frac{\partial \theta_H^*}{\partial r_2}.$$

If the value of the deposit claim increases in the deposit rate, then $\partial \Pi/\partial r_2 < 0$ must hold in equilibrium. The intuition is as follows. If the participation constraint were to bind for a deposit rate at which marginal profits still increase in the deposit rate, the bank would voluntarily pay higher deposit rates (in order to benefit from the beneficial effect via lower fragility). Then, the participation constraint would be slack and the equilibrium deposit rate pinned down by zero marginal profits. Since we focus on parameters for which the participation constraint binds in equilibrium, it must be that higher deposit rates reduce expected bank profits, as was to be shown.

Equipped with these results, we can turn to the planner's risk management choice. We maintain the assumption that the bank and the planner are at the participation constraint. Denote as $\hat{z}_P$ the equivalent of $\hat{z}$ for the planner, and similarly for $r_{2,P}$. The first-order condition that pins down $\hat{z}_P$ is:

$$\frac{d\Pi}{dz} \equiv \frac{\partial \Pi}{\partial z} + \frac{\partial \Pi}{\partial r_{2,P}} \frac{dr_{2,P}}{dz}, \tag{46}$$

where $\frac{dr_{2,P}}{dz}$ comes from the binding participation constraint and $\frac{\partial \Pi}{\partial z} = 0$ pins down $\hat{z}$ for the bank. Since $\frac{\partial \Pi}{\partial r_{2,P}} \frac{dr_{2,P}}{dz} > 0$, we must have $\frac{\partial \Pi}{\partial z}|_{z=\hat{z}_P} < 0$. By the concavity of Π in z from the bank's problem, it immediately follows that $\hat{z}_P > \hat{z}$. In words, when abstracting from the fixed cost, there exists a solution to the planner's problem with a higher risk management than the bank because the planner internalizes its benefit for reducing deposit rates.

Turning to risk management failures, we see that the planner's commitment to future risk management also has implications for whether the bank engages in any risk management in the first place. The result is straightforward from the following considerations. The planner maximizes welfare $SW(z)$, while the bank maximizes $\Pi(z)$. Therefore, it is immediate that $SW(\hat{z}_p) \geq SW(\hat{z})$, with the inequality strict whenever $\hat{z}_p \neq \hat{z}$. Now, $\Pi(z) = SW(z) - V(z)$. Since the participation constraint binds throughout, we have that $V(\hat{z}_p) = V(\hat{z})$, which then implies that $\Pi(\hat{z}_p) \geq \Pi(\hat{z})$, with the inequality again strict whenever $\hat{z}_p \neq \hat{z}$. This implies that for any given ℓ_L, if the differential in bank profits between hedging and not is exactly equal to zero when $F = \hat{F}$ and the bank chooses $z = \hat{z}$, they must be strictly positive when the planner chooses $z = \hat{z}_p$. Hence, $\hat{F}_P > \hat{F}$. $\square$